A Robust H_∞ Filter Design for Uncertain Nonlinear Singular Systems

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***Abstract* — For a type of uncertain generalized time-delay systems with Nonlinear perturbation, the robust H_∞ filter design of such forms the focus of this paper. It is proved that the filtering error dynamic system to be robustly asymptotically stable and has a sufficient condition of H_∞ norm bounded with Lyapunov functional method and linear matrix inequalities with unknown parameters. Expressions of the robust H_∞ filter of such generalized system are given on the basis of the corresponding feasible solution of linear matrix inequality. A numerical example shows the feasibility and validity of this method.**

Key words - generalized system; nonlinear; linear matrix inequality; filter

I. INTODUCTION

In the past few decades, $\boldsymbol{H}_\infty$ filtering problem has always been a hot research topic in control theory and application[1–3]. When the noise is any energy limited signal, $\boldsymbol{H}_\infty$filter can ensure that the level of noise suppression is given. Literature [4] uses $\boldsymbol{H}_\infty$ filter to eliminate the wave perturbation so that the stable filtering effect is obtained. The generalized system widely exists in engineering systems, such as electric power systems, nuclear reactors, limited robots and so on. While the uncertainty, time delay and nonlinearity in the actual system is inevitable. Uncertainty and time delay are always the main factors which lead to the system instability or the performance degradation. While the nonlinear brings greater difficulty for system stabilization, filtering, fault detection and so on. Therefore, to study uncertain robust filtering problem of nonlinear generalized systems with time delay has important theoretical significance and application value. Literature [5-6] studies H_∞ filter design problem for generalized systems, but it doesn't involve the time-delay and uncertain situation. Literature [7] studie the robust $\boldsymbol{H}_\infty$ reduced-order filter problem of the uncertain multiple time-delay singular systems but it doesn't consider $\boldsymbol{H}_\infty$ filter problem of generalized systems which contains the nonlinear perturbation. At present, we do not see the related reports about this kind of problems.

We adopt the tool of Lyapunov functional method and the linear matrix inequality to study the robust $\boldsymbol{H}_\infty$ filter design problem of uncertain generalized systems with nonlinear perturbation in the article. And we give the sufficient condition when robust $\boldsymbol{H}_\infty$ filter problem has the solution. We also give the structural method of robust $\boldsymbol{H}_\infty$ filter based on linear inequalities.

II. PROBLEM DESCRIPTION AND PREPARATION

We can consider the following uncertain nonlinear generalized time-delay systems.

$$\begin{cases} E\dot{x}(t)=(A+\Delta A)x(t)+(A_1+\Delta A_1)x(t-d)+ \\ f(t,x(t))+B\omega(t); \\ y(t)=(C+\Delta C)x(t)+(C_1+\Delta C_1)x(t-d)+ \\ D\omega(t); \\ z(t)=(H+\Delta H)x(t)+(H_1+\Delta H_1)x(t-d)+ \\ D_1\omega(t); \\ x(t)=\varphi(t),t\in[-d,0],\varphi\in C([-d,0],R^n). \end{cases} \quad (1)$$

Among them, $x(t)\in R^n$ is the state vector of system. $z(t)\in R^q$ is control output vector. $\omega(t)\in L_2[0,\infty)$ is p -dimensional perturbation input vector. $f=f(t,x(t))\in R^n$ is nonlinear vector function, and it satisfies $f(t,0)=0$ and the following Lipschitz condition

$$\left\|f(t,x(t))-f(t,\tilde{x}(t))\right\|\le\alpha\left\|F(x(t)-\tilde{x}(t))\right\|, \quad \forall x(t),\tilde{x}(t)\in R^n. \quad (2)$$

Among them, $\alpha>0$ is a positive number, $F\in R^{n\times n}$ is constant matrix. The norm $\|\cdot\|$ represents the distance function in R^n -dimensional space. According to the last equation, we get

$$\left\|f(t,x(t))\right\|\le\alpha\left\|Fx(t)\right\|. \quad (3)$$

The matrix $E,\ A,\ A_1,\ B,\ C,\ C_1,\ D,\ H,\ H_1$ and D_1 are known constant matrixes with proper dimensions. d is delay time and $d>0$. $rankE = r \le n$; $\Delta A,\ \Delta A_1,\ \Delta C,\ \Delta C_1,\ \Delta H,\ \Delta H_1$ are uncertain time-varying matrixs with proper dimensions. Suppose that the following forms exsit.

