Rectification of Railway Track Detecting Signal Utilizing Digital Variable Filter

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Abstract — Anti-aliasing pre-filters are generally used to cut off the bandwidth of certain railway track detecting signals that are analog signals in time domain. Track irregularities are distributed along the track in space and could be detected accurately by using inertial measurement sensors mounted on a running vehicle. The detecting signals are specially sampled evenly in space along a railway track instead of in time domain that most signals are in usual conditions. Thus, the detecting signals are processed in space and, however, changed as the vehicle speed varies due to the application of anti-aliasing pre-filters in time domain. In this paper, a new filtering method is presented to rectify the detecting signals utilizing a digital variable filter (VF) designed based on FSS (frequency spectrum sampling) and VAD (vector-array decomposition) algorithm. Firstly, the frequency response characteristic of an anti-aliasing pre-filter in space was sampled with regard to different vehicle speeds. Secondly, the sampled points were transformed as the frequency response characteristic of the variable filter and then constructed as a matrix. Thirdly, by utilizing VAD, the matrix was decomposed into a series of array and vector. Fourthly, the vectors were used as frequency responses and designed as invariable filters, while the arrays were fitted as polynomials associated with vehicle speed. Finally, invariable filters and polynomials were combined to form a variable filter realizing the rectification of the track detecting signals. Simulation experiments were conducted and show that the variable filter designed based on the FSS and VAD has satisfying performances with high precise of frequency response in space and can restore the railway track detecting signals accurately and suppress noise effectively.

Keywords - Digital Variable Filter, Signal Rectification, Railway Track Irregularity, Detection

I. INTRODUCTION

Track inspection car is the large-scale mobile inspection equipment used for inspecting track irregularities, guiding track maintenance, thus, to ensure train safety. It’s also an important means to realize track’s scientific management [1]. Currently, in track traffic, we usually use inertia measurement to obtain track dynamic irregularities. Its main feature is that the sensor has good frequency response with high vehicle speed. Because the track irregularity is spatially distributed, for trains driving at different speed, the irregularity wave length of the same track generates different frequency response. So the time frequency of the measurement of the irregularity wave length in the sensor distributed in space coordinates changes with the speed [2].

In order to get rid of the influence of speed on the detection results, on one hand, when collecting information, we use equal space interval sampling instead of equal time interval sampling, and the filter is designed based on space domain; on the other hand, we introduce anti-aliasing filters at the front end of detection system in time domain. The change in train’s speed prompts the anti-aliasing filter’s response feature makes variant in space domain, which make the detected information in space domain become distorted, thus influences the accuracy of the detection results [3]. Therefore, we need to adopt spatial variable filter to correct it.

II. THE SPATIAL VARIANT FEATURE OF TRACK DYNAMIC DETECTION INFORMATION

When the track inspection car inspects the irregularities, the sensor record the signals in space coordinates instead of in time coordinates, that is to say it maps the spatial displacement coordinates $x$ as time coordinates $t$. When the speed is constant, the mapping from space to time is linear; however, when the speed changes, the mapping is non-linear. Let detected information be $f[x(t)]$. We can get the following two formulas relating time domain to space domain

\[
\Omega = 2\pi f = 2\pi \nu \nu
\]  

(1)

\[
\omega = \Omega T_i = 2\pi \nu \nu T_i = 2\pi \nu L_i
\]  

(2)

In formula (1), $\Omega$ is angular frequency; in formula (2), $\omega$ is circular frequency; $T_i$ is the time interval of two adjacent samples; $L_i$ stands for the space interval of two adjacent samples.

Former research adopts the AS (analytical solution) to design the spatial variable restoration filter [3]. The way is to cascade the anti-aliasing filter and digital restoration AF and set the overall amplitude-frequency response as 1 all the time in space domain. The formula is as follows:

\[H_s(j\nu) = H_s(j2\pi \nu T_i) \cdot H_s(e^{j2\pi \nu L_i}) \equiv 1\]  

(3)

In the formula, $H_s(j2\pi \nu T_i)$ and $H_s(e^{j2\pi \nu L_i})$ is the frequency response function of anti-aliasing filter and digital
VF in space domain respectively. From the design method, we know that when we need to control the gain of certain frequency points in the filter, or when the order of the anti-aliasing filter is relatively high, we need to increase the order of restoration filter. Under this condition, it’s hard to work out the expression of filter’s transfer function. And the transfer function of anti-aliasing filter must be known clearly. At the same time, when the space sampling has high frequency, the restoration of this method will have relatively big error. Form formula (3), we know that this method doesn’t consider the phase change neither, which has to be avoided in track detection required high precision.

III. THE DESIGN OF VARIABLE FILTER

There are many methods to design VF. Dividing the filter’s index into a series of sub-filters and work out the sum of products of the function used in adjusting variant features to approach the ideal filter have become the mainstream ideology in VF’s design theory [4]. One of the most representative methods is the design method based on frequency spectrum sampling and combining data fitting technology. Deng [5] did in-depth research on the features of the multidimensional array based on samples from this method. He put forward VAD algorithm based on SVD (singular value decomposition), which made the design of amplitude-frequency and phase-frequency variable filter become possible.

