The Non-equidistant Linear Weighting Grey Model NWGM (1,1) and Its Application

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***Abstract* - To apply the modelling problem that the weighting grey model GM (1,1) not dealing with non-equidistant sequence and meet the requirement of non-equidistant sequence in engineering system, the non-equidistant linear weighting grey model NWGM (1,1) was put forward. Taking the weighting coefficient and the correction values of response function initial values as design variables, the minimum mean relative error and the minimum mean square value respectively as objective function, two optimization models were established, and Matlab programs were written. Example shows the proposed non-equidistant linear weighting grey model NWGM (1,1) has higher precision. This model is not only suitable for equidistant modelling, but also for non-equidistant modelling. It is worth widely applying in engineering and other related fields.**

Keywords – NWGM (1,1); Background value; Optimization; Linear weighting

I. INTRODUCTION

J.L. Deng first proposed grey system theory in 1982 [1]. The grey model GM (1,1) as an important part of grey system theory is widely used in economics, management, engineering field [2-4]. In order to improve the accuracy of GM (1,1), a large number of research has done at home and abroad for over 30 years, and these research focus on the fields such as initial conditions [5-6], the improvement for background values [7-8] , strengthening or weakening buffer operator [9-10], the improvement for differential equations [11-12] and weighting WGM (1,1) [13-15]. On the basis of the effect of new information greater than one of old information, the concept about the weighting accumulated generating operation was firstly proposed in [13]. The weighting function was put forward in [14], but the weight factor was still artificially given. The linear weighting function was proposed and the equidistant model based on weighting function was established in [15]. However, the existing weighting models are all based on equidistant sequence. For non-equidistant sequence, the weighting model can not be established to solve. The raw data obtained in the engineering system are often non-equidistant sequence, so it has a wide range of practical significance to establish a non-equidistant sequence weighted grey NWGM (1,1) model. To meet the requirement of non-equidistant sequence in engineering system and make up for lack of the existing weighting grey model WGM(1,1), the non-equidistant linear weighting grey model NWGM(1,1) was put forward. Taking the weighting coefficient and the correction values of response function initial values as design variables, the minimum mean relative error and the minimum mean square value respectively as objective function, two optimization models were established. Matlab programs were written, and the example in engineering application was given. Example shows the proposed non-equidistant linear weighting grey model NWGM (1,1) has higher precision and stronger adaptability. This model is not only suitable for equidistant modelling, but also for non-equidistant modelling. It is worth widely applying in engineering and other related fields.

II. NON-EQUIDISTANT LINEAR WEIGHTING GREY MODEL NWGM (1,1)

Definition 1: Supposed the non-negative sequence:

$X^{(0)}=[x^{(0)}(t_1),\cdots,x^{(0)}(t_m)]$, if $\Delta t_i = t_i - t_{i-1} \neq const$ where $k=2,\cdots,m$, $X^{(0)}$ is called as non-equidistant sequence [16].

Definition 2: Supposed the sequence:

$X^{(1)}=[x^{(1)}(t_1),x^{(1)}(t_2),\cdots,x^{(1)}(t_m)]$, if $x^{(1)}(t_1)=x^{(0)}(t_1)$ and $x^{(1)}(t_{k+1})=x^{(1)}(t_k)+x^{(0)}(t_{k+1})\cdot\Delta t_{k+1}$,

where $k=1,\cdots,m-1$, $X^{(1)}$ is first-order accumulated generation operation of non-equidistant sequence $X^{(0)}$, and it is denoted by 1-AGO [16].

Definition 3: Supposed the non-negative sequence:

$X^{(0)} = [x^{(0)}(t_1),\cdots,x^{(0)}(t_m)]$, if $\Delta t_i = t_i - t_{i-1} \neq const$ where $\Delta t_1 = 1$ and $i = 2,3,\cdots,m$, $X^{(1)} = [x^{(1)}(t_1) \quad x^{(1)}(t_2) \quad \cdots \quad x^{(1)}(t_m)]$

is called as first-order weighting accumulated generation operation, where

$$x^{(1)}(t_k) = \sum_{i=1}^{k} \rho(t_i) x^{(0)}(t_i)\Delta t_i \qquad (1)$$

where $\rho(t_i) = \gamma + i\delta$ is weighting function where $\gamma \geq 0$, $\delta \geq 0$ and $\gamma\delta \neq 0$.