$$\begin{bmatrix} \Delta A & \Delta A_1 \\ \Delta C & \Delta C_1 \\ \Delta H & \Delta H_1 \end{bmatrix} = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} F(t)\begin{bmatrix} M_1 & M_2 \end{bmatrix}. \quad (4)$$

Among them, $L_1,\ L_2,\ L_3,\ M_1,\ M_2$ are known constant matrixes with proper dimensions. $F(t)$ is bounded uncertain function matrixn and satisfies $F^T(t)F(t) \le I.$

The purpose of this article is to design a full order linear time invariant filter.

$$\begin{cases} E_c\dot{\xi}(t) = A_c\xi(t) + B_c y(t), \xi(0) = 0; \\ \hat{z}(t) = C_c\xi(t). \end{cases} \quad (5)$$

So we can obtain the estimation of signal $z(t)$. Combine the states of system (1) and (5) . And the dynamic system of augmented filter error is

$$\begin{cases} \bar{E}\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{A}_1\bar{x}(t-d) + \bar{f}(t,\bar{x}(t)) + \\ \bar{B}\omega(t); \\ \bar{z}(t) = \bar{C}\bar{x}(t) + \bar{C}_1\bar{x}(t-d) + \bar{D}\omega(t); \\ \bar{x}^T(t) = \left[\varphi^T(t), 0\right]; \forall t \in [-d, 0]. \end{cases} \quad (6)$$

Among them,

$$\bar{z}(t) = z(t) - \hat{z}(t); \bar{x}^T(t) = \left[x^T(t), \xi^T(t)\right];$$

$$\bar{f}^T = \bar{f}^T(t,\bar{x}(t)) = \left[f^T(t,x(t)), 0\right];$$

$$\left\|\bar{f}^T(t,\bar{x}(t))\right\| \le \alpha\left\|\bar{F}\bar{x}(t)\right\|; \bar{E} = \begin{bmatrix} E & 0 \\ 0 & E_c \end{bmatrix};$$

$$\bar{A} = \begin{bmatrix} \tilde{A} & 0 \\ B_c\tilde{C} & A_c \end{bmatrix}; \bar{A}_1 = \begin{bmatrix} \tilde{A}_1 & 0 \\ B_c\tilde{C}_1 & 0 \end{bmatrix}; \bar{B} = \begin{bmatrix} B \\ B_c D \end{bmatrix};$$

$$\bar{C} = \begin{bmatrix} \tilde{H} & -C_c \end{bmatrix}; \bar{C}_1 = \begin{bmatrix} \tilde{H}_1 & 0 \end{bmatrix}; \bar{D} = D_1;$$

$$\bar{F} = \begin{bmatrix} F & 0 \\ 0 & 0 \end{bmatrix}; \tilde{A} = A + \Delta A;\ \tilde{A}_1 = A_1 + \Delta A_1;$$

$$\tilde{C} = C + \Delta C;\ \tilde{C}_1 = C_1 + \Delta C_1;\ \tilde{H} = H + \Delta H;$$

$$\tilde{H}_1 = H_1 + \Delta H_1. \quad (7)$$

The purpose of this article is to design the robust filter as the equation (5) . Therefore, for the uncertainty which satisfies the allowable conditions, the dynamic system （6） of augmented filter error is asymptotically stable. Under zero initial conditions, for all non-zero $\omega \in L_2[0,\infty)$, there is $\left\|\bar{z}(t)\right\|_2 < \gamma\left\|\omega(t)\right\|_2, \forall\omega \in L_2[0,\infty).$

