

## Development of Urban Public Transit Network Structure Integrating Multi-Class Public Transit Lines and Transfer Hubs

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**Abstract** — Urban public transit consists of multiple transit modes, such as railway, BRT, and bus. The integration of multi-class urban public transit lines is very important to achieve a more efficient transit system for the cities that advocate the public transit priority. This paper presents an urban public transit network structure for the grid transit network form. The urban public transit network structure consists of primary routes, feeder routes and accessorial routes, which are comprised of different class transit lines, and transfer hubs are the key nodes connecting multi-class public transit lines. Primary routes spacing, MRT stops spacing and the size of transfer hub service zone are the key design variables. From the points of view of the traveler, operator and authority respectively, the optimization objective functions for the key design variables are formulated, and the numerical analysis is carried out to find the optimal solution. To a homogenous region, the recommended values of the key design variables are put forward. The result can provide the foundation for routes planning and transfer hubs location.

**Keywords** - public transit; network structure; transfer hubs; network optimization; location planning

### I. INTRODUCTION

Public transit consists of multiple transport modes, such as bus, railway, taxi and bus rapid transit (BRT). Some capitals of the provinces in China are offering the public transit service, only bus mode. The current bus lines network has an irrational structure: low density, high repetition rate, and even some blind spots for service, which result in the low level of service (LOS): over-crowded, the long travel time, the low operating speed, the long transfer time between different bus lines, etc. Some cities are starting to plan and construct the mass rapid transit (MRT) lines, aim to offer variety of public transit service and meet the passengers' different demand. Public transit network contains multi-class transit lines, and transfer hubs are the key nodes of the public transit network and they connect different class public transit lines. Transfer hubs also play a very important role to decrease the passenger travel time. It is very important and significant to organize a rational and efficient public transit network, and the public transit network structure should be designed.

Researchers from all over the world use systematic engineering methods such as genetic algorithm, ant-colony algorithm, optimization theory and graph theory to solve public transit network optimization problems. Zhao et al. [1] put forward a mathematical methodology for transit route network optimization and an integrated simulated annealing, tabu, and greedy search algorithm are used to find the expected global optimal result. Fan and Machemehl [2] presented a bi-level optimization model for solving the Public Transportation Network Redesign Problem (PTNRP), and Genetic Algorithm (GA) is developed to solve this PTNRP model. Shai Jerby and Avishai Ceder [3] presented

an optimal routing design method for shuttle bus service, and the optimal model and its heuristic alternative were compared in different scenarios on a small real-life road network, and the research shown that the heuristic algorithm indeed provides good (optimal in the test case) results.

Yu et al. [4, 5] proposes a mixed integer optimal location model for urban transit hubs, with the objective to minimize the demand-weighted total travel time; a cluster-based hierarchical location model for the selection of the proper locations and scales of urban transit hubs was developed with the objective of minimizing the demand-weighted total travel time. Liu [6] takes into account that the transport hub has a reaction to the transport network, and proposes simultaneously the optimization problems of transportation hub location and network design establishing a two-step decision-making model. Lü [7, 8] puts urban transfer hub into five categories and establishes the passenger transfer hubs location model for the urban central district; the author presents that the transfer hub should be located in the point that serves the maximum amount of people and jobs within the reasonable walking area.

Other authors have studied structural transit questions. For example, Byrne [9], radial systems, Wirasinghe et al. [10], corridors and Newell [11]. Daganzo [12] presents a hybrid concept and these hybrid networks combine a grid structure in the city center with a hub and spoke pattern in the periphery. Estrada [13] presents and tests a method to design high-performance transit networks. The method produces the hybrid network conceptual plans for geometric idealizations of a particular city that are later adapted to the real conditions. Felix Laube has characterized the physical configuration of transit service in Zurich as an integrated network of three components: (a) a primary net, a radially

oriented line-haul system, (b) a secondary net, a timed-transfer network, (c) a fine-grained grid of mainly tram lines that circulate within dense, built-up areas. The regional configuration of PT service offers a reference for this research [14]. Van Nes *et al.* (15) discuss and analyze possible objectives using analytical models for optimizing stop spacing and line spacing in urban transit networks, and the results of these analytical models for two typical city types are analyzed by comparing performance characteristics (i.e., travel time, operator costs, and patronage).

