Design Optimization of a Hybrid Energy System Through Fast Convex Programming

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***Abstract*—In this paper, a methodology for the optimal economic design of a hybrid energy system is presented. The novelty of the proposed methodology lies in the fact that a convex optimisation approach is used - allowing a solution to be found very efficiently compared to existing techniques. First, a simulation model for a grid-connected PV/wind/diesel hybrid system is implemented in Mathworks Simulink (TM). Using this simulation model, it is shown that the design problem under consideration is indeed convex. An interior-point convex solver is then used to design a sample PV/wind/diesel system for a small university campus and the results compare favourably to those obtained using the closed-source HOMER software package which is based on slow discrete combinatorial optimization.**

Keywords – convex optimization; hybrid system; renewable energy; PV; wind

I. INTRODUCTION

The problem of optimal hybrid energy system sizing has been well studied in the literature.

Seeling-Hochsmuth addresses the problem of designing an optimal hybrid energy system for a rural environment at length in his PhD thesis [1]. Like most of the other studies on optimal system design, he chooses a genetic algorithm to perform the system optimisation. The main motivation given for this choice is the derivative-free nature of the algorithm. This allows for the use of an accurate and complex simulation of the underlying system leaving the optimisation algorithm to cope with a non-smooth objective function. Even so, although the simulation models used are adequately complex to capture the demands of the application, they capture the operation of the system only at the single-phase, steady state active power level ie. three-phase sources, active, reactive and apparent power are not accounted for. A simple, heuristic control strategy is used to dispatch the battery and diesel generator.

A number of other authors have also used genetic algorithms. For example, [2] investigates the impact of a battery energy storage and high RES penetration (wind and PV) on an islanded microgrid, while [3] focusses on the implementation of distributed medium and small scale storage in a smart grid - both authors using genetic algorithms for optimal sizing of components.

Hong et al. present an interesting method that uses a markov-chain based probabilistic model instead of a timedomain simulation in an effort to reduce computation time [4]. They apply this method to a wind-PV-diesel system - also using a genetic algorithm for optimisation.

Ter-Gazarian et al. address the problem of optimising the design of a distribution system [5]. They include conventional and renewable sources as well as storage elements. Each type of component is modelled using a generic form which allows for a linear programming formulation of the design problem.

A similar method is followed by Atwa et al., however, they use a non-linear formulation of the design problem which they solve using mixed integer nonlinear programming [6].

There are also a number of commercial software packages that address the problem of optimal hybrid energy system design. One such package is HOMER.

A. HOMER Energy

HOMER is a software package designed at the National Renewable Energy Laboratory (NREL) to facilitate the design of hybrid renewable energy systems [7]. The package allows the user to simulate and optimise the sizing of various types of components and includes tools to perform sensitivity analysis to determine the effect of changing capital equipment and running costs and how these affect the optimal system design [7].

HOMER's simulation model is based on a time-domain simulation run at the energy-flow level with discrete timesteps of 1 hour. HOMER uses a discrete combinatorial optimisation approach to determine the optimal system configuration. The main disadvantages of this package include:

- Combinatorial optimisation is very slow for a moderately large number of configurations
- Peak demand charge is not considered

- The software is closed-source and simulation models of components cannot be altered

However, the function of the HOMER package is closely related to the problem considered in this paper and will thus be used as a baseline to verify the results obtained using the proposed method.

II. Convex Optimisation

The convex optimisation problem considered in this paper is a specific form of the more general non-linear optimisation problems. Non-linear optimisation problems take the form [8]

$$\text{minimize } f(x) \qquad (1) \text{ subject to } g_i(x) \leq 0$$

Such an optimisation problem is convex when the objective function, $f(x)$, and constraints, $g_i(x)$ are all convex ie. they satisfy the constraint

$$f(ax + by) \leq af(x) + bf(y)$$

Convex optimisation problems can be further divied into a number of subclasses. These include:

- Linear programming: where $f(x)$ and $g_i(x)$ are affine ie. they take the form $a_ix + c_i$
- Quadratic Programming: where $f(x)$ and $g_i(x)$ are convex and quadratic $x^TQx + r_i^Tx + b_i$

Convex optimisation problems can be solved more efficiently than general non-linear problems which do not have the convexity property. Thus, in many cases where the objective function is convex or approximately convex, it is advantageous to approximate these non-convex problems as a suitable convex problem.