Lemma 1[8] singular system

$$E\dot{x}(t) = Ax(t).$$

It is regular and impulse -free. It will exsit when and only when the matrix P s

$$E^T P = P^T E \ge 0;\ A^T P + P^T A < 0.$$

As far as Lemma 2[9] for any matrix $F(t)$ is concerned, if $F^T(t)F(t) \le I$, for any $\varepsilon > 0$, there is

$$2x^T PBF(t)Ex \le \varepsilon x^T PBB^T Px + \varepsilon^{-1}x^T E^T Ex.$$

For the real matrixes $\Pi,\ H,\ \Gamma$ with appropriate dimension given by lemma 3, among them Π is symmetrical, then $\Pi + \Gamma F(t)H + H^T F^T(t)\Gamma^T < 0;$ If all the matrixes $F(t)$ which satisfy $F^T(t)F(t) \le I$ is established, when and only when a constant $\varepsilon > 0$ exsits, it will make $\Pi + \varepsilon^{-1}\Gamma\Gamma^T + \varepsilon H^T H < 0.$

III. THE PERFORMANCE ANALYSIS OF ROBUST H_∞

We study H_∞ filtering problem in this section, H_∞ performance adjustment of the dynamic system （6） of filtering error can be given in the following theorem.

Theorem 1 For uncertain nonlinear generalized time-delay system (1) and the given positive constant $\gamma > 0$, if the symmetric positive definite matrix P exsits, $S \in R^{n\times n}$ and the scalar $\varepsilon > 0$, it should satisfy the inequation

$$\bar{E}^T P = P^T \bar{E} \ge 0; \quad (8a)$$

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} + S + \dfrac{\alpha^2}{\varepsilon}\bar{F}^T\bar{F} + \varepsilon P^T P & P\bar{A}_1 & P\bar{B} & \bar{C}^T \\ * & -S & 0 & \bar{C}_1^T \\ * & * & -\gamma^2 I & \bar{D}^T \\ * & * & * & -I \end{bmatrix} < 0. \quad (8b)$$

Then the dynamic system (6) of filtering error is asymptotically stable, and it satisfies H_∞ noise suppression level γ .

The Proof: First we prove that the dynamic system (6) of filtering error is regular and pulse-free. The equation (8a) and equation(8b) are established and $S>0$.Therefore,

$$\bar{E}^T P = P^T \bar{E} \geq 0;\ \bar{A}^T P + P\bar{A} < 0.$$

From the Lemma 1 we know that the dynamic system (6) of filtering error is regular and pulse-free.

Next, we will prove that the dynamic system (6) of filtering error is asymptotically stable. So we take the Lyapunov function,

$$V(\bar{x}_t) = \bar{x}^T(t)P\bar{E}\bar{x}(t) + \int_{t-d}^{t} \bar{x}^T(s)S\bar{x}(s)ds;$$

We take the derivative of $V(\bar{x}_t)$ along the dynamic system (6) of filtering error, so we get

$$\dot{V}(\bar{x}_t) = \dot{\bar{x}}^T(t)P\bar{E}\bar{x}(t) + \bar{x}^T(t)P\bar{E}\dot{\bar{x}}(t) + \bar{x}^T(t)S\bar{x}(t) - \bar{x}^T(t-d)S\bar{x}(t-d);$$

$$= \bar{x}^T(t)\left[\bar{A}^T P + P\bar{A} + S\right]\bar{x}(t) + 2\bar{x}^T(t)P\bar{A}_1\bar{x}(t-d) + 2\bar{x}^T(t)P\bar{f} + 2\bar{x}^T(t)P\bar{B}\omega(t) - \bar{x}^T(t-d)S\bar{x}(t-d).$$

From the Lemma 2 we get

$$\dot{V}(\bar{x}_t) \leq \bar{x}^T(t)\left[\bar{A}^T P + P\bar{A} + S + \frac{\alpha^2}{\varepsilon}\bar{F}^T\bar{F} + \varepsilon P^T P\right]\bar{x}(t) + 2\bar{x}^T(t)P\bar{A}_1\bar{x}(t-d) + 2\bar{x}^T(t)P\bar{B}\omega(t) - \bar{x}^T(t-d)S\bar{x}(t-d).$$

When $\omega(t)=0$, there is

$$\dot{V}(\bar{x}_t) \leq \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T \begin{bmatrix} & P\bar{A}_1 \\ * & -S \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix};$$

Among them

$$\bar{A}^T P + P\bar{A} + S + \frac{\alpha^2}{\varepsilon}\bar{F}^T\bar{F} + \varepsilon P^T P$$

From the Schur complement lemma of matrix we know that if matrix inequality (8b) is established, there is

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} + S + \frac{\alpha^2}{\varepsilon}\bar{F}^T\bar{F} + \varepsilon P^T P & P\bar{A}_1 \\ * & -S \end{bmatrix} < 0.$$

That is to say, $\dot{V}(\bar{x}_t) < 0$ negative definition. According to the Lyapunov theorem, the dynamic system (6) of filtering error is asymptotically stable.

