Analysis of Elastic Lateral-Resistant Stiffness of Steel Plate Shear Wall

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***Abstract*—The main function of the steel plate slotted in the steel plate shear wall is resisting horizontal forces, Therefore, estimating its lateral-resistant stiffness plays a very important role in analyzing the reaction of the whole structure under horizontal forces. This paper analyzes the research on the elastic lateral-resistant stiffness of steel plate shear wall. Firstly, basing on analyzing the interaction of the slotted-in steel plates and the boundary frame, the theoretic expression of the lateral-resistant stiffness of steel plate Shear wall is derivated. Next, some finite element models of samples are given by using Abacus. Finally, The results of analysis based on Abaqus and the theoretical were compared, it shows that the theoretic expression of the lateral-resistant stiffness of steel plate Shear wall is accurate and reliable.**

Keywords-Steel plate shear wall; Lateral-resistant stiffness; Finite element analysis

I. INTRODUCTION

Steel plate shear wall can be classified into stiffened steel plate shear wall and unstiffened steel plate shear wall based on whether stiffened or not; and it can be classified into combined steel plate shear wall and buckling-restrained steel plate shear wall based on the method combining embedded steel plate with the concrete slabs on both sides. Unstiffened steel plate shear wall is composed of embedded steel plate and edge frame; while as for buckling-restrained steel plate shear wall, there are concrete slabs equipped on both sides of steel plate to prevent embedded steel plate buckling. Unstiffened steel plate shear wall easily buckles under the action of horizontal loads, but it still could provide stiffness and shear capacity after buckling; so during design, steel plate's post-buckling properties can be used, so as to adequately play steel plate's role during design. At present, several researchers have carried out studies regarding unstiffened steel plate shear wall's post-buckling properties, and Canada is the first country that uses unstiffened thin steel plate shear wall [1]. Thorburn [2] first put forward the concept using unstiffened steel plate shear wall's post-buckling property in 1983. In consideration of the interaction between embedded steel plate and edge frame, Sabouri [3] put forward frame-steel plate interaction model. Tromposch and Kulak [4] obtained unstiffened thin steel plate shear wall's hysteretic curve by low frequency cyclic loading test. Cem [5] studied steel plate shear wall's elastically lateral stiffness, and gave two lateral stiffness calculation methods. Kharrazi [6] studied steel plate shear wall's analysis and design methods, and obtained steel plate shear wall's elastically lateral stiffness according to its shear force-displacement relation curve. Zhouming [7] put forward unstiffened steel plate shear wall's unified equivalent model, and studied shear proportionality coefficient and gave specific expression by viewing embedded steel plate's mechanical characteristics as a superposition of "only shearing filed" and "only tension field". Guo Yanlin, Zhouming and Dong Quanli [3] gave buckling-restrained steel plate shear wall's shear capacity formula based on numerical analysis of finite element; provided its elastically lateral stiffness expression based on theoretical analysis, and analyzed and compared it with finite element result. However, steel plate shear wall is not widely used in our country, and its design method and design theory need further improvement. As buckling-restrained steel plate shear wall's mechanical properties have been proved to approximate to that of unstiffened thick steel plate shear wall, in this paper, buckling-restrained steel plate shear wall and unstiffened steel plate shear wall are classified into one category and taken as the study object. [7-8] Steel plate shear wall's elastically lateral stiffness is studied in this paper. First, steel plate shear wall's theoretical lateral stiffness expression is deduced based on analyzing the interaction between embedded plate and edge frame. Then, several calculation cases' finite element model is established using ABAQUS software; and at last, finite element analysis results are compared with theoretical results.

II. THEORETICAL ANALYSIS OF STEEL PLATE SHEAR WALL'S ELASTICALLY LATERAL STIFFNESS

Unstiffened steel plate shear wall can be classified into three categories according to common height-thickness ratio. That is:

$$\lambda_n = \sqrt{\tau_y / \tau_{cr}} \in \begin{cases} (-\infty, 0.8] Thick\ plate \\ (0.8, 1.2] Moderate-thick\ plate \\ (1.2, +\infty) Thin\ plate \end{cases}$$

Where: λ_n is common height-thickness ratio, τ_y is embedded steel plate's shear yield stress, and τ_{cr} is embedded steel plate's critical yield stress.

Analysis of thin plate's lateral stiffness.

Extremely thin steel plates get shear buckling very easily under the action of horizontal shear, steel plate's force can be seen as a "only tension field", and effect of steel plate on outside frame's force is as shown in Figure 1.