This allows an approximate solution to the original problem to be obtained in very little time with high accuracy (depending on the nature of the approximation) as convex optimisation problems can be solved using one of a wide range of fast and efficient algorithms. These include interior-point methods, bundle methods or subgradient projection methods. These algorithms have a number of advantages over nonconvex optimisation algorithms which are usually based on heuristics such as genetic algorithms, simulated annealing and sequential quadratic programming. These advantages include:

- With convex algorithms convergence is guaranteed
- The solution can be found in a finite (polynomial) time
- The solution is always globally optimal

Therefore, the use of convex optimisation procedures is highly desirable if the problem can be reformulated as a convex problem or even approximately be shown to be of a convex nature.

Convexity can be proved using the following proposition.

Proposition 1. *[8] If f is a twice-differentiable function on* $[a,b]$*, then f is convex if*

• f^0 is non-decreasing on the interval $[a,b]$ *• f^{00} is non-negative on the interval* $[a,b]$

A number of operations exist that preserve convexity. These operations allow the construction of new functions which are themselves convex. One such operation which will be used in this paper is the non-negative weighted sum of convex functions [8].

Proposition 2. *If the functions $f_i, i = 1...n$ are convex then [8]*

$$f(x) = w_1f_1(x) + \cdots + w_nf_n(x) \text{ is also}$$

convex where each $w_i \geq 0$.

A. Data Synthesis

In the most of the literature on optimal sizing of hybrid energy systems, it is often assumed that the necessary data has been recorded and is readily available to be used as inputs for the optimisation procedure. The issue of data availability is therefore often overlooked. In most practical cases, the data sets are not complete and significant portions may be missing due to false readings, communication issues or meter downtime.

To recover this data, simple interpolation or estimation schemes are often used which include mean substitution, regression substition and analysis of variance (ANOVA) techniques. Another popular and fairly accurate method for the synthesis of both load and resource data involves the use of a stationary statistical model fitted using maximum likelihood methods to capture the distribution of data values combined with a auto-regressive model to capture the time dependence or temporal structure of the data. These techniques, however, only capture the characteristics of the underlying dataset in a very simplistic manner. This can significantly impact on the accuracy of the simulation where detailed features, such as daily peak demand values contribute to the objective function.

In this paper, missing elements in the data are recovered using the Support Vector Regression technique presented in [9] as this method can provide near state-of-the-art prediction of missing values.

III. Main Contributions

The first contribution presented in this paper is the development of a phasor time-domain hybrid simulation model designed to allow extremely long duration simulations (year length data) while still providing detailed information of the system including:

1) Three-phase voltage and currents
2) Reactive, real and apparent power
3) Implementation a custom cost tariff structure that depends on high resolution usage data (such as the 30 min peak demand charge used in South Africa) as well as long-term data ie. monthly active power totals

The main contribution, however, is the postulation and motivation for a convex approach to the optimal design of a

PV/wind/diesel system. It is shown that the problem is indeed convex and thus its optimisation carried out using a standard, fast convex optimisation algorithm.

IV. ELECTRICAL MODEL

The system simulation is implemented in the Matlab / Simulink environment. The system components are mostly implemented as Matlab functions interfaced to the SimPowerSystems environment. The model uses a mixed-mode simulation consisting of both phasor, $f(j\omega)$, and time domain, $f(t)$, signals.

Simulink - time domain signals SimPowerSystems - phasor domain, by mixing both environments computational complexity and simulation times are reduced.