When $\gamma = 1$ and $\delta = 0$, the linear weighting AGO is transformed into 1-AGO. The linear weighting AGO is an expansion of 1-AGO, and 1-AGO is an exception of the linear weighting AGO.

Definition 4: The non-equidistant weighting accumulated grey model GM(1,1) is referred to as NWGM(1,1), whose differential equation is defined as:

$\frac{dx^{(1)}(t_k)}{dt} + az^{(1)}(t_k) = b$ where $k = 2,3,\cdots,m$,

$\frac{dx^{(1)}(t_k)}{dt} = \frac{x^{(1)}(t_k) - x^{(1)}(t_{k-1})}{\Delta t_k}$,

a is the development coefficient, b is the action of the model, and $z^{(1)} = [z^{(1)}(t_2) \quad z^{(1)}(t_3) \quad \cdots \quad z^{(1)}(t_m)]$ is referred to as the background value. When the mean is generated, $z^{(1)}(t_k) = 0.5x^{(1)}(t_{k-1}) + 0.5x^{(1)}(t_k)$.

The mean generation to background value is to approximately calculate the enclosed area of $\frac{dx^{(1)}(t_k)}{dt}$ among t axis and $[t_{k-1}, t_k]$ by the trapezoidal rule. When the time interval is small, that is, the sequence data changes smoothly, the constructed background value is suitable, while the model error is greater when the sequence data changes abruptly. So it is necessary to research the construction of background value in weighting grey model NWGM (1,1).

The albino equation $\frac{dx^{(1)}}{dt} + ax^{(1)} = b$ is integrated and simplified in $[t_{k-1}, t_k]$:

$$\int_{t_{k-1}}^{t_k} dx^{(1)}(t_k) + a\int_{t_{k-1}}^{t_k} x^{(1)} dt = b\Delta t_k \qquad (2)$$

Comparing $\frac{dx^{(1)}(t_k)}{dt} + az^{(1)}(t_k) = b$ and Eq.(2), it is better adapted to the albino equation that:

$z^{(1)}(t_k) = \int_{t_{k-1}}^{t_k} x^{(1)} dt$ is used as background value.

In Eq. (2), it is found that the solution to:

$\int_{t_{k-1}}^{t_k} dx^{(1)}(t_k) + a\int_{t_{k-1}}^{t_k} x^{(1)} dt = b\Delta t_k$ satisfies the inhomogeneous exponential form. Setting

$x^{(1)}(t_k) = Ge^{-a(t_k - t_1)} + C$,

where G and C are unknown variables, the regressive sequence is:

$$x^{(0)}(t_k) = \frac{x^{(1)}(t_k) - x^{(1)}(t_{k-1})}{\Delta t_k} = \frac{G(1 - e^{a\Delta t_k})}{\Delta t_k} e^{-a(t_k - t_1)} \qquad (3)$$

$e^{a\Delta t_i}$ is expanded and the first two items of the expansion are selected when a and Δt_k are smaller. The following formula can be obtained:

$$1 - e^{a\Delta t_k} = -a\Delta t_k, \quad \frac{x^{(0)}(t_k)}{x^{(0)}(t_{k-1})} = \frac{e^{-a(t_k - t_1)}}{e^{-a(t_{k-1} - t_1)}} = e^{-a\Delta t_k}$$

then:

$$a = -\frac{\ln x^{(0)}(t_k) - \ln x^{(0)}(t_{k-1})}{\Delta t_k} (k = 2,3,\cdots,m) \qquad (4)$$

Eq.(4) is substituted into Eq.(3):

$$G = \frac{x^{(0)}(t_k)\Delta t_k [x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_k - t_1}{\Delta t_k}}}{1 - \frac{x^{(0)}(t_{k-1})}{x^{(0)}(t_k)}} \qquad (5)$$

According to initial condition as:

$x^{(1)}(t_1) = Ge^{-a(t_1 - t_1)} + C = G + C$, we can obtain:

$$C = x^{(1)}(t_1) - G = x^{(0)}(t_1) - \frac{x^{(0)}(t_k)\Delta t_k [x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_k - t_1}{\Delta t_k}}}{1 - \frac{x^{(0)}(t_{k-1})}{x^{(0)}(t_k)}} \quad (6)$$