At last we prove that under zero initial condition H_∞ noise suppression level γ of dynamic system (6) of filtering error makes the given $\gamma > 0$ be brought in ***H***$_\infty$ performance index:

$$J = \int_0^\infty \left[\bar{z}^T(t)\bar{z}(t) - \gamma^2\omega^T(t)\omega(t)\right]dt.$$

We make use of the structural Lyapunov function $V(\bar{x}_t)$ and zero initial condition. For any nonzero $\omega(t) \in L_2[0,\infty)$, there is

$$J \leq \int_0^\infty \left[\bar{z}^T(t)\bar{z}(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(x_t)\right]dt + V(0) - V(\infty);$$

$$\leq \int_0^\infty \left[\bar{z}^T(t)\bar{z}(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(x_t)\right]dt.$$

And

$$\bar{z}^T(t)\bar{z}(t) = \left[\bar{C}\bar{x}(t) + \bar{C}_1\bar{x}(t-d) + \bar{D}\omega(t)\right]^T * \left[\bar{C}\bar{x}(t) + \bar{C}_1\bar{x}(t-d) + \bar{D}\omega(t)\right]$$

$$= \bar{x}^T(t)\bar{C}^T\bar{C}\bar{x}(t) + 2\bar{x}^T(t)\bar{C}^T\bar{C}_1\bar{x}(t-d) + 2\bar{x}^T(t)\bar{C}^T\bar{D}\omega(t) + \bar{x}^T(t-d)\bar{C}_1^T\bar{C}_1\bar{x}(t-d) + 2\bar{x}^T(t-d)\bar{C}_1^T\bar{D}\omega(t) + \omega(t)\bar{D}^T\bar{D}\omega(t).$$

So

$$J \leq \int_0^\infty \begin{bmatrix} x(t) \\ x(t-d) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} \Xi_{11} & P\bar{A}_1 + \bar{C}^T\bar{C}_1 & P\bar{B} + \bar{C}^T\bar{D} \\ * & -S + \bar{C}_1^T\bar{C}_1 & \bar{C}_1^T\bar{D} \\ * & * & -\gamma^2 I + \bar{D}^T\bar{D} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \\ \omega(t) \end{bmatrix} dt;$$

Among them

$$\Xi_{11} = \bar{A}^T P + P\bar{A} + S + \frac{\alpha^2}{\varepsilon}\bar{F}^T\bar{F} + \varepsilon P^T P + \bar{C}^T\bar{C}.$$

According to the Schur complement lemma of matrix we know that if matrix inequality (8b) is established, there is

$$\begin{bmatrix} \Xi_{11} & P\bar{A}_1 + \bar{C}^T\bar{C}_1 & P\bar{B} + \bar{C}^T\bar{D} \\ * & -S + +\bar{C}_1^T\bar{C}_1 & \bar{C}_1^T\bar{D} \\ * & * & -\gamma^2 I + \bar{D}^T\bar{D} \end{bmatrix} < 0.$$

So that $J \leq 0$, that is to say, $\|\bar{z}(t)\|_2 < \gamma\|\omega(t)\|_2$. Therefore, the dynamic system (6) of filtering error satisfies H_∞ noise suppression level γ. The proof is over.

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IV. THE DESIGN OF ROBUST FILTER

Filter design method is given in the following theorem.

