Inverse Kinematics of Multi-Joint Robot based on Improved Firefly Algorithm

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***Abstract* — The optimization of the inverse kinematics of robots is the most important part of robot kinematics. Parameters for each link of the robot need to be identified according to the position and orientation in both of teaching programming and interpolation. In view of the fact that to get the analytic solutions of the inverse kinematics of robots must meet special conditions, this paper used a general solution, improved firefly algorithm, to get the optimization of inverse kinematics without any of special conditions. Simulation results show that, the algorithm can be effectively applied to the robot inverse kinematics, and is able to achieve sufficient accuracy.**

Keywords - Kinematics algorithms optimization

I. INTRODUCTION

There are two aspects of kinematics of multi-joint robot, forward kinematics and inverse kinematics. The forward kinematics is to obtain the robot pose based on the robot joint variables, while the inverse kinematics is to obtain the robot joint variables based on the robot pose. [1] The forward kinematics solution can be obtained by the pose matrix, and the solution is analytical, determined, and unique. The best method for inverse kinematics is to obtain the analytical solution of joint variables according to the inverse kinematics equation, but the analytical solution can only be obtained in the special conditions [2], otherwise, the optimal solution can just be obtained by various optimization algorithms.

Since 1980s, many bionic optimization algorithms developed rapidly, such as artificial neural network (ANN), genetic algorithm (GA), ant colony (ACO), simulated annealing (SA), firefly algorithm (FA). Their common features are developed by simulating or revealing certain phenomena or processes of nature. They are also called intelligent optimization algorithms.

Firefly algorithm (FA) [3, 4] is a heuristic algorithm which is developed in 2008 by Yang Xinshe. The brightness of the firefly is a signal system which will attract other fireflies. Because FA is simple, high efficient and easy to realize, it has caused the attention of scholars, and hundreds of papers about FA have been published. FA has been applied successfully in many fields. However, there are no papers about optimization of robot inverse kinematics based on FA. This paper tries to obtain the optimization of inverse kinematics with FA. Simulation results show that, the algorithm can be applied to the robot inverse kinematics effectively, and is able to achieve sufficient accuracy.

II. D-H PARAMETERS EXPRESSION OF MULTI-JOINT ROBOT

There are several robot pose expressions, such as Euler angle, rotation matrix and four element method, etc.. The rotation matrix is the most common one, and D-H parameters expression is the most famous one of the rotation matrix method. D-H is developed by Denavita and Hartenberg in 1995. [5]

In D-H, the D-H parameters of the link i must be determined first. The 4 parameters are link length a_i, link twist a_i, link offset d_i and joint angle θ_i. Then homogeneous coordinate transformation matrix T_i will be calculated with D-H parameters. T_i is a primary coordinate transformation matrix, which describes the relative translation and rotation between the link coordinate systems. T_i is as follow:

$$T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

For rotating joints, a_i, α_i and d_i are constant, while θ_i is variable. Instead for sliding joints, a_i, α_i and θ_i are constant, while d_i is variable.

For a 6 links robot, the terminal pose matrix T is a relative translation between the hand (the 6th joint) and base coordinate system. T is as follow: [5]

$$T = T_1T_2T_3T_4T_5T_6. \tag{2}$$

Assuming the position vector of the hand coordinate origin in the base coordinate is p, while the direction vectors are n, o and a, T can be described by a 4×4 matrix as follow:

$$T=\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (3)$$

Fig. (1) is the 3D simulation diagram of a low-cost 6 joints robot, and Fig. (2) is a schematic diagram of the robot and the corresponding coordinate systems. The pose matrix T of the robot is shown as Eq. 3. p、n、o and a in this equation are as follow:

$$\begin{aligned}
p_x &= c_1\left[-c_{23}\left(d_4c_4s_5-d_5s_4\right)+s_{23}\left(d_6c_5+d_4\right)+a_1+a_2c_2\right] \\
&-s_1\left[d_6s_4s_5+d_5c_4\right], \\
p_y &= s_1\left[-c_{23}\left(d_4c_4s_5-d_5s_4\right)+s_{23}\left(d_6c_5+d_4\right)+a_1+a_2c_2\right] \\
&+c_1\left[d_6s_4s_5+d_5c_4\right], \\
p_z &= -s_{23}\left(d_4c_4s_5-d_5s_4\right)-c_{23}\left(d_6c_5+d_4\right)+a_2s_2, \\
n_x &= c_1\left[c_{23}\left(c_4c_5c_6-s_4s_6\right)+s_{23}s_5c_6\right]+s_1\left(s_4c_5c_6+c_4s_6\right), \\
n_y &= s_1\left[c_{23}\left(c_4c_5c_6-s_4s_6\right)+s_{23}s_5c_6\right]-c_1\left(s_4c_5c_6+c_4s_6\right), \\
n_z &= s_{23}\left(c_4c_5c_6-s_4s_6\right)-c_{23}s_5c_6, \\
o_x &= -c_1\left[c_{23}\left(c_4c_5s_6+s_4c_6\right)+s_{23}s_5s_6\right]-s_1\left(s_4c_5s_6-c_4c_6\right), \\
o_y &= -s_1\left[c_{23}\left(c_4c_5s_6+s_4c_6\right)+s_{23}s_5s_6\right]+c_1\left(s_4c_5s_6+c_4c_6\right), \\
o_z &= -s_{23}\left(c_4c_5s_6+s_4c_6\right)+c_{23}s_5s_6, \\
a_x &= -c_1\left(c_{23}c_4s_5-s_{23}c_5\right)-s_1s_4s_5, \\
a_y &= -s_1\left(c_{23}c_4s_5-s_{23}c_5\right)+c_1s_4s_5, \\
a_z &= -s_{23}c_4s_5-c_{23}c_5.
\end{aligned} \quad (4)$$

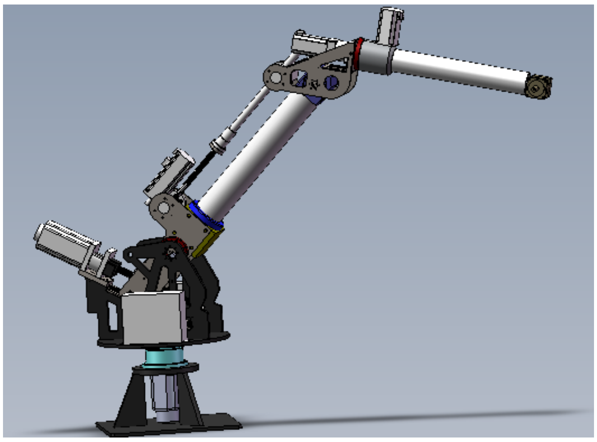

Fig. (1). 6 joints robot 3D simulation diagram

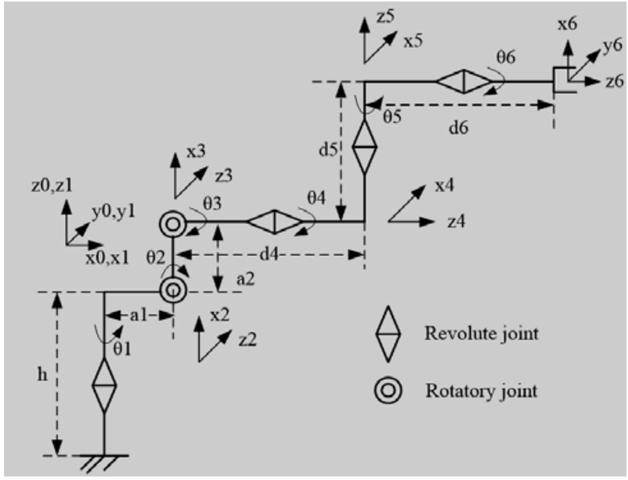

Fig. (2). 6 joints robot schematic diagram

In Eq. 4 $s_i=\sin(\theta_i)$, $c_i=\cos(\theta_i)$, $s_{ij}=\sin(\theta_i+\theta_j)$, $c_{ij}=\cos(\theta_i+\theta_j)$.

According to Eq. (1) and (4), because all the 6 joints of the robot are rotary joints, the variables of D-H parameters are $\theta_i(i=1\sim6)$. If all the θ_i are known, it is easy get T with Eq. (4).