This paper focuses on developing an urban public transit network structure that integrates multi-class public transit lines and transfer hubs. The optimization model for the key design variables of public transit network structure is formulated, and the satisfactory solution of key design parameters is proposed.

II. PUBLIC TRANSIT NETWORK STRUCTURE

Public transit network consists of the lines and the nodes, the lines are composed of railway lines, BRT lines and bus lines and so on, the nodes are composed of transfer hubs, transfer stations and stops. Most of transit models usually focus on single-class transit networks. Exceptions can be found in literature on airline-network design. In these studies a two-layered network structure is assumed, in which the higher-level network connects the major airports or the hubs and the lower-level network connects the other airports with the hubs (the spokes) [16]. The hub and spoke network structure offers a solution for the multi-class public transit network structure design.

Passengers need different class public transit lines to meet the different trip demand: the different trip distance, speed and passenger volume. The main PT service in China is the fixed-route bus lines. In some metropolises, such as Beijing, Shanghai and Guangzhou, the subway service is becoming the main trip mode. The BRT also is being widely applied in China, such as Beijing, Jinan, Kunming, and Hangzhou. Public transit lines fall into four classes: the subway lines, the BRT lines, the trunk bus lines, and the shuttle bus lines. The design standards for every class public transit lines are shown in Table I.

There are several network forms for public transit network, such as grid, triangular, radial and radial/arc. To the grid network form, the ideal urban public transit network structure is shown in figure 1, and it comprises of different functional routes, transfer hubs and transfer hubs service zones. The urban area is divided into some transfer hubs service zones, and only one transfer hub is located in the center of the transfer hubs service zone. Transfer hubs offer the transfer service among multi-class public transit lines, and may combine subway lines and BRT lines with local bus services, taxi. The size of transfer hub service zone is an acceptable range around the transfer hub.

The public transit routes consist of primary routes, feeder routes and accessorial routes:

Primary routes connect the transfer hubs and take on the function of transit corridor service between transfer hubs service zones. Primary routes contain one or multiple class public transit lines, such as railway lines, BRT lines and

trunk bus lines, and offer various and differential transit service for the passengers traveling between transfer hubs service zones.

TABLE I. DESIGN STANDARD FOR EVERY CLASS PUBLIC TRANSIT LINES

Lines type	Operation speed (km/h)	Departure interval (min)	Length (km)	Stop spacing (km)	Capacity (passengers/h)
Railway lines	25~50	3~5	>15	>1.0	25000~50000
BRT lines	>25	4~8	>15	>1.0	≤30000
Trunk bus lines	18~20	4~10	10~15	0.5~0.8	≤5000
Shuttle bus lines	15~20	4~10	<8	0.3~0.5	≤3000

Feeder routes deliver the travelers from and to MRT stops or transfer hubs. On the other hand, feeder routes offer the transit service within the transfer hubs service zone. Feeder routes contain the shuttle bus lines operating within the transfer hubs service zone and the extending segment of trunk bus lines that extend from the transfer hub to the inside of the transfer hubs service zone.

Accessorial routes are regarded as the complement of primary routes and take on the service function of secondary transit corridor between transfer hubs service zones. Accessorial routes connect the travelers' concentration districts and offer the nonstop transit service for the neighboring transfer hubs service zones. Accessorial routes comprise of trunk bus lines.

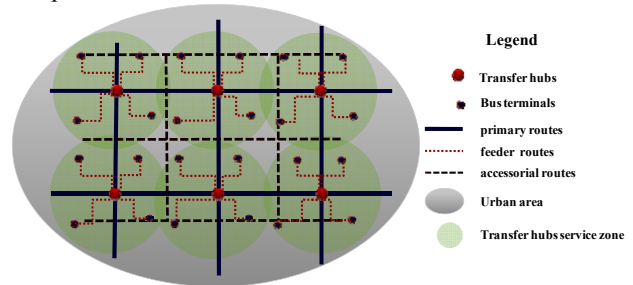


Figure 1. Ideal urban public transit network structure.

Figure 2 shows an urban public transit corridor that is comprised of multi-class public transit lines. Primary route is comprised of the segments of MRT line and trunk bus line between transfer hub *i* and transfer hub *j*. Feeder routes are comprised of the segments of trunk bus line between bus stop *k* and *i* and between bus stop *l* and *i* and the shuttle bus lines. Accessorial route is comprised of the segments of trunk bus line between bus stop *m* and *n*.