A. PV Model

Maximum power point tracking is implemented using a simple root finding technique as presented in [10]. First an analytical expression for the derivative of the produced power with respect to the output voltage is determined:

$$\frac{dP(V,T)}{dV} = I_{PV} - I_0\left(e^{\frac{qV}{N_s kT}} - 1\right) - \frac{I_0 e^{\frac{qV}{N_s kT}}}{\left(\frac{q}{N_s kT}\right)V}$$

The maximum power point is found by equating this derivative to zero. This is done by finding the roots of the resulting nonlinear algebraic equation.

$$\frac{dP(V^\star,T)}{dV} = 0$$

V. GRID / INFINITE BUS

The grid is modelled as an AC voltage source at a constant frequency. As the grid impedance losses are negligible compared to those of the other elements of the system, No limitations are placed on the power that can be drawn or supplied to the grid.

VI. WIND TURBINE

A standard mathematical model of a VAWT is used which is identical to that implemented in the *SimPower Systems* package [11] with a significant modification - the dynamical induction generator has been removed to allow the simulation to run with a large time

VII. DIESEL GENERATOR

The model published in [12] is used to model the diesel generator. It is based on a simple non-linear fuel consumption model:

$$\text{Fuel} = f_0 + f_1 P + f_2 P^2$$

where $f_0 \ldots f_2$ are the fuel-consumption coefficients and P is the power being supplied by the generator. It should be noted that as the electrical power is used as an input, the coefficients account for the efficiency of the generator.

The diesel generator is scheduled to run when the cost of the fuel consumed is less than the cost of purchasing energy from the grid. In this case, the generator supplies all the net energy. The power output of the generator is therefore determined as follows:

IF (Fuel Per Hour $*$ Fuel Price) < Grid Price (2)

$P = P_{net}$

ELSE

$P = 0$

VIII. ECONOMIC MODEL

The economic model forms the basis of the optimisation procedure. Every element of the system has an associated capital cost which is a once-off cost for provisioning the equipment. This cost is assumed to be linearly dependent on the capacity of the unit which is modelled by a separate coefficient for each device. These coefficients are:

- C_w - Capacity of the WECS (kW)
- C_{pv} - Capacity of the photovoltaic array (kW)
- C_g - Capacity of the diesel generator (kW)
- p_w - Cost of the wind WECS (per kW)
- p_{pv} - Cost of the photovoltaic array (per kW)
- p_g - Cost of the diesel generator (per kW)

The capital cost is then calulated as

$$\text{CapitalCost} = p_w C_w + p_{pv} C_{pv} + p_g C_g + p_s C_s \tag{3}$$

Each unit also has an associated operational cost. The units are divided into two categories: free sources and non-free sources. The free sources include the PV units and the WECS. Although, in reality these elements have maintenance costs, they are assumed negligible in this paper.

$$\text{RunningCost} = \text{kwhCharge} \times \text{totalKWh} + \text{peakCharge} \times \text{totalPeak} + \text{dieselCost} \times \text{totalDiesel} \tag{4}$$

IX. DESIGN OPTIMISATION PROBLEM

The sizing of components is essentially a non-linear, discrete optimisation problem that seeks to find the component sizes that give the lowest total cost which is a combination of Equation 3 and 4. For example, the prospective PV arrays might consist of a number of 1kW panels strung together, in either series or parallel, to achieve the desired output capacity. A naive solution to this problem is to enumerate all possible combinations of components and evaluate the objective function for each combination. However, this becomes computationally intractable for even a small number of possible configurations.

However, in this paper, focus is placed on obtaining a fast solution and as such two approximations are made:

1) A continuous solution to the optimisation is found

2) A convex approximation to the objective function is used

The motivation for using a convex approximation is two-fold

1) The capital cost increases monotonically with an increase in PV, wind and diesel capacity and is thus a convex function
2) The running costs (kWh charge) decrease monotonically with an increase in renewable capacity and is thus convex
3) The peak charge decreases monotonically with an increase in storage and renewable capacity making it convex

The total cost, which is a strictly positively-weighted sum of these functions is therefore also convex according to Proposition 2.