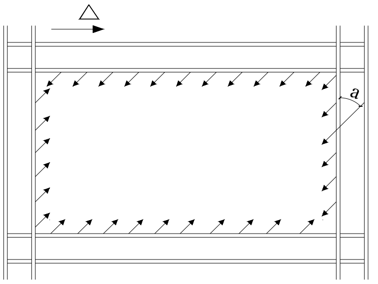

Figure 1. The force of edge frame

Compared with general frame columns, there is additional inter-column load for edge columns' force. As force of the tension zone formed by tension fields is uneven, the inter-column load of edge columns is not even load. Now let the inter-column load become equivalent to uniformly distributed load by introducing edge columns' uneven loading influence coefficient α_c, then edge columns' simplified force figure is shown in Figure 2. It is worth noting that the pre-assumed structure's force in this paper is rightward, thus, in design, any edge column should be calculated for bearing capacity as right column and left column, respectively. When determining the overall stiffness, calculation should be done with one edge column of steel plate shear wall taken as left column and the other as right column.

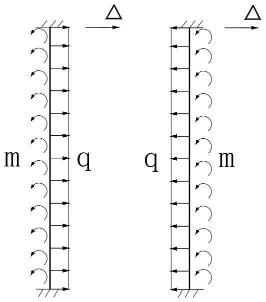

Figure 2. The force of edge pillar

Where:

$$q = \alpha_c \frac{Et_w \sin^2 \alpha \cos \alpha}{h} \Delta$$

$$m = \alpha_c \frac{Et_w h_c \sin \alpha \cos^2 \alpha}{2h} \Delta$$

E is steel plate's elastic modules, t_w is steel plate's thickness, α is tension zone's dip angle, and 45° can be taken for it, h is frame column layers' height, h_c is the height of outside frame column section, and Δ is column top's side displacement

The relation between column bottom's shear force Fleft, Fright and column top's displacement Δ can be obtained as follows by force method:

$$F_{\text{左}} = \frac{12EI_c}{(1+\beta)h^2}\Delta + \frac{\alpha_c Et_w \sin\alpha\cos\alpha}{2h}\left(h\sin\alpha + \frac{h_c\cos\alpha}{1+\beta}\right)\Delta$$

$$F_{\text{右}} = \frac{12EI_c}{(1+\beta)h^2}\Delta + \frac{\alpha_c Et_w \sin\alpha\cos\alpha}{2h}\left(\frac{h_c\cos\alpha}{1+\beta} - h\sin\alpha\right)\Delta$$

Where:

$$\beta = \frac{12uEI_c}{GA_c h^2}$$

β is edge column's shear deformation influence coefficient, if it is not necessary to take its shear deformation into consideration, β can be set at 0; u is section shape factor, its value is 1.2 and 10/9 for rectangular section and circular section, respectively. I_c is edge column's section inertia moment.

According to the concept of lateral stiffness, left and right edge column's lateral stiffness can be obtained:

$$k_{\text{左}} = \frac{12EI_c}{(1+\beta)h^3} + \frac{\alpha_c Et_w \sin\alpha\cos\alpha}{2h}\left(h\sin\alpha + \frac{h_c\cos\alpha}{1+\beta}\right)$$

$$k_{\text{右}} = \frac{12EI_c}{(1+\beta)h^3} + \frac{\alpha_c Et_w \sin\alpha\cos\alpha}{2h}\left(\frac{h_c\cos\alpha}{1+\beta} - h\sin\alpha\right)$$

For steel plate, steel plate tension filed effect is obvious, force of the tension zone formed by tension fields is uneven. Embedded steel plate's lateral stiffness can be obtained as follows after steel plate's uneven loading influence coefficient α_w is introduced:

$$k_w = \alpha_w \frac{Et_w b}{4h}$$

In this paper, it is recommended that both edge column and steel plate's uneven loading influence coefficient are 0.6.

Therefore, thin steel plate shear wall's overall lateral stiffness is:

$$k_{total} = k_{left} + k_{right} + k_w = \frac{24EI_c}{(1+\beta)h^3} + \frac{\alpha_c Et_w \sin\alpha\cos^2\alpha}{2h(1+\beta)} + \alpha_w \frac{Et_w b}{4h}$$

Analysis of thick plate's lateral stiffness.

Under the action of horizontal shear, thick steel plate's shear yield is antecedent to overall yield, steel plate presents a state that the plane is loaded with shear stress evenly; only additional eccentric force is applied by steel plate to edge columns, and the effect of steel plate on outside frame's force is shown in Figure 3. Ignore the influence generated by edge columns' axial deformation, then edge columns' simplified force figure is given in Figure.4.