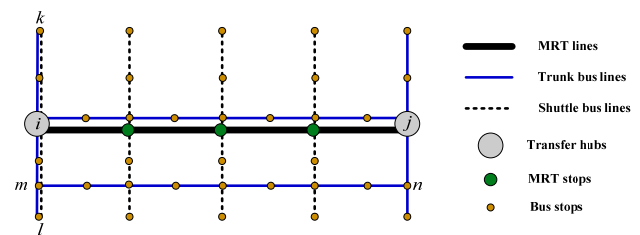


Figure 2. Urban public transit corridor with multi-class lines.

III. KEY DESIGN VARIABLES FOR URBAN PUBLIC TRANSIT NETWORK

Figure 3 is the sketch map of key design variables of urban public transit network, the key design variables are the primary routes spacing  $D_l$ , the MRT stops spacing  $D_s$  and the size of transfer hub service zone  $D_l \times D_l$ .  $D_l$  and  $D_s$  with different values will result in different passengers' travel time, and operational cost.

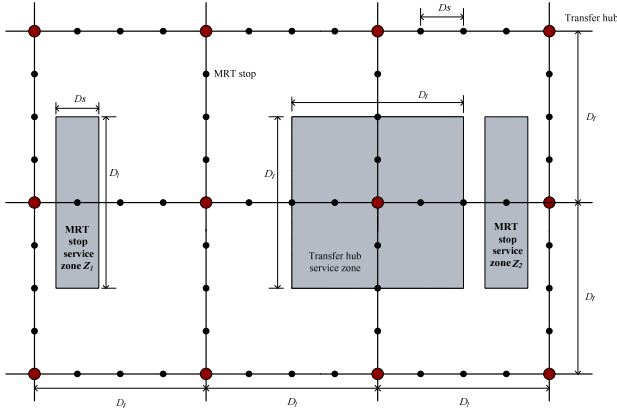


Figure 3. Illustration of key design variables of public transit network structure.

The main dilemma in transit-network design is the controversy between the traveler and the operator. Simply put, the traveler wants to travel at any time and as fast as possible to his or her destination. This requires a network having a high network density and high frequencies. Such a network is clearly too expensive for an operator, who would like to have a network of profitable lines only. Thus, regarding the traveler, the main characteristic of a transit network is total travel time. This travel time can be divided into access time, waiting time, in-vehicle time, transfer time, and egress time. From the point of view of the operator, the main characteristics are the revenues and the operational costs. In practice, there is also a third party involved in transit-network design—the local authorities. They might opt for the traveler, for instance by subsidizing transit, or they might try to find a balance between the traveler's and operator's interests using the concept of social welfare [15].

On the assumption that the trips whose origin and destination is within the same transfer hub service zone are completed by taking feeder routes or accessorial routes; the trips whose origin and destination belong to the adjacent two transfer hub service zones are completed by taking accessorial routes; the trips whose origin and destination are not belong to the adjacent two transfer hub service zones (span-multi-zone trips) are completed by taking the combination of primary routes and other feeder modes, such as feeder routes, walking and bicycle, and the trips need to transfer at the transfer hubs or MRT stops. The trips between MRT stop service zone  $Z_1$  and  $Z_2$  belong to the span-multi-zone trips (Figure 3). The following sections present the optimization model for the key design variables from the points of view of the traveler, operator and authority respectively for the span-multi-zone trips.

A. Traveler's Objective

The total travel time for span-multi-zone trip is divided into access time (from the origin to MRT stops or transfer hubs), waiting time (at the MRT stops or transfer hubs), in-vehicle time (taking MRT lines), egress time (from MRT stops or transfer hubs to the destination) and time lost at a transfer hub for transferring between MRT lines.

1) Access time from origin to MRT stops or transfer hubs

The main trip modes from origin to MRT stops or transfer hubs are walking, bicycle and feeder bus. Access time ( $T_a$ ) can be defined as

$$T_a = T_{aw} \cdot P_{aw} + T_{ab} \cdot P_{ab} + T_{af} \cdot P_{af} \quad (1)$$

where  $T_{aw}$  is the average walking travel time access to MRT stops or transfer hubs,  $T_{ab}$  is the average travel time taking bicycle access to MRT stops or transfer hubs, and  $T_{af}$  is the average travel time taking feeder bus access to MRT stops or transfer hubs,  $P_{aw}$  is the walking mode split proportion,  $P_{ab}$  is the bicycle mode split proportion, and  $P_{af}$  is the feeder bus mode split proportion. The modes split proportion can be described using the logit mode-choice model.

