Study on Characteristics of Stress Field around the Crack-tip of Volume Fracturing Based on Chaos Theory

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***Abstract* - Volume fracturing of horizontal well in unconventional reservoir is a hot problem in global research, and is a major technical means of high-efficient mining. In order to describe crack networks evolution behavior in volume fracturing, the paper studies the distribution regularities of the stress field around the crack-tip, circumferential stress based on chaos theory, the axial stress around the crack-tip is used as characteristic indicators to describe crack evolutionary. Correlation dimension, Lyapunov index and Kolmogorov entropy are introduced as chaos characteristic quantities of crack evolution system, and the process of coal-rock fracturing crack damage evolution could be calculated and described. Shale, coal and tight sandstone are chosen as research objects to calculae and analyze the chaos characteristics of different kinds of rock mass. The conclusion has provided a new way for further research on coal-rock fracturing crack evolution regularities.**

Keywords - Distortion rock mass fracturing; Stress field around the crack-tip; Chaos characteristic quantities; Unconventional reservoir

I. INTRODUCTION

Hydrafracturing is a major technology of Coal-Bed Methane (CBM) production. Due to the systematical development and complicated structure, the hydraulically created fracture of coal petrography cracked, extended and broadened along with the weak structure surfaces at different levels. In recent years, several corresponding researches about coal-bed fracture have been explored by scholars at home and abroad. Olovyanny developed a mathematical model of hydraulic fracturing of coal seam, and confirmed the pressure and location of fracture initiation[1]. Lekontsev researched the oriented fracture initiation and expanding principle with Berezovskaya Mine as study object[2]. Tonglin Li explored the fundamental theories of hydraulic fracturing technology, including basic properties of coal, formation conditions of crack generated by hydraulic fracturing of coal seam, crack patterns and crack propagation direction[3]. Bingxiang Huang proposed a calculation model of hydrafracturing based on the fracture mechanics method, which *Corresponding Author

was used to calculate the minimum water pressure in fissure and extension length of hydraulic airfoil branch crack[4]. In allusion to influence of stress on initiation fracture pressure and fissure distribution, Yishan Pan established a mechanical model integrating theoretical analysis with numerical modeling[5]. Jinsu Ding found out the necessary condition of hydraulic fracturing test with thin-wall cylinder and soil as testing material[6]. Mian Chen employed the large-sized true triaxial experimental system to simulate formation condition, and they researched the fracture strike and variation of fracture width with cement mortar as test material[7]. Guangzhi Yin did a series of researches on bifurcation and chaotic characteristics in micro-fracture process and micro-crack evolution of rock based on non-linear theory[8,9,10].

On account of the physical cellular automata model (PCA), Hui Zhou researched the chaotic features of mining quake, and developed the nonlinear forecasting theory[11]. Fangcai Zhu proposed a rock-mechanics system, which comprised a rock and a tester, to study the chaotic features of rock uniaxial compression process[12]. Yongdong Jiang analyzed the chaotic characters in rock stress-strain procedure in the basis of evolvement in acoustic emission event[13]. Chi Ai found out the chaotic features when researching the evolution properties of micro-crack in rock hydraulic fracturing process[14]. Hui Tao analyzed the chaos characteristic of microseism time series of rock bust. In this manuscript, in view of the distribution characteristics of natural fracture of coal rock, a basic model of evolvement in fissure of rock fracture is proposed[15]. Moreover, the chaotic characteristic analysis of fracture evolution is achieved by calculating the chaotic feature values of three characteristic indexes. The results show that the fracture evolution of coal rock is chaotic, which provides the new ideas for further studies.

II. CRACK EVOLUTİON MODEL OF COAL AND ROCK MASS FRACTURİNG

The evolution of micro fracture is essential to a dynamic process of, propagation and coalescence of micro cracks in the interior. It has a great relationship with the crack morphology and micro crack growth rate. The stress of nonlinear tensor radial damage near the crack tip is:

$$\sigma = \frac{K_d \tilde{l}}{\sqrt{2\pi(\tilde{l}+a)}}\sqrt{\frac{1}{r_1}} - \sigma_h \qquad (1)$$

Where σ is the coal and rock fracture process, fracture tip damage stress; K_d is the stress intensity factor; r_1 is the distance from the crack tip, m; σ_k is the minimum horizontal principal stress in the vertical direction of the crack wall; $\tilde{l}$ is the average characteristic value of micro cracks in rock mass; a is micro crack growth length.