X. System Overview

An overview of the entire system is shown in Figure X which shows how the simulation model, the optimisation algorithm and the peak shaving algorithm are used. The base of the optimisation is the simulink simulation model. The output of this model is used as the input to the optimal peak shaving algorithm and the output of the peak shaving algorithm is used as the objective function for the optimisation.

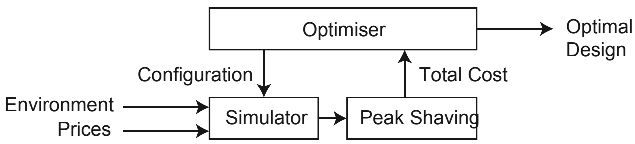

Figure 2. An overview of the optimisation system.

XI. Testing Conditions

In order to test the simulation model, the prices shown in Table XI have been used. These prices are realistic at the time of publication but are meant to demonstrate the main results, not to provide an accurate sizing for a particular application. Insolation and wind data for the University of the Witwatersrand, Johannesburg, South Africa have been used.

TABLE I
TABLE SHOWING THE PRICES FOR VARIOUS EXPENSES USED DURING TESTING

Cost	Price	Unit
PV unit	R51077	per kW
Wind Turbine	R19885	per kW
Diesel Generator	R5735	per kW
Fuel (diesel)	R12.48	per liter
kWh (Utility)	R0.78	per kWh
Peak Demand (Utility)	R11.2	per kVA

A summary of the aspects of the testing conditions are:

- Mean wind speed: 9 *m/s*
- Weibull factor: 1.5
- Mean solar insolation: 6 $kWh/m^2/d$
- Mean load: 1.925 MVar
- Peak load: 3.09 MVar

The convex optimisation problem is solved using the sequential quadratic programming (SQP) solver of the fmincon function in MATLAB. SQP is an algorithm for non-linear optimisation problems that uses convex quadratic approximations to the objective function to converge towards a solution.

At each iteration, a quadratic approximation is found at the current position and solved using simple quadratic programming. This values is then used to update the decision variables. Although SQP is not strictly for convex problems, it converges extremely fast for such problems due to the nature of the approximations used at each step. As such, SQP was used as the optimisation method of choice in this paper.

XII. Results

TABLE II
TABLE COMPARING THE OPTIMISATION MODEL PRESENTED IN THIS PAPER TO HOMER

Feature	HOMER	This paper
Optimisation	Combinatorial	SQP
Costs	No peak charge	Peak charge
Storage dispatch	Simple [1]	Peak shaving
Missing data	Mean sub	SVR

Figure XII shows the result of running the simulation of a test configuration using a wind and PV capacity of 100 kW, a 1 MW diesel generator set and a storage element with 4 MWh of capacity. The simulation is run using a sampling time of 30 min over a 1 yr timeframe and the quantities extended to represent a 20 yr lifetime. The total simulation time is 16.25s.

From these results it can be seen that in Johannesburg, South Africa, a near maximum peak PV output is achieved, while the wind power output is significantly less than the peak.

Total P: 165549.834 kWh
Peak: 481.5521 kVA
Diesel: 0 litres
Peak PV: 95.0692 kWp
Peak Wind: 39.8006 kWp

Capital Cost: R 7096200 Peak Charge:
R 5393.3833 kWh Charge: R
20492000.0

Figure 3. Output of running a 20 yr simulation for C_w = 100 kW, C_{PV} = 100 kW and C_g = 1 MW.

A plot of the one-dimensional objective function (obtained by varying C_{pv} and setting all other capacities to 0) is shown in Figure XII. This function is clearly convex in the range [0,2500] kWp according to Proposition 2, as the first derivative is monotonically increasing and the second

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derivative is always greater than zero. Similar results are obtained for the other variables C_w,C_g and C_s.

It then follows from Proposition 2, that, as all the constituent functions are convex and the total cost is merely a nonnegative weighted sum of each component cost, the overall cost function is also convex.