Eq.(11) and Eq.(13) are substituted into the calculation formula of background value as $\int_{t_{k-1}}^{t_k} x^{(r)} dt$:

$$z^{(1)}(t_k) = \int_{t_{k-1}}^{t_k} x^{(1)} dt = -\frac{\Delta t_k x^{(0)}(t_k)}{a} + C\Delta t_k = \frac{(\Delta t_k)^2 x^{(0)}(t_k)}{\ln x^{(0)}(t_k) - \ln x^{(0)}(t_{k-1})} + x^{(0)}(t_1)\Delta t_k - \frac{x^{(0)}(t_k)(\Delta t_k)^2 [x^{(0)}(t_k)/x^{(0)}(t_{k-1})]^{\frac{t_1 - t_k}{\Delta t_k}}}{1 - \frac{x^{(0)}(t_{k-1})}{x^{(0)}(t_k)}} \quad (7)$$

Least squares estimation parameter of $\int_{t_{k-1}}^{t_k} dx^{(1)}(t_k) + a\int_{t_{k-1}}^{t_k} x^{(1)} dt = b\Delta t_k$ is:

$$\hat{\mathbf{a}} = (\mathbf{B}^{\mathrm{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Y} \quad (8)$$

where

$$\mathbf{B} = \begin{bmatrix} -z^{(1)}(t_2) & \Delta t_2 \\ -z^{(1)}(t_2) & \Delta t_3 \\ \cdots & \cdots \\ -z^{(1)}(t_m) & \Delta t_m \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} x^{(1)}(t_2) - x^{(1)}(t_1) \\ x^{(1)}(t_3) - x^{(1)}(t_2) \\ \cdots \\ x^{(1)}(tm) - x^{(1)}(t_{m-1}) \end{bmatrix} \quad (9)$$

Definition 5: $\frac{dx^{(1)}}{dt} + ax^{(1)} = b$ is defined as the albino differential equation in the weighting accumulated grey model.

Given the initial conditions $x^{(1)}(t_1) = \rho(t_1)x^{(0)}(t_1)$, the albino differential equation is obtained as:

$$\hat{x}^{(1)}(t_k) = (x^{(1)}(t_1) - \frac{b}{a})e^{-a(t_k - t_1)} + \frac{b}{a} (k = 1,2,\cdots,m) \quad (10)$$

Amending the initial value can obtain the albino differential equation:

$$\hat{x}^{(1)}(t_k) = (x^{(1)}(t_1) - \frac{b}{a} + C_1)e^{-a(t_k - t_1)} + \frac{b}{a} (k = 1,2,\cdots,m) \quad (11)$$

$\hat{x}^{(1)}$ is operated by 1-AGO generation, and then $\hat{x}^{(1)}$ is restored to the original sequence $\hat{x}^{(0)}$.

The absolute error of the fitting data:

$$q(t_k) = \hat{x}^{(0)}(t_k) - x^{(0)}(t_k) \quad (12)$$

The relative error (%) of the fitting data:

$$e(t_k) = \frac{\hat{x}^{(0)}(t_k) - x^{(0)}(t_k)}{x^{(0)}(t_k)} \quad (13)$$

The mean relative error of the fitting data:

$$f(\gamma,\delta,C_1) = \frac{1}{m}\sum_{k=1}^{m}\frac{\hat{x}^{(0)}(t_k) - x^{(0)}(t_k)}{x^{(0)}(t_k)} \quad (14)$$

III. OPTIMIZATION MODEL OF NON-EQUIDISTANT LINEAR WEIGHTING GREY MODEL NWGM (1,1) AND SOLUTION

A. Optimization Model of Non-equidistant Linear Weighting Grey Model NWGM (1,1)

Taking $\rho(i)=\gamma+\delta i$ and the correction value C_1 of initial value as design variables, that is:

$$x = [\gamma, \delta, C_1]$$

the minimum mean relative error as objective function, non-equidistant weighting grey model NWGM(1,1) is to solve the following optimization problem, referred to as Model I:

$$\min \quad f(x) = \frac{1}{m}\sum_{k=1}^{m}\left|\frac{x^{(0)}(t_k) - \hat{x}^{(0)}(t_k)}{x^{(0)}(t_k)}\right| \quad (15)$$

Taking $\rho(i)=\gamma+\delta i$ and the correction value C_1 of initial value as design variables, that is, $x = [\gamma, \delta, C_1]$, the minimum mean square value as objective function, non-equidistant weighting grey model NWGM(1,1) is to solve the following optimization problem, referred to as Model II:

$$\min \quad f(x)=\frac{1}{m}\sum_{k=1}^{m}(x^{(0)}(t_k)-\hat{x}^{(0)}(t_k))^2 \tag{16}$$

If not weighted accumulating and not amending the initial value, that is, $x=[1,0,0]$, this model is referred to as Model III.