Step 1: conduct equal interval sampling on the given variable filter’s frequency response index to build a multidimensional plurality array;

Step 2: divide the multidimensional plurality array into a series of sum of products of plurality vector and real number array;

Step 3: set the plurality vector as the invariable filter’s frequency response index by designing invariable filter to approach the index;

Step 4: set the multidimensional real number array as discrete value of the multivariate polynomial and work out the optimized multivariate polynomial through data fitting technology to approach the multidimensional real number array;

Step 5: combine a series of invariable filter and multivariate polynomial to form a digital variable filter.

Set the ideal digital variable filter’s frequency response under one dimensional multivariate as:

$$H_1(\omega, \psi_1, \psi_2, \ldots, \psi_K) = M_1(\omega, \psi_1, \psi_2, \ldots, \psi_K) e^{j\Phi_1(\omega)} \quad (4)$$

In the formula, $\omega \in [-\pi, \pi]$ is circular frequency; $K$ is variable; $\psi_k \in [\psi_{k\min}, \psi_{k\max}]$, $k = 1, 2, \ldots, K$ is spectrum parameter to adjust frequency response.

We conduct equal interval sampling to the frequency spectrum parameter, the samples of the spectrum is as follows

$$\omega(l) = -\pi + \frac{2\pi(l-1)}{L-1}, \quad l = 1, 2, \ldots, L \quad (5)$$

$$P(\psi_1(m_1), \psi_2(m_2), \ldots, \psi_K(m_K)) = \sum_{j_1=0}^{J_1} \sum_{j_2=0}^{J_2} \ldots \sum_{j_K=0}^{J_K} c(j_1, j_2, \ldots, j_K) \psi_1^{j_1}(m_1) \psi_2^{j_2}(m_2) \ldots \psi_K^{j_K}(m_K) \quad (10)$$

$$\psi_k(m_k) = \psi_{k\min} + \frac{(\psi_{k\max} - \psi_{k\min})(m_k - 1)}{M_k - 1}, \quad k = 1, 2, \ldots, K, \quad m_k = 1, 2, \ldots, M_k \quad (6)$$

In the formula, $L$ is the sampling number of circular frequency $\omega$; $M_k$ is sampling number of the number $k$ spectrum parameter $\psi_k$ . We can work out discrete parameter of the frequency response

$$\tilde{a}(l, m_1, m_2, \ldots, m_K) = H_1(\omega(l), \psi_1(m_1), \psi_2(m_2), \ldots, \psi_K(m_K)) \quad (7)$$

We can build a $K + 1$ dimensional plurality array $\tilde{A} = [\tilde{a}(l, m_1, m_2, \ldots, m_K)]$. If we decompose array $\tilde{A}$, we will get the following decomposition:

$$\tilde{A} \approx \bigotimes_{i=1}^{r} C_i(l) \otimes R_i(m_1, m_2, \ldots, m_K) \quad (8)$$

In the decomposition, $\otimes$ are the products of vector quantity and multidimensional array; $C_i$ is plurality conjugate symmetry vector quantity; $R_i$ is $K$ dimensional real number array. Decomposition is called as VAD. $C_i$ and $R_i$ can be worked out through VAD algorithm. $C_i$ and $R_i$ are set as the invariable filter’s frequency response index and discrete value of $K$ dimensional polynomial respectively, then the formula of one dimensional variable filter is as follows

$$H(z, \psi_1, \psi_2, \ldots, \psi_K) = \sum_{l=1}^{L} H_1(z \omega(l), \psi_1, \psi_2, \ldots, \psi_K) \quad (9)$$

From the formula, we know that as long as we adjust the spectrum parameter $\psi_k$ ($k = 1, 2, \ldots, K$), we can change the value of polynomial $P(\psi_1, \psi_2, \ldots, \psi_K)$ ($i = 1, 2, \ldots, r$), thus realizing the filter’s variant features. Therefore, variable filter can be efficiently realized by using parallel structure of Fig.1

![Figure 1 Parallel structure of VF based on VAD](image)

Figure 1 Parallel structure of VF based on VAD

After decomposed by VAD, the element of $K$ dimensional vector quantity $R_i$ is a real number, and, therefore, can be realized by fitting through real coefficient $K$ dimensional polynomial as follow.
IV. SIMULATED ANALYSIS

A. The design of spatial variable filter

The common second-order low-pass active filter’s transfer function is:

\[ A_p(s) = \frac{A_{up}}{a_0 + a_1s + s^2} \]  \hspace{1cm} (11)

In the function, \( A_{up} \) is the gain of pass band, \( s = j\Omega = j2\pi f \). When \( a_0 \) and \( a_1 \) has different values, we can design different types of filter. Since the Butterworth filter has flat amplitude-frequency characteristic and has linear phase characteristic in pass band, the Butterworth filter is the most widely used as anti-aliasing filter. When substitute \( s = j\Omega = j2\pi f v \) into second order Butterworth filter’s transfer function, we can get its space frequency response as follows:

\[ H(j\psi,v) = \left( j\psi \right)^2 + \left( \frac{\Omega}{2\pi v} \right) \left( j\psi \right) + \left( \frac{\Omega}{2\pi v} \right)^2 \]  \hspace{1cm} (12)

In the formula, \( \Omega = 2\pi f_c \) is the filter’s cut-off frequency. From formula (12), we can see that the frequency response in space domain of the anti-aliasing filter in time domain changes with the speed. It also has space domain variant characteristic. If we set cut-off frequency \( f_c = 10 \) Hz, we can get the filter’s space amplitude-frequency characteristic as Fig. 2. From Fig. 2, we can see that the increase of the train’s speed, the filter’s cut-off frequency moves towards left, which means that it is decreasing.