To simplify the problem, on the assumption that the travelers whose origins are within the 400m radius around the transfer hubs or MRT stops arrive to the transfer hubs or MRT stops by walking, and other traveler by bicycle and feeder bus. The mode split proportion of bicycle and feeder bus is determined according to the mode split forecasting for the planning years. The ratio of feeder bus mode to bicycle mode is  $p$ , and

$$\begin{cases} P_{aw} = (\frac{1}{2} \cdot 400 \cdot 400) / (\frac{D_s}{2} \cdot \frac{D_l}{2}) = \frac{80000}{D_s \cdot D_l} \\ P_{ab} = \frac{1 - P_{af}}{p + 1} \\ P_{af} = \frac{p \cdot (1 - P_{af})}{p + 1} \end{cases} \quad (2)$$

The average walking time access to MRT stops or transfer hubs is

$$T_{aw} = D_{aw} / V_w = 2 \times \frac{1}{3} \times 400 / 1 = 267 \text{ (s)} \quad (3)$$

where  $D_{aw}$  is the average walking distance access to MRT stops or transfer hubs, and  $V_w$  is the walking speed, 1.0m/s; The average travel time taking bicycle access to MRT stops or transfer hubs is

$$T_{ab} = 3.6 \times D_{ab} / V_b + T_{tb} \quad (4)$$

where  $D_{ab}$  is the average travel distance of bicycle mode access to MRT stops or transfer hubs, and

$$D_{ab} = \frac{\frac{1}{16} \cdot D_s \cdot D_l^2 + \frac{1}{16} \cdot D_s^2 \cdot D_l - 21280000}{\frac{1}{4} \cdot D_s \cdot D_l - 80000} \quad (5)$$

where  $V_b$  is the average travel speed of bicycle mode, 16km/h, and  $T_{tb}$  is the time lost during bicycle parking at the MRT stops or transfer hubs. The average travel time taking feeder bus access to MRT stops or transfer hubs is

$$T_{af} = 3.6 \times D_{af} / V_f + T_{vf} + T_{tf} \quad (6)$$

where  $D_{af}$  is average travel distance of feeder bus mode access to MRT stops or transfer hubs, and  $D_{af} = D_{ab}$ ,  $V_f$  is the

average operation speed of feeder bus lines,  $T_{wf}$  is the sum of walking time and waiting time from origin to bus stop, and  $T_{tf}$  is the transfer time lost between feeder bus and MRT lines.

2) *Waiting time at the transfer hubs or MRT stops*

Waiting time mainly depends on the frequency of the service; the waiting time can be defined easily by

$$T_w = f_w \cdot \frac{3600}{F} \tag{7}$$

where  $f_w$  is the factor for the waiting time, and  $F$  is the frequency of the MRT lines service.

3) *in-vehicle time of MRT lines*

The in-vehicle time is determined by the average span-multi-zone travel distance, the maximum speed of MRT lines and the time lost at each stop:

$$T_i = \frac{1000 \cdot D}{D_s} \left( \frac{3.6 \cdot D_s}{v} + T_s \right) \tag{8}$$

Where,  $D$  is the average travel distance of span-multi-zone trips,  $v$  is the maximum speed of MRT lines, and  $T_s$  is the time lost at a transfer hub or a MRT line stop.

4) *Egress time from the MRT stop or the transfer hub to the destination*

To each trip mode, the egress time from the MRT stop or the transfer hub to the destination is equal to the access time from origin to the MRT stop or the transfer hub:

$$\begin{cases} T_{ew} = T_{aw} \\ T_{eb} = T_{ab} \\ T_{ef} = T_{af} \end{cases} \tag{9}$$

where  $T_{ew}$ ,  $T_{eb}$  and  $T_{ef}$  are the average walking time, the average bicycle travel time and the average feeder bus travel time from the MRT stop or the transfer hub to the destination respectively.

The average egress time from the MRT stop or the transfer hub to the destination can be defined as

$$T_e = T_{ew} \cdot P_{ew} + T_{eb} \cdot P_{eb} + T_{ef} \cdot P_{ef} \tag{10}$$

where  $P_{ew}$ ,  $P_{eb}$  and  $P_{ef}$  are the mode split proportion of walking, the bicycle and the feeder bus from the MRT stop or the transfer hub to the destination respectively.