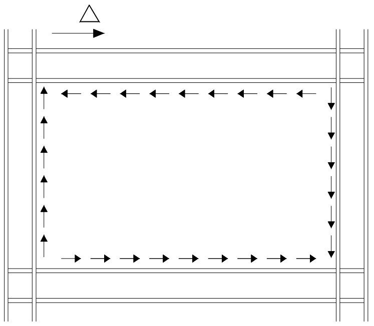

Figure 3. the force of edge frame

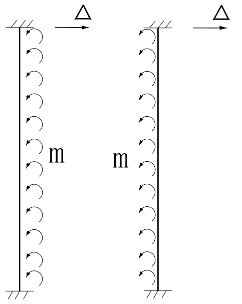

Figure 4. the force of edge pillar

Where:

$$m = \frac{Gt_w h_c}{2.4h} \Delta$$

G is steel plate's shear modulus, t_w is steel plate's actual thickness, h_c is the height of outside frame column section, Δ is column top's side displacement.

The relation between column bottom's shear force Fleft, Fright and column top's displacement Δ can be obtained by force method:

$$F_{left} = F_{rigth} = \frac{12EI_c}{(1+\beta)h^3}\Delta + \frac{5Gt_w h_c}{12(1+\beta)h}\Delta$$

According to the concept of lateral stiffness, left and right edge column's lateral stiffness can be obtained:

$$k_{left} = k_{rigth} = \frac{12EI_c}{(1+\beta)h^3} + \frac{5Gt_w h_c}{12(1+\beta)h}$$

It is found by analysis that steel plate's tension field effect disappears, and steel plate is sheared evenly. Reference [7] gives embedded plate's lateral stiffness as follows:

$$k_w = \frac{Gt_w b}{1.2h}$$

The above formula is actually steel plate's shear lateral stiffness; however, in steel plate shear wall, embedded steel plate acts together with edge frame, steel plate shear wall's deformation is divided into edge frame's bending deformation and steel plate's shear deformation. While in general, set overall inter-layer side displacement as an unknown quantity, then steel plate's lateral stiffness is:

$$k_w = \chi \frac{Gt_w b}{1.2h}$$

Where, χ is steel plate shear wall's shear deformation ratio. Regarding χ 's value, deduction process is as follows:

Steel plate shear wall's deformation:

$$\Delta = \Delta_f + \Delta_s$$

Where, Δ is steel plate shear wall's overall inter-layer side displacement, Δ_f is inter-layer bending deformation, and Δ_s is inter-layer shear deformation. Inter-layer bending deformation:

$$\begin{aligned}\Delta_f &= \frac{2h}{b+h_c}\int_0^h \frac{N_c + Gt_w z/(1.2h)\Delta_s}{EA_c}dz \\ &= \frac{2N_c h^2}{EA_c(b+h_c)} + \frac{Gt_w h^2}{1.2EA_c(b+h_c)}\Delta_s\end{aligned}$$

Where, Nc is the axial force generated under action of the horizontal force at top of edge column, Ac is edge column's sectional area, and rest parameters are the same as above.

The axial force at top of outside frame column is:

$$N_c = \frac{D_c h}{b+h_c}\Delta$$

Where:

$$D_c = \frac{12EI_c}{(1+\beta)h^3}$$

So:

$$\chi = \frac{\Delta_s}{\Delta} = \frac{1-B}{1+A}$$

Where, parameter A and B are as follows:

$$A = \frac{Gt_w h^2}{1.2EA_c(b+h_c)}$$

$$B = \frac{2D_c h^3}{EA_c(b+h_c)^2}$$

Therefore, thick steel plate shear wall's overall lateral stiffness is:

$$\begin{aligned}k_{total} &= k_{left} + k_{right} + k_w \\ &= \frac{24EI_c}{(1+\beta)h^3} + \frac{5Gt_w h_c}{6h(1+\beta)} + \chi\frac{Gt_w b}{1.2h}\end{aligned}$$

Analysis of moderate-thick plate's lateral stiffness.

As for moderate-thick plate, its force can be seen as a superposition of "only shear" and "only tension field" by a certain ratio. That is, moderate-thick plate's force= η ×"only shear field"+ (1- η)×"only tension field, where, η is "only shear field" participation factor. Zhouming gave shear proportionality coefficient's expression:

$$\eta = \begin{cases} 1.0, & \lambda_n \le 0.8 \\ 1 - 0.88(\lambda_n - 0.8), & 0.8 < \lambda_n \le 1.2 \\ 0.94/\lambda_n^2, & \lambda_n > 1.2 \end{cases}$$

With the edge column and embedded steel plate's force characteristics under both "only field" and "only tension filed" mechanism that have been deduced in this paper and the "only shear filed" participation factor, moderate-thick steel plate shear wall's force characteristics can be easily obtained.