It is postulated that the evolution of the crack is related to the characteristics of the rock mass and the evolution velocity. The stress intensity factor of the crack evolution $K_{I,i+1}^{dyn}$ is defined as

$$K_d = k_1 \cdot K_D + k_2 \cdot K_V \cdot K_I \qquad (2)$$

Where:

$$\begin{cases} K_D = f(\omega,\alpha,\sigma) \\ K_V = g(v) \end{cases}$$

III. PHASE SPACE RECONSTRUCTİON OF STRESS FİELD AROUND THE CRACK-TIP

A. Setting up the Model of Phase Space Reconstruction

It is postulated that n is the embedded dimension of crack tip stress distribution system, delay time method is used to reconstruct m-dimensional system characteristic quantity difference form of phase space:

$$\xi(t) = (x(t), x(t+\tau), \dots x(t+(m-1)\tau)) \qquad (3)$$

Where $x(t)$ is the change parameter of crack tip stress distribution system, t is the delay time.

B. Delay Time and Embedded Dimension

Introducing function $S_1 = (m,r,t)$, define the monitoring statistics as:

$$S_1(n,N,r,t) = \frac{2}{M(M-1)}\sum_{j=1}^{M}\sum_{i\neq j+1}^{M}\Theta(r - \|\xi_i - \xi_j\|) - \frac{2}{N(N-1)}\sum_{j=1}^{N}\sum_{i\neq j+1}^{N}\Theta(r - \|\xi_i - \xi_j\|) \qquad (4)$$

N is the size of data set, $M = N-(n-1)t$ is phase amount in m-dimension phase space, $\|\dots\|$ is Euclidean Distance. Determine the delay time is to investigate the dependence of $S_1 = (n,r,t)$ to delay time index.

Dividing each time series equally, and calculating each $S(n,N,r,t)$

$$S_2(n,N,r,t) = \frac{1}{t}\sum_{s=1}^{t}\left[\frac{2}{M(M-1)}\sum_{j=1}^{M}\sum_{i\neq j+1}^{M}\Theta(r - \|\xi_i - \xi_j\|) - \frac{2}{N(N-1)}\sum_{j=1}^{N}\sum_{i\neq j+1}^{N}\Theta(r - \|\xi_i - \xi_j\|)\right] \qquad (5)$$

IV. STUDY ON CHAOS CHARACTERİSTİCS OF STRESS FİELD AROUND THE CRACK-TİP

A. Calculation of Lyapunov Index

(1) By the time queue of crack system $(\delta_1, \delta_2, \dots \delta_i, \dots \delta_N)$ phase space R^n can be reconstructed as:

$$\xi_i = [\delta_i, \delta_{i+t}, \delta_{i+2t}, \dots, \delta_{i+(n-1)t}]$$
$$(i = 1,2,\dots,N-(n-1)t) X_i \in R^n \qquad (6)$$

(2) Searching ξ_j nearest to each ξ_j, i.e.

$$d_j(0) = \min\|X_j - X'_j\|, |j - j'| > P \qquad (7)$$

Where P is the average period of time series.

(3) For every ξ_j in the phase space, calculating distance $d_{j(t)}$ after t discrete time step:

$$d_{j(t)} = \min\|\xi_{i+1} - \xi'_{i+1}\| \qquad (8)$$
$$t = 1,2,\dots,\min(M-j, M-j')$$

(4) Calculating average $x(t)$ of $\ln d_{j(t)}$ of all j :

$$x(t) = \frac{1}{q\Delta t}\sum_{j=1}^{q}\ln d_{j(t)} \qquad (9)$$

Where q is the amount of nonzero $d_{j(t)}$.

(5) Drawing the curve of $x(t) \sim t$, the maximum Lyapunov index λ can be received from the straight slope via least square method.

B. Calculation of Correlation Dimension

Reconstructing phase space R^n by time series, and calculate correlation integral:

$$C(n,r) = \frac{2}{M(M-1)}\sum_{j=1}^{M}\sum_{i\neq j+1}^{M}\Theta(r - \|\xi_i - \xi_j\|) \qquad (10)$$

An interval of r has following scale relation:

$$\frac{2}{M(M-1)}\sum_{j=1}^{M}\sum_{i\neq j+1}^{M}\Theta(r - \|\xi_i - \xi_j\|) \propto r^y \qquad (11)$$

The index γ in formula 12 is a dimension. Correlation dimension is defined as:

$$D=\lim_{t\to 0}\frac{\ln[\frac{2}{M(M-1)}\sum_{j=1}^{M}\sum_{i\neq j+1}^{M}\Theta(r-\|\xi_i-\xi_j\|)\propto r^{\gamma}]}{\ln r} \quad (12)$$

Where D is correlation dimension.