A. Case 1: Simulation Only

In Figure XII-A, the results of running the simulation for a PV-Wind system are plotted. From the Figure, the structure of the various constituents of the total costs can be seen. These behave as expected. For example, the net grid energy decreases non-linearly and monotonically with an increase in the capacity of the renewable sources. Also, the capital cost correctly increases linearly with capacity as a linear capital cost model has been used. It should also be noted that at the prices used to plot this figure, the optimal sizing for the PVwind system lies on the boundary of the domain - with 20 kWp of PV capacity and 0 kWp of Wind capacity. This is typical of a low-wind speed, high insolation environment such as that found in South Africa.

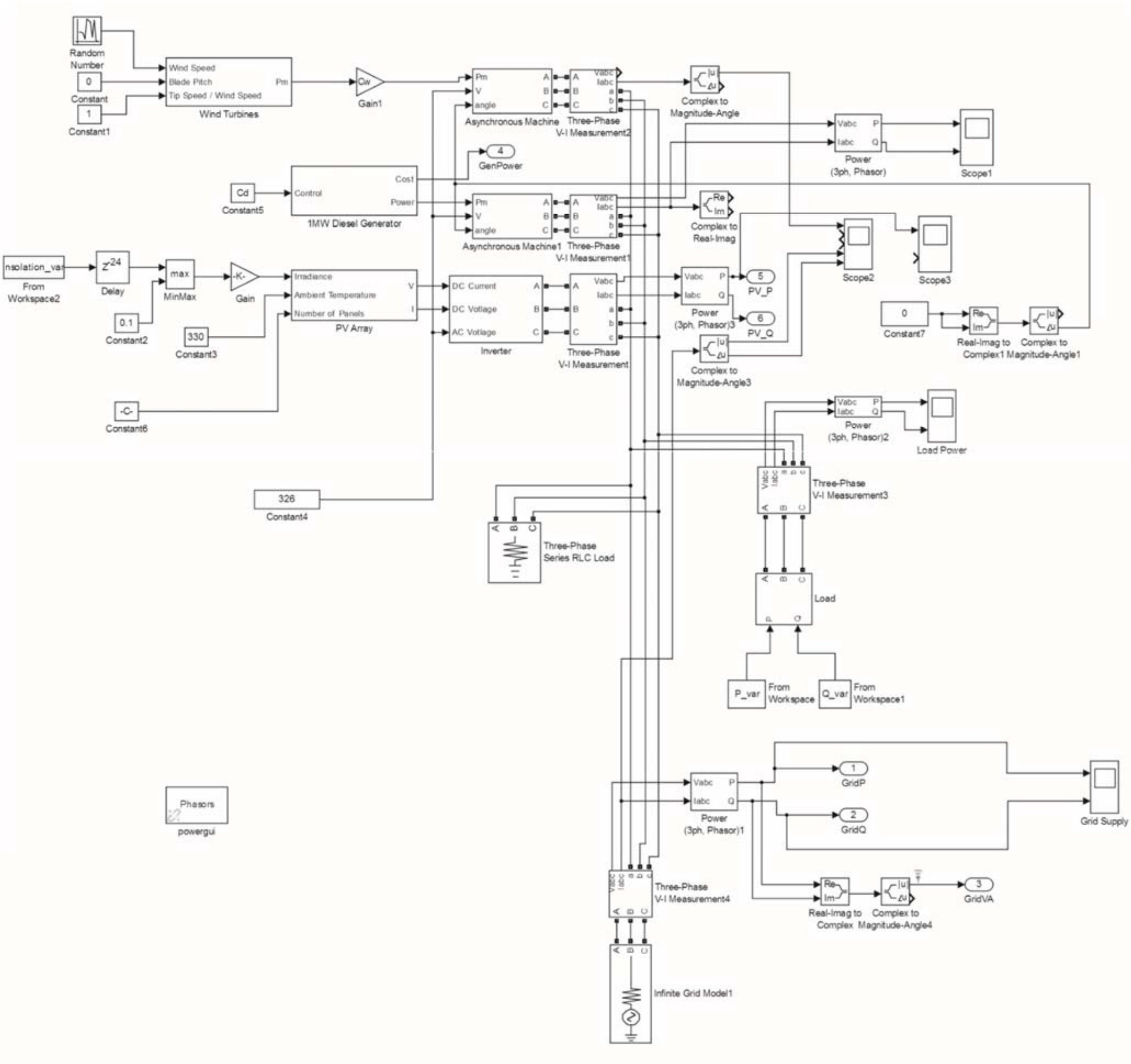

Figure 1. An overview of the Simulink model that was used to carry out a detailed time domain simulation of the hybrid energy system configuration. The input variables include a year's worth time domain load data, P_{var}, Q_{var}, along with wind and solar insolation time data, Ins_{var} and $Wind_{var}$ and the sizes of the system elements, C_d, C_{pv}, C_w and C_s, describing the diesel generator set, PV array and wind turbines, respectively.