The relation between column bottom's shear force Fleft, Fright and column top's displacement Δ can be obtained by force method:

$$F_{左} = [\frac{12EI_c}{(1+\beta)h^2} + \frac{(1-\eta)\alpha_c E t_w \sin\alpha\cos\alpha}{2h}(h\sin\alpha + \frac{h_c\cos\alpha}{1+\beta}) + \frac{5\eta G t_w h_c}{12(1+\beta)h}]\Delta$$

$$F_{右} = [\frac{12EI_c}{(1+\beta)h^2} + \frac{(1-\eta)\alpha_c E t_w \sin\alpha\cos\alpha}{2h}(-h\sin\alpha + \frac{h_c\cos\alpha}{1+\beta}) + \frac{5\eta G t_w h_c}{12(1+\beta)h}]\Delta$$

According to the concept of lateral stiffness, its lateral stiffness can be obtained as follows:

$$k_{左} = \frac{12EI_c}{(1+\beta)h^2} + \frac{(1-\eta)\alpha_c E t_w \sin\alpha\cos\alpha}{2h}(h\sin\alpha + \frac{h_c\cos\alpha}{1+\beta}) + \frac{5\eta G t_w h_c}{12(1+\beta)h}$$

$$k_{右} = \frac{12EI_c}{(1+\beta)h^2} + \frac{(1-\eta)\alpha_c E t_w \sin\alpha\cos\alpha}{2h}(-h\sin\alpha + \frac{h_c\cos\alpha}{1+\beta}) + \frac{5\eta G t_w h_c}{12(1+\beta)h}$$

As for steel plate, its force can be seen as a superposition of "only shear" and "only tension filed" by a certain ratio, so embedded steel plate's lateral stiffness can be obtained as follows:

$$k_w = (1-\eta)\alpha_w \frac{E t_w b}{4h} + \eta\chi \frac{G t_w b}{1.2h}$$

Therefore, moderate-thick steel plate shear wall's overall lateral stiffness is:

$$\begin{aligned} k_{total} &= k_{left} + k_{right} + k_w \\ &= \frac{24EI_c}{(1+\beta)h^2} + \frac{(1-\eta)\alpha_c E t_w h_c \sin\alpha\cos^2\alpha}{h(1+\beta)} + \\ &\quad \frac{5\eta G t_w h_c}{6(1+\beta)h} + (1-\eta)\alpha_w \frac{E t_w b}{4h} + \eta\chi \frac{G t_w b}{1.2h} \end{aligned}$$

Analysis of buckling-restrained steel plate shear wall's lateral stiffness.

As for buckling-restrained steel plate shear wall, since there are concrete slabs on both sides of embedded steel plate and there concrete slabs could bind steel plates, through reasonably designing the concrete slabs on both sides, it can be ensured that steel plate's yield lags behind steel plate's shear yield, then its force characteristics also can be seen as "only shear", like thick steel plate shear wall. So they won't be covered again here.

III. FINITE ELEMENT ANALYSIS

The theoretical expression of steel plate shear wall's overall lateral stiffness is deduced above. In order to verify the deduced results' correctness and reliability, use general finite element software ABAQUS to analyze results and compare them with theoretical results to verify their accuracy.

A. Finite Element Model

Since moderate-thick plate's force mechanism can be seen as a superposition of "only shear filed" and "only tension filed", reference [9] has verified its correctness and gave the shear proportionality coefficient, and also because buckling-restrained steel plate shear wall's force mechanism is the same as that of thick plate, then it is only necessary to verify theoretical formula's correctness under "only tension filed (thin plate)" and "only shear field (thick plate)".

Take the model provided in reference [10] as the basic model, and change other parameters to form comparison models. Basic model (model 1-1)'s model parameters are as follows: frame column's size is 420 mm×350 mm×20 mm×15 mm; as edge beam is bound by upper and lower steel plates, its stiffness can be seen as infinity; beam's section is 500 mm×500 mm; steel plate size is 2700 mm×1800 and 4500 mm×1800 mm; steel's constitutive model is an ideal intensification model, with yield strength, elastic modulus Es and intensification modulus of 340 MPa, 2.06×105 MPa and 0.01Es, respectively. Thin plate's model parameters are given in TableI, and thick plate's model parameters are given in Table II. Establish 12 models using ABAQUS finite element analysis software, respectively. Outside frame uses entity element, adopt equivalent pull rod model for thin steel plate, that is, truss element is used; adopt shell element for thick steel plate. Embedded steel plate is connected with edge frames by means of shell-entity coupling.