Theorem 2 If, for the given constants, symmetric positive definite matrix and appropriate dimension matrix exsit so as to make the following matrix inequality be established.

$$EX = X^T E^T; \tag{9a}$$

$$E^T Y = Y^T E; \tag{9b}$$

$$\begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix} > 0; \tag{9c}$$

$$\Sigma = \begin{bmatrix} (\Sigma_1)_{6\times 6} & (\Sigma_2)_{6\times 8} \\ * & (\Sigma)_{8\times 8} \end{bmatrix} < 0. \tag{9d}$$

Among them

$$\Sigma_1 = \begin{bmatrix} X^T A + XA & A + \hat{H}^T & A_1 & 0 & B & \Sigma_{16} \\ * & \Sigma_{22} & \Sigma_{23} & 0 & \Sigma_{25} & H^T \\ * & * & -S_1 & -S_2 & 0 & H_1^T \\ * & * & * & -S_3 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & D_1^T \\ * & * & * & * & * & -I \end{bmatrix};$$

$$\Sigma_2 = \begin{bmatrix} \alpha X^T F^T & 0 & I & 0 & X^T & M & X^T M_1^T & L_1 \\ \alpha F^T & 0 & Y^T & N & I & 0 & M_1^T & \Sigma_{214} \\ 0 & 0 & 0 & 0 & 0 & 0 & M_2^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_3 \end{bmatrix};$$

$$\Sigma_3 = \begin{bmatrix} -\varepsilon I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -\varepsilon I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon^{-1} I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon^{-1} I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\tilde{S}_1 & -\tilde{S}_2 & 0 & 0 \\ * & * & * & * & * & -\tilde{S}_3 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_1^{-1} I \end{bmatrix};$$

$$\Sigma_{16} = X^T H^T - G^T;$$

$$\Sigma_{22} = Y^T A + A^T Y + \hat{F} C + C^T \hat{F}^T; \ \Sigma_{23} = Y^T A_1 + \hat{F} C_1;$$

$$\Sigma_{25} = Y^T B + \hat{F} D;$$

$$\Sigma_{25} = Y^T B + \hat{F} D; \ \Sigma_{214} = Y^T L_1 + \hat{F} L_2.$$

Then the filter problem of system (1) has the solutions and filter parameters safisfy

$$B_c = N^{-1} \hat{F}; \ C_c = G M^{-T};$$

$$\begin{bmatrix} X_1^T \bar{A}^T X_2 + X_2^T \bar{A} X_1 & X_2^T \bar{A}_1 & X_2^T \bar{B} & X_1^T \bar{C}^T & \alpha X_1^T F^T & X_2^T & X_1^T \\ * & -S & 0 & \bar{C}_1^T & 0 & 0 & 0 \\ * & * & -\gamma^2 I & \bar{D}^T & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & * & -\varepsilon^{-1} I & 0 \\ * & * & * & * & * & * & -S^{-1} \end{bmatrix} < 0 \tag{14}$$

In the equation（14）, we take

$$S = \begin{bmatrix} S_1 & S_2 \\ * & S_3 \end{bmatrix} > 0; \ S^{-1} = \begin{bmatrix} \tilde{S}_1 & \tilde{S}_2 \\ * & \tilde{S}_3 \end{bmatrix} > 0;$$

At the same time, we make

$$\hat{F} = N B_c; \qquad G = C_c M^T;$$

$$\hat{H} = Y^T A X + N B_c C X + N A_c M^T. \tag{15}$$

We take the forms of matrix $S > 0,\ S^{-1} > 0,\ X_1$ and X_2 be substituted into the equation (7) and lemma 3. And then we can get the equation(9d) from the equation(7) and lemma 3.

At last we give the design steps for $\boldsymbol{H}_\infty$ filter of system (1). And we show them as follows.

We solve for the matrix $X,\ Y,\ G,\ F$ and $\hat{H}$ by matrix inequality（9d）.

We take the right side of equation (11) to make matrix decomposition to solve for the nonsingular matrix M and N.

We make $X,\ Y,\ G,\ F,\ \hat{H},\ M$ and N be substituted the equation (15) so as to get the solutions, $A_c,\ B_c,\ C_c$ and E_c. The proof is over.