C. Calculation of Kolmogorov Entropy

It was postulated that $P(i_1,i_2,\dots,i_n)$ is the joint probability of $X=(t=\tau)$ in i_1 box ... $X=(t=\tau)$ in in box. So, Kolmogorov entropy is:

$$K=-\lim_{t\to 0}\lim_{\tau\to 0}\lim_{n\to\infty}\frac{1}{n\tau}\ln\sum_{i_1,\dots i_n}p(i_1,i_2,\dots i_n)\log_2 P(i_1,i_2\dots i_n) \quad (13)$$

We can estimate K_1 in virtue of 2-steps Renyi entropy. 2-steps Renyi entropy is defined as:

$$K=-\lim_{t\to 0}\lim_{\tau\to 0}\lim_{n\to\infty}\frac{1}{n\tau}\ln\sum_{i_1,\dots i_n}p^2(i_1,i_2,\dots i_n) \quad (14)$$

Grassberger and Procaccia come to the conclusion that Correlation integral $C(n,r)$ and $\sum_{i_1,\dots i_n}p^2(i_1,i_2,\dots i_n)$ has the following relationship:

$$C(n,r)\approx\sum_{i_1,\dots i_n}p^2(i_1,i_2,\dots i_n) \quad (15)$$

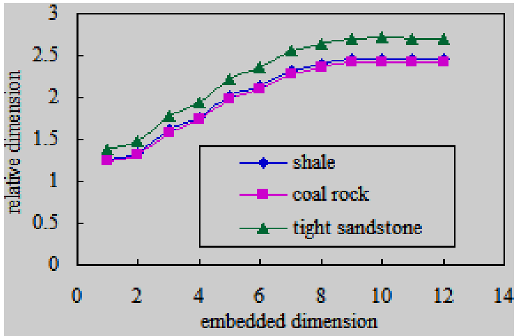
Select proper delay time τ to reconstruct n-dimension phase space, so:

$$K_2(n,r)=\frac{1}{\tau}\frac{\sum_{i_1,\dots i_n}p^2(i_1,i_2,\dots i_n)}{\sum_{i_1,\dots i_n}p^2(i_1,i_2,\cdots i_n,i_{n+1})} \quad (16)$$

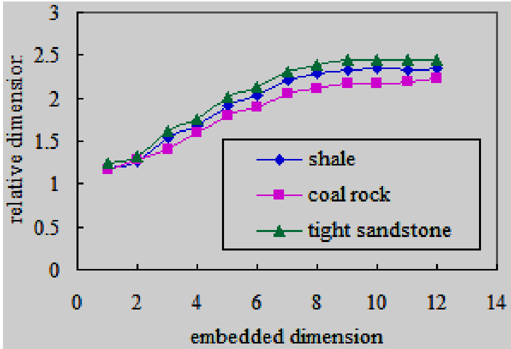
$$K_2=\lim_{\substack{n\to\infty\\ \tau\to 0}}K_2(n,r) \quad (17)$$

V. EXAMPLE VERIFICATION

Choosing shale sample (Qingshankou formation in Caiyuanzi, Dehui, Jilin); Coal sample (Jixi mining area, Heilongjiang. Buried depth is 901-910m); Tight sandstone sample (box 8, Ordos basin), measuring mechanical parameters. Sample size is 300×300×300mm according to crack distribution regularities, surrounding rock pressure is 10MPa. According to measuring results and mechanical parameters, calculating tip stress distribution regularities of different lithology, results of phase reconstruction, chaos characteristic quantities, correlation dimension, Lyapunov index, Kolmogorov entropy were shown on Fig.(1).-Fig.(3).