5) *Transfer time lost between MRT lines*

Some travelers need to transfer at a transfer hub between two MRT lines. On the assumption that the maximum transfer times for one trip is one time, the average transfer time lost can be defined as

$$T_t = p_t \cdot t_t \tag{11}$$

where  $t_t$  is the transfer time lost at a transfer hub,  $p_t$  is the ratio of the travelers who need to transfer to all travelers.

The total subjective travel time is defined:

$$T_c = (w_a \cdot T_a + w_w \cdot T_w + T_t + w_e \cdot T_e + w_t \cdot T_t) \tag{12}$$

where  $T_c$  is the total weighted travel time, and  $w_x$  is the weight for time element  $x$ ,  $w_k$  is the weight for time element  $k$ .

From the point of view of traveler, the objective is minimizing weighted travel time:

$$\min(w_a \cdot T_a + w_w \cdot T_w + T_t + w_e \cdot T_e + w_t \cdot T_t) \tag{13}$$

B. *Operator's Objective*

The operational costs in terms of costs per unit area served are determined by the vehicle density, which might be assumed to be fixed or which might depend on the stop spacing and line spacing [15]. Operators want to maximize the operational efficiency that is the income per cost. The objective function for the operational efficiency maximum can be formulated:

$$\max \left\{ \frac{(r_t + r_s) \cdot P}{c_o \cdot \frac{1000 \cdot 1000}{D_s \cdot D_l} \cdot \frac{D_s}{1000}} \right\} = \max \left\{ \frac{(r_t + r_s) \cdot P}{c_o \cdot \frac{1000}{D_l}} \right\} \tag{14}$$

where  $r_t$  is the fare paid by the traveler,  $r_s$  is the subsidy paid by the authorities per traveler,  $P$  is the number of travelers per km<sup>2</sup>, and  $c_o$  is the operational cost per unit length of MRT lines.

On the other hand, operators want to maximize the operational profit that is equal to the income minus the operational cost. The objective function for the operational profit maximum can be formulated:

$$\max \left\{ (r_t + r_s) \cdot P - c_o \cdot \frac{1000}{D_l} \right\} \tag{15}$$

Chinese government doesn't constitute a rational subsidy system, and the standard of subsidy is very different to the different cities. This paper presents the objective function of operators is to minimize the operational cost, which can be formulated:

$$\min \left\{ c_o \cdot \frac{1000}{D_l} \right\} \tag{16}$$

C. *Authority's Objective*

Local authorities might try to find a balance between the traveler's and the operator's interests. One way to do so is to determine the net benefit of the transit system. From an economic point of view, the concept of maximizing social welfare, defined as the sum of consumer surplus and producer surplus, would be best. The formulation for the consumer surplus, however, is rather complicated [15]. Therefore, a somewhat simpler approach is chosen. The objective function of authorities is to minimize the cost associated with traveling can be formulated:

$$\min \left\{ \frac{c_t \cdot T_c \cdot P}{3600} + c_o \cdot \frac{1000}{D_l} \right\} \tag{17}$$

where  $c_t$  is the value of time for trips by public transit.

D. *Numerical Analysis*

Your goal is to simulate the usual appearance of papers in a Journal of the Academy Publisher. We are requesting that you follow these guidelines as closely as possible.

The statistics is finished for the cities in China with over 3,000,000 populations, and the density of population is between 11,000 and 14,000 per km<sup>2</sup>, and the travelers travel 2.0~2.4 times per day. The value of time for trips by public transit is 6~15 Chinese Yuan per hour. An assumption was made to reduce complexities: the density of trip distribution is homogenous.

The input parameters values (shown in Table II) are used for the numerical analysis.