V. THE SIMULATION EXAMPLE

We consider uncertain nonlinear generalized time-delay systems (1). Among them all the coefficients can be seen as follows.

$$E=\begin{bmatrix}1&0\\0&0\end{bmatrix};\ A=\begin{bmatrix}-7&0\\2&-4\end{bmatrix};\ A_1=\begin{bmatrix}3.1&1\\0&1\end{bmatrix};$$

$$B=\begin{bmatrix}1.1&2.2\\1.1&0\end{bmatrix};\ C=\begin{bmatrix}16&0\\2.3&15\end{bmatrix};$$

$$C_1=\begin{bmatrix}-5.5&-7.5\\-2&3\end{bmatrix};\ D=\begin{bmatrix}5&0\\0&2\end{bmatrix};$$

$$D_1=\begin{bmatrix}2&1\end{bmatrix};\ H=\begin{bmatrix}-1&1\end{bmatrix};\ H_1=\begin{bmatrix}0&2\end{bmatrix};$$

$$L_1=\begin{bmatrix}0.02&0\\0&0.02\end{bmatrix};\ L_2=\begin{bmatrix}0.02&0\\0&0.02\end{bmatrix};$$

$$L_3=\begin{bmatrix}0.2&0\end{bmatrix}\ M_1=M_2=I;$$

$$f\left(t,x(t)\right)=\begin{bmatrix}\sin x_1(t)\\ \sin\left[x_1(t)+x_2(t)\right]\end{bmatrix};$$

$$x(t)=\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix}.$$

Then there is

$$\begin{aligned}\left\|f\left(t,x(t)\right)\right\|^2&=\sin^2 x_2(t)+\sin^2\left[x_1(t)+x_2(t)\right]\\&\le x_1^2(t)+\left[x_1(t)+x_2(t)\right]^2\\&=x^T(t)Fx(t).\end{aligned}$$

Among them

$$F=\begin{bmatrix}2&1\\1&1\end{bmatrix}.$$

We take $\alpha=0.2$. According to theorem 2, we use the toolbox of MATLAB to get a feasible solution. It is

$$X=\begin{bmatrix}6.3597&1.1405\\1.1405&11.6442\end{bmatrix};$$

$$Y=\begin{bmatrix}4.6594&-0.6628\\-0.6628&11.1796\end{bmatrix};$$

$$G=\begin{bmatrix}-5.2192&10.5037\end{bmatrix};$$

$$\hat{F}=\begin{bmatrix}-0.5798&-2.3881\\0.4424&0.0937\end{bmatrix};$$

$$\hat{H}=\begin{bmatrix}7.0000&0\\-2.0000&4.0000\end{bmatrix}.$$

We take the right side of equation (11) to make matrix decomposition to solve for the nonsingular matrix M and N.

$$M=\begin{bmatrix}-0.7071&-0.7071\\-0.7071&0.7071\end{bmatrix};$$

$$N=\begin{bmatrix}2.1213&0.7071\\-2.1213&0.7071\end{bmatrix}.$$

We take X, Y, G, F, $\hat{H}$, M and N be substituted the equation (15) so as to get the solutions, A_c, B_c, C_c and E_c.

$$B_c=\begin{bmatrix}-0.2410&-0.5850\\-0.0972&-1.6224\end{bmatrix};$$

$$C_c=\begin{bmatrix}-3.7367&11.1177\end{bmatrix};$$

$$A_c=\begin{bmatrix}-5.1333&72.2000\\235.4000&25.4000\end{bmatrix};$$

$$E_c=\begin{bmatrix}0.1667&0.1667\\0.5000&0.5000\end{bmatrix}.$$

VI. CONCLUSION

Aiming at a kind of uncertain generalized time-delay systems with nonlinear perturbation, we study the design method of $\boldsymbol{H}_\infty$ filter in this article. We use the inequality tool of Lyapunov function and linear matrix to get that the robust of filtering error dynamic system is asymptotically stable and have the sufficient condition that $\boldsymbol{H}_\infty$ norm is bounded. And then we make it be transformed into the linear matrix inequality without parameter uncertain matrix. Based on the corresponding feasible solution of linear matrix inequality, we give this kind of expressions about generalized system robust $\boldsymbol{H}_\infty$ filter. The given results all be given in the form of strict linear matrix inequality so that the solution process will be convenient and simple if LMI toolbox of Matlab is adopted.

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