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B. Case 2: Current Prices

The optimal hybrid energy system for the University of the Witwatersrand at current costs is shown in Figure XII-B. It is clear that this is a trivial solution as the capital cost of the renewable sources cannot compete with the price of electricity from the grid.

The optimal system design obtained using HOMER is similar with a PV, wind and diesel capacity of 0 kW.

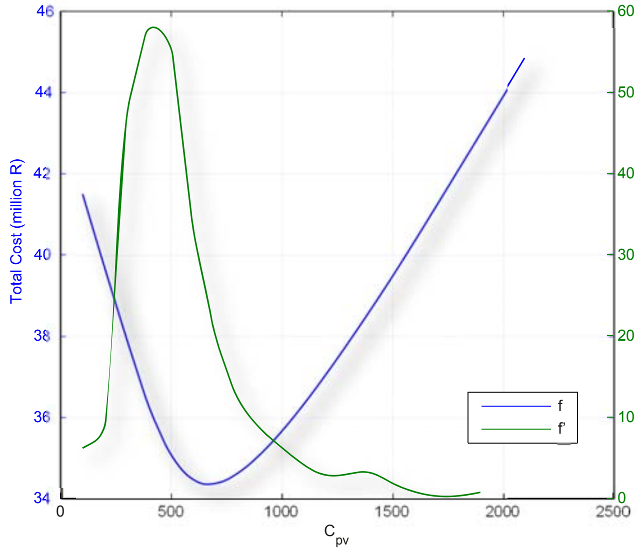

Figure 4. Plot of the objective function and its second derivative for various values of C_{pv} at 1/2 the current PV cost. The capacity of all other elements are set to zero. The objective function is convex according to Proposition 2, as the second derivative is always greater than zero in the range [0,2500].

Total P: 184287.8797 kWh
Peak: 511.176 kVA
Diesel: 0 litres
Peak PV: 0 kWp
Peak Wind: 0 kWp
---------------Capital Cost: R 0.00 Peak Charge: R 5725.1707 kWh Charge: R 39920360.368

Figure 7. Optimal system parameters at current costs.