TABLE II. INPUT PARAMETER VALUES FOR THE NUMERICAL ANALYSIS

parameter	symbol	values	unit
Span-multi-zone travel distance	$D$	6, 8, 10	km
Walking time access to a bus stop and waiting time	$T_{wf}$	180	s
Time lost during bicycle parking at a MRT stop of a transfer hub	$T_{tb}$	180	s
Transfer time lost between feeder bus and MRT lines	$T_{tf}$	180	s
Walking speed	$V_p$	1	m/s
Bicycle speed	$V_b$	16	km/h
Feeder bus operational speed	$V_f$	15	km/h
Maximum operational speed of MRT lines	$v$	40	km/h
Service frequency of MRT lines	$F$	15	vehicles/h
Factor for the waiting time	$f_w$	0.5	
Time lost at a MRT stop	$T_s$	30	s
Ratio of feeder bus mode to bicycle mode	$p$	7	
Ratio of the travelers who need to transfer to all travelers	$p_t$	0.5	
Transfer time lost at a transfer hub	$t_t$	270	s
Number of travelers per unit area	$P$	2160	travelers/km <sup>2</sup>
Value of time for trips by public transit	$c_t$	15	Chinese Yuan/h
Operational cost per unit length of MRT lines	$c_o$	30000	Chinese Yuan /km
Weight for the segment of travel time	$W_a, W_w, W_i, W_e, W_l$	1.0	

When the span-multi-zone travel distance is 6km and MRT stops spacing is 1,200m, the relationship between the objective function values and the primary routes spacing is illustrated in Figure 4, which is similar to the one when the MRT stops spacing is 2,000m(Figure 5). As the primary routes spacing increase, the total weighted travel time increase linearly, while the operational cost and the cost associated with traveling per square kilometer decrease rapidly when the primary routes spacing is less than 2,000m, and the two objective function values are basically changeless when the primary routes spacing is more than 3,600m. When the span-multi-zone travel distance is 8km or 10km, the relationship between the objective function values and the primary routes spacing is similar to the one that the span-multi-zone travel distance is 6km. From the point of view of travelers, they hope the less primary routes spacing, but the government or investor can't support the large-scale MRT lines

construction. From the point of view of operator and authority, the cost should be limited within an acceptable range. So, the author thinks that the acceptable satisfactory solution for the primary routes spacing is between 2,000m and 3,600m.

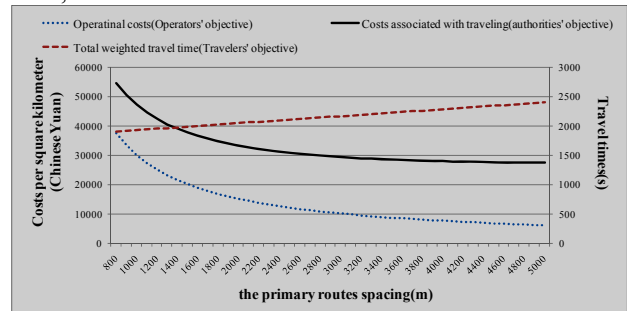


Figure 4. Relationship between the objective function values and the primary routes spacing (average travel distance 6km and MRT stop spacing 1200m).

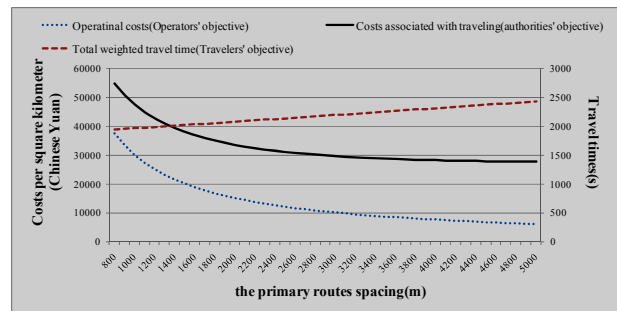


Figure 5. Relationship between the objective function values and the primary routes spacing (average travel distance 6km and MRT stop spacing 2000m).

When the span-multi-zone travel distance is 6km and the primary routes spacing is 2,500m, the relationship between the objective function values and the MRT stops spacing is illustrated in Figure 6. There is an optimum for the total weighted travel time and the cost associated with traveling in the case that the MRT stops spacing equals 1,250m, and when the primary routes spacing is 3,500m, there is an optimum in the case that the MRT stops spacing equals 1,300m(Figure 7).

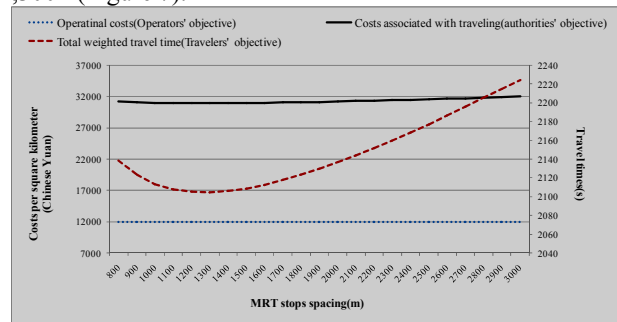


Figure 6. Relationship between the objective function values and the MRT stop spacing (average travel distance 6km and primary routes spacing 2,500m).