![Figure 2 Frequency response of anti-aliasing filter in space](image)

We set the ripple quantity \( \delta_x = -30 \) dB in compensation band of the final overall filter (cascade of the anti-aliasing filter and digital restoration VF) as the design target. The chosen filter’s number of channels \( r = 3 \) meets the design’s demand. The order of every channel is listed in Table. 1.

<table>
<thead>
<tr>
<th>( N_{11} )</th>
<th>( N_{12} )</th>
<th>( N_{21} )</th>
<th>( N_{22} )</th>
<th>( N_{31} )</th>
<th>( N_{32} )</th>
<th>( J_1 )</th>
<th>( J_2 )</th>
<th>( J_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-256</td>
<td>256</td>
<td>-237</td>
<td>237</td>
<td>-245</td>
<td>245</td>
<td>8</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

Fig. 3 and fig. 4 show the spatial frequency response characteristics of the ideal target VF and the designed VF respectively. And fig. 5 gives the overall spatial frequency response characteristic of anti-aliasing filter and VF. From the figure, we can see the overall frequency response has uniformed characteristic in space domain even if the vehicle speed is changed.

![Figure 3 Spatial frequency response characteristic of the ideal VF](image)

![Figure 4 Spatial frequency response characteristic of the designed VF](image)
B. Analysis of the rectification effect of spatial information

We assume there is an ideal track. Its irregularity value is $A = \sin(2\pi \psi_c d)$, among which, $\psi_c = 1/\lambda_c$, $\lambda_c = 6.192 \text{ m}$, $d$ is the spatial distance and the function of time $t$. In order to detect the rectification performance of the spatial VF in compensation band based on FSS, we set the train’s speed, $v = at$, acceleration $a = 40 \text{ m/s}^2$. In the train’s real operation, considering riding comfort, the train’s acceleration is controlled within $\pm 1.2 \text{ m/s}^2$. The simulated train’s speed increases from 0 km/h to the normal operating speed 430 km/h.

The change in train’s speed incentivizes the change in frequency due to the irregularities of the track. Acceleration signal sensing the irregularity waveform is a variable frequency signal, similar as frequency modulated signal, which is shown in Fig. 6(a). We conduct spatial sampling from time domain signal to get irregularity signal in space domain. Irregularity signal in space domain remains the ideal sinusoidal signal, shown in Fig. 6(b), which is also the expected irregularity signal in track detection. After the irregularity signal goes through the anti-aliasing filter, with the increase of frequency response, the attenuation degree of its amplitude also increases, shown in Fig. 6(c) and Fig. 6(d). Here, we used the designed spatial compensation VF to restore the distorted irregularity signal. Fig. 6(e) and fig. 6(f) are the waveform of spatial irregularity after it was rectified by VF based on FSS and VF based on AS respectively.

Besides, variable filter based on spectrum sampling has a group delay, in this research $\tau_g = 256$, therefore, the restored waveform has $\tau_g$ number of delay. Since the filter’s original quantity is 0, the front end of the VF will get a section of zero output, shown in fig. 6(e).

Figure 5 Overall spatial frequency response characteristic of anti-aliasing filter and VF

![Figure 5 Overall spatial frequency response characteristic of anti-aliasing filter and VF](image)

![Figure 6(a) Irregularity signal in time domain](image)

![Figure 6(b) Irregularity signal in space](image)

![Figure 6(c) Irregularity signal passed through the anti-aliasing filter in time domain](image)

![Figure 6(e) Irregularity signal processed by the anti-aliasing filter](image)
V. CONCLUSION

In most practical measurements, anti-aliasing filters are usually applied to suppress high frequency noise of certain signals before they are sampled and quantized. Due to the application of anti-aliasing pre-filter, data acquisition of the railway track detecting signals with even intervals in space results in that the frequency response characteristic in space is changed as the speed of vehicle varies, and signal distortion consequently.

A new variable filtering method based on FSS and VAD is presented to rectify the distorted signals in this paper. As we discussed above, compared with VF designed by AS, the filtering method for the railway track detecting signals is more universal, flexible and easy to use. It doesn’t have to require the transfer function of anti-aliasing pre-filter exactly but just the frequency response, which can be measured if its transfer function is unknown or calculated out otherwise. Once the frequency response characteristic in space is obtained, the variable filter can be designed easily with satisfying performances. Therefore, the distorted detecting signals can be rectified with high precise, high effectiveness of de-noising, and low sensibility of sampling time interval associated with the speed of vehicle.

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