According to Ref. [2], the solution of the inverse kinematics of the 6 DOF robot can be obtained only when one of the two conditions in the Pieper criterions is met. The two Pieper criterions are:

(1) Three adjacent joint axes cross the same point;

(2) Three adjacent joint axes are parallel to each other.

The robot described in this paper can't meet the above two criterions, so the optimal solution can be obtained only by numerical method. This paper will introduce how to obtain the solution of the inverse kinematics based on improved FA.

II. BASIC FIREFLY ALGORITHM

Firefly algorithm is a heuristic algorithm which is developed in 2008 by Yang Xinshe. The algorithm mimics the firefly's behavior to find food by emitting light. Some firefly characteristics are abandoned. For example, the sex of the firefly is ignored. The firefly uses light emission features to search for a nearby area and moves to the position which is better. In the optimizing process, the brightness and attraction of firefly are two key factors for the firefly and position evolution. The position of the firefly depends on its brightness. The higher brightness, the better position. The attraction of the firefly determines the moving direction of the firefly. The more attractive the firefly is, the easier it is to attract the firefly to its own position. The brightness and attraction of the firefly are inversely proportional to the distance between two fireflies, and they decrease with the increase of the distance. The effect of the algorithm is achieved by updating the attraction and brightness constantly. The mathematical description of the firefly algorithm is as follows. [3, 4, 6]

Supposed there are m fireflies and the dimension of search space is N, the initial position of the firefly i is $x_i = \left(x_{i1}, x_{i2}, \cdots, x_{iN}\right)$.

Definition 1. The relative brightness of the firefly i is as follow:

$$I_i = I_{0i} \cdot e^{-\gamma \cdot r_{ij}}. \tag{5}$$

Where γ is the attraction factor of the light intensity, which value is related to the scale area and set constant in generally. r_{ij} is the distance between firefly i and j, where $r_{ij} = \left|x_i - x_j\right|$, The value I_i decreases with the increasing of r_{ij}, when $r_{ij} = 0$, I_i will reach the maximum value I_{0i} which value is generally proportional to the objective function.

Definition 2. The attractiveness is as follow:

$$\beta = \beta_0 \cdot e^{-\gamma \cdot r_{ij}^2}. \tag{6}$$

where β_0 is the attractiveness at r = 0.

Definition 3. The movement of firefly i attracted to another more attractive (brighter) firefly j is determined by:

$$x_i\left(t+1\right) = x_i\left(t\right) + \beta \cdot \left(x_j\left(t\right) - x_i\left(t\right)\right) + \alpha \cdot \left(rand - 0.5\right). \tag{7}$$

where, α is the step size, rand is a random factor that is subject to the uniform distribution [0, 1]. In order to increase the spatial range of the search space, improve the diversity of the population and prevent premature convergence, a random disturbance $\alpha \cdot \left(rand - 0.5\right)$ is brought in.

The step of basic firefly algorithm is as follows: [3, 4, 6, 7]

(1) Initialize the population of the fireflies randomly;

(2) Calculate the brightness of each firefly;

(3) Take comparison of the brightness. If firefly j is brighter than firefly i, firefly i will move to the area of firefly j based on Eq. (7).

(4) Update and save the best firefly position and brightness;

(5) If the stopping criterion is met, then the best global position and brightness are found, otherwise return to step (2).

IV. IMPROVEMENT OF BASIC FIREFLY ALGORITHM

Basic FA is easy to realize, but it exists premature phenomenon. For multi-dimensional and multi-peak procedure, the basic algorithm easily converges to the local optimum. In order to overcome the disadvantage, a lot of literatures are proposed to improve the algorithm [6-10]. This paper will use several improvements below.

(1) Set the exponential declining step factor α

The term $\alpha \cdot \left(rand - 0.5\right)$ is in order to increase the diversity of the population. When α is large, it is advantageous to increase the sample diversity, expand the search scope, and avoid quickly to enter the local optimal value. When the α is small, it is good for fine search. In the algorithm, α should be diminishing. The iteration of α is as follow:

$$\alpha = \alpha \cdot \left(10^{-4} / 0.9\right)^{1/MaxIteration}. \tag{8}$$

where, MaxIteration is the maximum iterations.