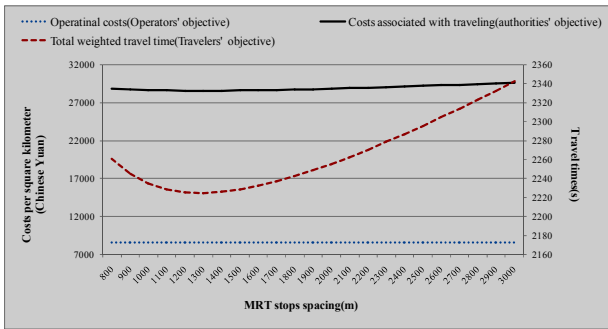


Figure 7. Relationship between the objective function values and the MRT stop spacing (average travel distance 6km and primary routes spacing 3,500m).

When the primary routes spacing equals 2,500m and 3,500m, the relationship between the optimal MRT stops spacing and average travel distance is illustrated in Figure 8. As the average travel distance increase, the optimal MRT stops increase linearly.

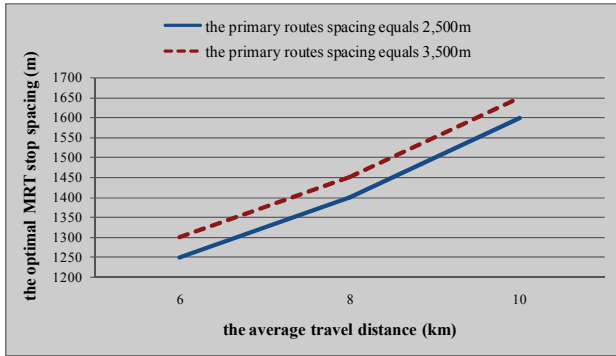


Figure 8. Relationship between the average travel distance and the optimal MRT stops spacing.

In order to illustrate the sensitivity to the assumptions, the density of population has been considered varying  $\pm 15\%$ , the result is invariable for the optimal MRT stops spacing. So, the distribution of population has no direct effect on the optimal MRT stops spacing, and the average travel distance is the key factor for determine the optimal MRT stops spacing. This paper presents the recommended values for the key design variables according to the different average travel distance (shown in table III).

TABLE III. RECOMMENDED VALUES FOR THE KEY DESIGN VARIABLES

Span-multi-zone travel distance (km)	Primary routes spacing(m)	MRT stops spacing(m)	Size of transfer hub service zone (m×m)
≤4	2000	1000	2000×2000
6	2500	1250	2500×2500
8	3000	1425	3000×3000
10	3500	1600	3500×3500

IV. CONCLUSION

The author presents the urban public transit network structure. From the points of view of the traveler, operator and authority respectively, the optimization objective functions for the key design variables are formulated, and the numerical analysis is carried out to find the optimal solution for the key design parameters. For the given input parameters values, the recommended values of the key design variables are put forward.

The methodology proposed here can be implemented in designing a multi-class urban public transit network structure. The result of the key design variables can provide the foundation for the following routes planning and transfer hub layout. The transfer hub service zones are clearly marked off on the urban area, and only one transfer hub is located within every transfer hub service zones. Transfer hubs are regarded as the key nodes for organizing the multi-class urban public transit network, the different class public transit lines should be located according to the travel demand, which will be the following research direction. Furthermore, the authors think that several key issues should be discussed in the future research. Some recommendations can be stated:

- (1)The public transit network structure is designed more suitable for a grid road network form, and further study is necessary to design a public transit network structure for the radial, triangular, or circular road network form.
- (2)The recommended results for the key design variables are concluded basing on the given input parameters values. For other cases, the optimal model and the analytical process can be applied to design an urban transit network structure, and the input parameters values are different from the ones that this paper refers to.
- (3)The location method for the railway lines, BRT lines, and bus lines need to be discussed basing on the proposed urban public transit network structure.

ACKNOWLEDGMENT

This work was supported in part by a grant from Natural Science Foundation of Hebei Province (No.E2014402003) and Hebei University of Engineering Youth Academic support project. We also thank the anonymous reviewers for their helpful feedback and suggestions.

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