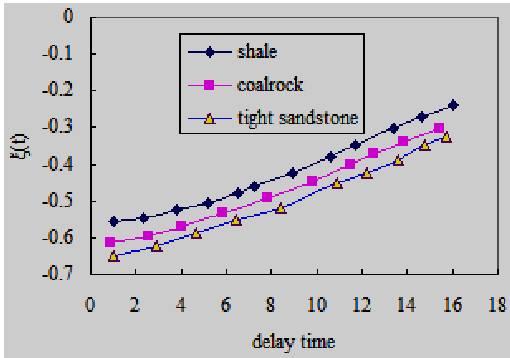


(a) Crack Tip Radial Stress

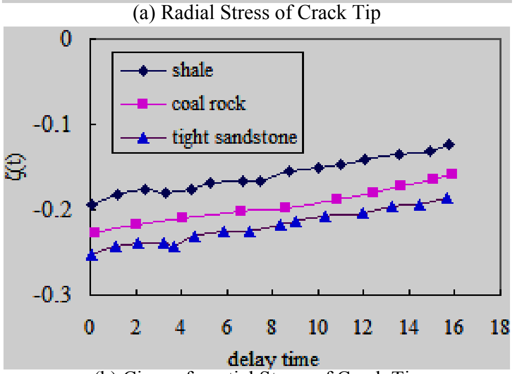


(b) Crack Tip Circumferential Stress

Fig.(1). Curve of Relative Dimension with Embedded Dimension

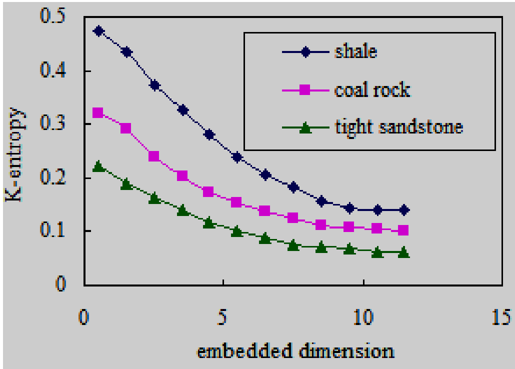


(a) Radial Stress of Crack Tip

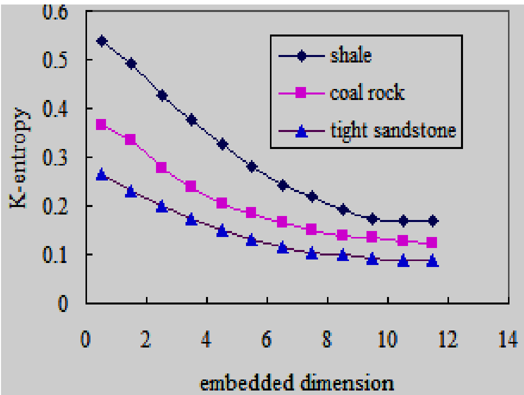


(b) Circumferential Stress of Crack Tip

Fig.(2). Curve of $\xi(t)$ with Delay time



(a) Radial Stress of Crack Tip



(b) Circumferential Stress of Crack Tip

Fig.(3). Curve of K_2 with m

As the calculation result shows, we can see that there are obvious linear segments in the $\ln C(n,r) \sim \ln r$ curve from Fig.1.

TABLE 1. CALCULATION RESULT OF COAL, SHALE AND SANDSTONE SAMPLE.

Type	No.	Correlation dimension	Lyapunov index	Kolmogorov entropy
Coal sample	Group 1	2.120	0.0625	0.179
	Group 2	2.140	0.0581	0.179
	Group 3	2.120	0.0613	0.176
	Average	2.127	0.0606	0.178
Shale sample	Group 1	2.600	0.0720	0.191
	Group 2	2.610	0.0613	0.186
	Group 3	2.630	0.0700	0.190
	Average	2.613	0.068	0.189
Tight sand-stone sample	Group 1	1.91	0.0411	0.110
	Group 2	1.88	0.0395	0.085
	Group 3	1.91	0.0403	0.130
	Average	1.90	0.0403	0.108

When value m is small, the gradient of the straight line is small and the separation is large. With the increasing of value m, the gradient of the straight line increases and the separation becomes small and gradually closer. When $m \geq 5$, the straight sections of each curve tend to be paralleled with each other and gradually intensive, and the gradient of the straight line that is almost no longer changed can be thought a certain value. This value is the dimension of relevance that characterizes these sequences.

From the dimension of relevance D_2 changed along with the embedding dimension m, we can obtain that the dimension of relevance of crack number evolution series is 2.12. And the time series of coal and rock cracks have chaotic characteristics. The maximum Lyapunov index of the time series of crack evolution number is 0.0625 by calculating, which indicates the maximum Lyapunov index is positive. And this indicates adjacent track is divergent and further shows that the system has chaotic characteristics.

VI. CONCLUSION

(1) Calculating model of stress field around the crack-tip is established, and stress field distribution of crack evolution is established by time delay method. Chaos characteristics correlation dimension, Lyapunov index and Kolmogorov entropy are established. Aiming at shale, coal and tight sandstone, calculating chaos characteristics of fracturing stress field around the crack-tip.

(2) By calculating chaos characteristics correlation dimension, Lyapunov index and Kolmogorov entropy of different types of rock mass, results shows that fracturing crack chaos characteristics of shale is more obvious than coal and tight sandstone, and chaos characteristics of shale after fracturing is obvious, too. Expanding directions of crack tip are complex, diagonal expansion of cracks is complex and easy to form crack networks. Disribing crack evolution and instability of judgment based on chaos theory is scientific.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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