When $\alpha_0 = 0.5$, the curve of Eq. (8) is as Fig. (3).

(2) Optimize position update equation

In order to improve the convergence of the algorithm, the position update equation is improved as follow:

$$x_i\left(t+1\right) = x_i\left(t\right) + \omega \cdot rand \cdot \left(x_{best}\left(t\right) - x_i\left(t\right)\right) + \beta \cdot \left(x_j\left(t\right) - x_i\left(t\right)\right) + \alpha \cdot \left(rand - 0.5\right). \tag{9}$$

Comparison with Eq. (7), Eq. (9) has an additional term $\omega \cdot rand \cdot \left(x_{best}\left(t\right) - x_i\left(t\right)\right)$, where $x_{best}\left(t\right)$ is the position of the firefly whose brightness is the best; rand is a random factor that is subject to the uniform distribution [0, 1]; ω is a weight factor, ω has a similar recursive equation with α shown as Eq. (10), where $\omega_0 = 0.9$.

$$\omega = \omega \cdot \left(10^{-4} / 0.9\right)^{1/MaxIteration}. \tag{10}$$

(3) Bring in simulated annealing mechanism

Simulated annealing algorithm (SA) is a random combination optimization method which developed in the early 1980s. It simulates the thermodynamic process of metal temperature cooling, and is widely used in combination optimization problem. The initial temperature is determined by the simulated annealing algorithm in the optimization, randomly select an initial state and investigate the objective function of the state; Attach a small perturbation to the current state and calculate the target function values for the new state; In the whole cooling process, the better state will be accepted in probability 1, while worse state will also be accepted in some probability which is less than 1. If the start temperature is high enough and the temperature falls slowly enough, SA can converge to the global optimum with probability 1. Because of its ability to accept the worse sample in some probability and improve the sample's diversity, it has the ability to jump out of the local optimal solution. [11]

In Eq. (8), in order to improve the diversity of the population, $x_{best}\left(t\right)$ is selected in the follow FA.

$$\left(\min\left(\mathbf{I}\right) - I_{best}\right) < \varepsilon \,\|\, e^{-\left(\frac{\min(\mathbf{I}) - I_{best}}{T}\right)} > rand. \tag{11}$$

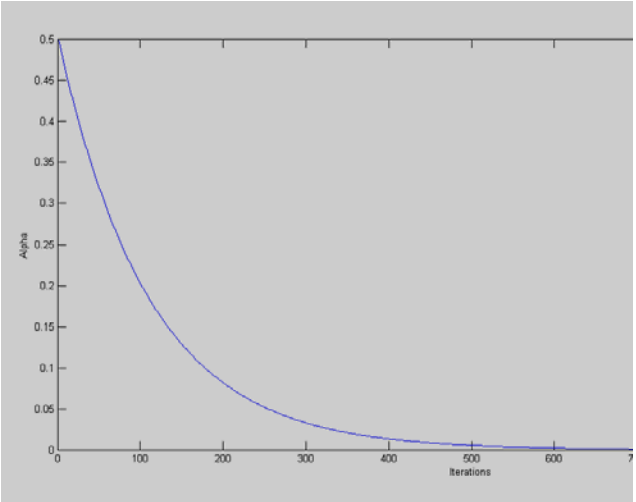

Fig. (3). α curve

Where, the brightness and the target function are positive ratio, the smaller the target function value indicates the higher the brightness. ε is a decimal in [0, 1] which is to improve the diversity of the population. rand is a random factor that is subject to the uniform distribution [0, 1]. T is the annealing temperature.

V. OBTAIN THE SOLUTION OF THE MULTI-JOINT ROBOT INVERSE KINEMATICS BASED ON IMPROVED FA

The solution T of forward kinematic is obtained with Eq. (1) ~ (4). The brightness function of FA is as follow:

$$I_i = \sum_{m=1}^{3}\sum_{n=1}^{4}\left|T_{mn} - T_{imn}\right| \tag{12}$$

where T is the target pose matrix which is the expected pose matrix of the robot in a certain time, T_i is the actual pose matrix of the certain particle, I_i is the deviation of the corresponding elements of the actual and expected pose matrix. Since the value of the last row of the matrix T_i in Eq. (3) is kept constant, last row is not within the range of computation.