TABLE I THE MODEL PARAMETERS OF THIN STEEL PLATE SHEAR WALLS

Model	1-1	1-2	1-3	1-4	1-5	1-6
Size/mm	2700×1800	1800×1200	2700×1800	4500×1800	3000×1200	4500×1800
ratio of height to thickness	400	400	350	400	400	350
span-depth ratio	1.5	1.5	1.5	2.5	2.5	2.5

TABLE II THE MODEL PARAMETERS OF THICK STEEL PLATE SHEAR WALLS

Model	2-1	2-2	2-3	2-4	2-5	2-6
Size/mm	2700×1800	1800×1200	2700×1800	4500×1800	3000×1200	4500×1800
ratio of height to thickness	50	50	80	50	50	80
span-depth ratio	1.5	1.5	1.5	2.5	2.5	2.5

B. *Comparison Between Theoretical Results and Finite Element Analysis Results*

Compare the theoretical results deduced in this paper with finite element analysis results. Comparison results of lateral stiffness of thin steel plate shear wall's edge column and embedded steel plate's lateral stiffness are given in Tale III, Table IV and Table V, respectively. And comparison results of lateral stiffness of thick steel plate shear wall's edge column and embedded steel plate's lateral stiffness are shown in Table VI and Table V,respectively.

TABLE III THE LEFT EDGE PILLAR'S LATERAL-RESISTANT STIFFNESS OF THIN STEEL PLATE SHEAR WALL

Model	1-1	1-2	1-3	1-4	1-5	1-6
The formula of this paper	177.5	297.2	184.4	177.5	297.2	184.4
Finite element results	177.1	273.6	181.8	180.8	316.1	190.2
Differ /%	0.3	8.6	1.4	1.8	5.9	3.0

TABLE IVTHE RIGHT EDGE PILLAR'S LATERAL-RESISTANT STIFFNESS OF THIN STEEL PLATE SHEAR WALL

Model	1-1	1-2	1-3	1-4	1-5	1-6
The formula of this paper	91.4	239.8	86.0	91.4	239.8	86.0
Finite element results	84.5	220.3	83.0	90.8	221.0	82.5
Differ /%	8.2	8.8	3.7	0.7	8.5	4.3

TABLE VTHE SLOTTED-IN STEEL PLATE'S LATERAL-RESISTANT STIFFNESS OF THIN STEEL PLATE SHEAR WALL

Model	1-1	1-2	1-3	1-4	1-5	1-6
The formula of this paper	20.8	13.9	23.8	34.7	23.2	39.5
Finite element results	21.5	15.3	22.7	32.2	25.6	36.4
Differ /%	3.3	9.2	4.8	7.8	9.4	8.5

TABLE VITHE EDGE PILLAR'S LATERAL-RESISTANT STIFFNESS OF THICK STEEL PLATE SHEAR WALL

Model	2-1	2-2	2-3	2-4	2-5	2-6
The formula of this paper	263.7	347.1	213.3	263.7	347.1	213.3
Finite element results	270.5	349.8	235.8	256.1	352.9	236.2
Differ /%	2.5	0.8	9.5	3.0	1.6	9.7

TABLE VIITHE SLOTTED-IN STEEL PLATE'S LATERAL-RESISTANT STIFFNESS OF THICK STEEL PLATE SHEAR WALL

Model	2-1	2-2	2-3	2-4	2-5	2-6
The formula of this paper	21.4	18.0	15.6	42.2	33.2	29.3
Finite element results	23.7	18.5	17.1	40.5	30.8	29.0
Differ /%	9.7	2.7	8.8	4.2	7.8	1.0

Results in Table III~Table VII show that there are certain errors between finite element analysis results and theoretical results in this paper, but they are very little and are all within 10%, which indicates that the lateral stiffness calculation formula of edge column and embedded steel plate of thin plate and thick steel plate shear wall deduced in this paper is correct and reliable.

IV. CONCLUSION

(1) In this paper, steel plate shear wall's force characteristics and the interaction between edge frame and

embedded steel are analyzed, and on this basis, the theoretical expressions of lateral stiffness of thin steel plate and thick steel plate shear wall's edge column and embedded steel are deduced, respectively, so that the theoretical expression of overall lateral stiffness of steel plate shear is obtained.

(2) A finite element model for steel plate shear wall is established, and the theoretical expression of elastically lateral stiffness of steel plate shear is verified to be correct by comparing the results analyzed by that finite element model with theoretically deduced results.

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