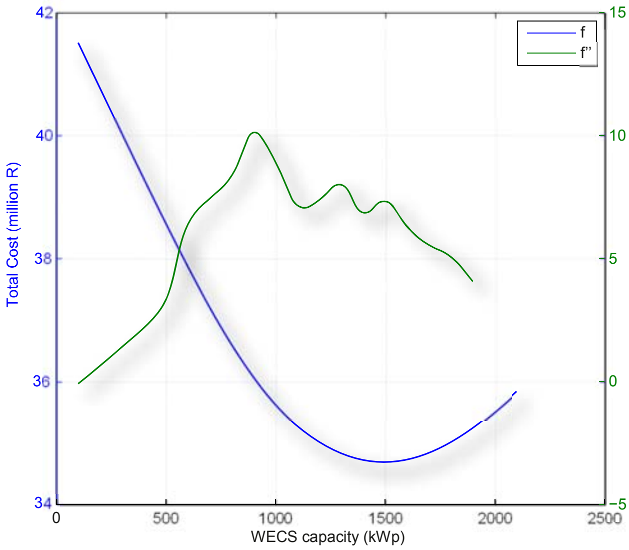

Figure 5. Plot of the objective function and its second derivative for various values of C_w at 1/20th the current WECS cost. The capacity of all other elements are set to zero. The objective function is convex according to Proposition 2, as the second derivative is always greater than zero in the range [0,2500].

C. Case 3: Half Current Prices

The result of running the optimisation with PV and Wind capital costs at half the current prices. As expected for a lowwind area, the optimal PV capacity is much greater than the wind capacity. In fact, even at these prices the optimal mix does not consist of any wind generation.

The optimal system design obtained using HOMER is

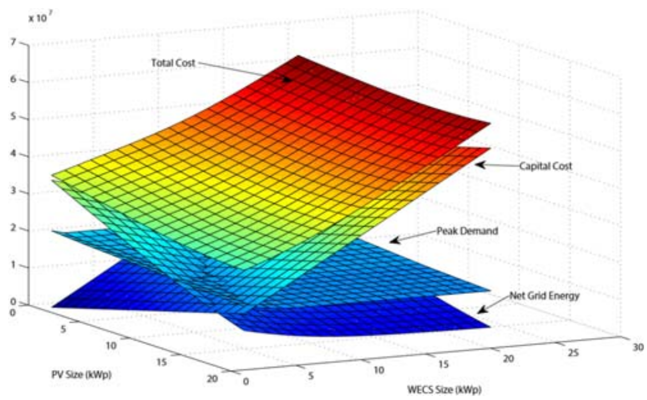

Figure 6. The results of running the simulation on a variety of PV C_{pv} and wind turbine C_w system sizes. The diagram shows a breakdown of the energy bill components (ie. the peak demand and total usage). As expected, the captial cost is a lienar hyperplane increasing with C_w and C_{pv}. The peak charge and total energy consumed, although non-linear, are still smooth, convex functions of the component sizes. As a result, the overall system cost is also a smooth and convex function.

Total P: 146101.4123 kWh
Peak: 488.8913 kVA

Diesel: 0 litres
Peak PV: 285.915 kWp
Peak Wind: 0 kWp

Capital Cost: R 15317424.1687 Peak
Charge: R 5475.5827 kWh Charge: R
31648424.4031

Figure 8. Optimal system parameters at current costs.

similar with a PV capacity of 1000 kW and a diesel and wind capacity of 0 kW.

XIII. Conclusions

In this paper a hybrid energy system simulation model was described and implemented in MATLAB/Simulink. An economic model taking into account the capital and running costs was outlined and used as the objective function for determining the optimal system design. The simulation model was used to show that, under the assumed constraints, the object function is convex and thus fast, convex optimisation algorithms can be used. Finally, SQP was used to carry out the optimisation and the results using this method were validated by those produced by the commercial HOMER software package.

XIV. Future Work and Improvements

Implement other simulation elements such as biogass, hydrogen cells and batteries. The simulation model could also be extended to include a thermal layer - ie. thermal generators and loads. Implement a ”hub-based” model of the microgrid. This will lead to a more generalised formulation of the problem allowing it to be extended to larger systems with different architectures etc.

Extend the optimisation to the case of dynamically reconfiguring a system. For example, in a microgrid environment.

Nomenclature and definitions

MPPT	Maximum Power Point Tracker
T	Temperature in Kelvin
q	Electron charge
k	Boltzmann constant
$V_?$	Voltage at MP Point
N_s	Number of series PV panels
I_0	Reverse saturation current
C_d	Capacity of diesel generator in kW
C_{pv}	Peak capacity of PV in kW
C_w	Peak capacity of wind turbines in kW
C_s	Peak output capacity of storage in kW

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