The steps of improved FA are as follows[9,10]:

(1) Randomly initialize the position of each firefly, save the expected pose matrix.

(2) Initialize the annealing start temperature T_0 and cooling rate K.

(3) Calculate the brightness I_i with Eq. (12) of each firefly.

(4) Use Eq. (10) to obtain the alternative firefly for the global optimum firefly.

(5) If the brightness of firefly j is better than brightness of firefly i, then firefly i move to the region near firefly j in a manner as Eq. (9).

(6) Calculate brightness again, and save the current best brightness and position.

(7) If the stopping criterion is met, then the best global brightness and position are found, otherwise, return to step (3).

VI. SIMULATION AND VERIFICATION

To verify the feasibility of the algorithm, the inverse kinematic matlab simulation for the 6-DOF robot shown in Fig. (1) and Fig. (2) is given. In order to make the p, n, o and a in the pose matrix T the same magnitude, take the unit of each link parameter as meter. In the simulation, $a_1 = 0.232$, $a_2 = 1.000$, $d_4 = 0.600$, $d_5 = 0.043$, $d_6 = 0.025$. The random expected pose matrix is as follow:

$$T = \begin{bmatrix} 0.612372 & 0.353553 & -0.707106 & 1.372025 \\ 0.500000 & -0.866025 & 0.000000 & 0.021500 \\ 0.612372 & 0.353553 & 0.707106 & 0.326852 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

In the improved FA, the population size is 40, the dimension is 6, iterations is 2000, $a_0 = 0.5$, $\omega_0 = 0.9$, $\varepsilon = 0.01$,annealing start temperature $T_0 = 1000$, temperature cooling rate $K = 0.99$. After 2000 iterations, the actual pose matrix is:

$$T' = \begin{bmatrix} 0.612371 & 0.353553 & -0.707106 & 1.372005 \\ 0.500000 & -0.866024 & 0.000000 & 0.021510 \\ 0.612372 & 0.353553 & 0.707106 & 0.326852 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

In the simulation, the minimum of I_i in Eq. (12) is 0.000032. The brightness curve is shown as Fig. (3).

According to Eq. (13) and (14), the maximum deviation of direction dimension is 0.000001, the maximum deviation of position dimension is 0.000010m=0.010mm. The result shows that the robot pose accuracy can meets the requirements. From Fig. (4) it can be seen that the brightness is quickly close to the global optimum in in the first 700 iterations, and continue to move to the global optimum very slowly after 700 iterations.

The solution T of forward kinematic is obtained with Eq. (1) ~ (4). The brightness function of FA is as follow:

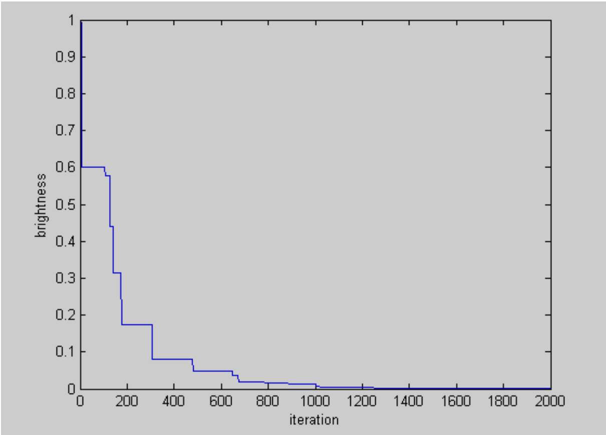

Fig. (4). Brightness curve

VII. CONCLUSION

The forward kinematics and inverse kinematics are two aspects of robot kinematics. Based on D-H, the forward kinematics equation and pose matrix can be obtained quickly. But the solution of the inverse kinematics is not easy to obtain. If one of the two conditions in the Pieper criterions is met, the analytical solution can be obtained by using the analytical method, otherwise, the optimal solution can only be obtained by numerical solution. If the pose matrix is known, the improved FA can search the angle values of every joint, and thus the expected position and direction can be obtained accurately enough. The simulation result shows that the algorithm can be applied to inverse kinematics of multi DOF robot.

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