B. Elite Multi-Parent Crossover Algorithm

Considering the following optimization problem:

$$\min f(\mathbf{x}), \mathbf{x}\in D, D=\{\mathbf{x}\in S; g_k(\mathbf{x})\le 0,\ k=1,2,\cdots,q, h_j(\mathbf{x})=0, j=1,2,\cdots,p$$

where, $S\subset R^n$ is the search space, $l_i \le x_i \le u_i (i=1,2,\cdots,n)$, f is the objective function, n is the number of variables, D is the set of feasible point, g_k is constraint functions and q is the number of constraints [18].

Noting

$$H(\mathbf{x})=h(\mathbf{x})(\sum_{k=1}^{q}\mu(\phi_k(\mathbf{x}))\phi_k(\mathbf{x})^{\delta(\phi_k(\mathbf{x}))}+\sum_{j=1}^{p}\mu(\varphi_j(\mathbf{x}))\varphi_j(\mathbf{x})^{\delta(\varphi_j(\mathbf{x}))})$$

where, $\phi_k(\mathbf{x})=\max\{0,g_k(\mathbf{x})\}$, $\varphi_j(\mathbf{x})=|h_j(\mathbf{x})|$, h is punishment function. $h(\bullet)$, $\mu(\bullet)$ and $\delta(\bullet)$ are depended on the specific situation.

a is a bigger positive number. The logical function is defined as:

$$better(\mathbf{x}_1,\mathbf{x}_2)=\begin{cases} Ture, \ if \quad H(\mathbf{x}_1)<H(\mathbf{x}_2) \\ False, \ if \quad H(\mathbf{x}_1)>H(\mathbf{x}_2) \\ Ture, \ if \quad (H(\mathbf{x}_1)=H(\mathbf{x}_2))\wedge (f(\mathbf{x}_1)\le f(\mathbf{x}_2)) \\ False, \ if \quad (H(\mathbf{x}_1)=H(\mathbf{x}_2))\wedge (f(\mathbf{x}_1)> f(\mathbf{x}_2)) \end{cases}$$

If $better(\mathbf{x}_1,\mathbf{x}_2)$ is true, $\mathbf{x}_1$ is better than $\mathbf{x}_2$, otherwise $\mathbf{x}_2$ is better than $\mathbf{x}_1$.

The algorithm is as follows:

Step 1: randomly generating initial population $P_0=\{\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_N\}$ in the search space S, where N is the number of initial population and $t=0$;

Step 2: P_0 sorted from best to worst in accordance with the principle of logic function as $better(\mathbf{x}_1,\mathbf{x}_2)$. The sorted sequence is still referred to as $P_0=\{\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_N\}$ where $\mathbf{x}_1$ the best individual and $\mathbf{x}_N$ is the worst;

Step 3: When $better(\mathbf{x}_{worst},\mathbf{x}_{best})$ is true, that is, the best individual is the worst, turning to Step 5;

Step 4: Selecting K best individuals $[\mathbf{x}'_1,\mathbf{x}'_2,\cdots,\mathbf{x}'_K]$ from P_t where $K\le M$, and randomly selecting $(M-K)$ individuals $[\mathbf{x}'_{K+1},\mathbf{x}'_{K+2},\cdots,\mathbf{x}'_M]$ from the rest individuals. M individuals where $M\le N$ are formed to the subspace $V=\left\{\mathbf{x}\mid \mathbf{x}\in S, \mathbf{x}=\sum_{i=1}^{M}a_i\mathbf{x}'_i\right\}$ where a_i meets $\sum_{i=1}^{M}a_i=1$ and $-0.5\le a_i\le 1.5$.

Randomly selecting L points in the subspace V gets L new individuals where the best point is referred to as $\bar{\mathbf{x}}$. If $better(\bar{\mathbf{x}},\mathbf{x}_{worst})$ is true, $\bar{\mathbf{x}}$ is substituted into $\mathbf{x}_{worst}$ and formed to P_{t+1}, otherwise, $P_{t+1}=P_t$. t is displaced to $t+1$ and the program is turned to step 2.

Step 5: Outputting the best solution that makes the function is equal to zero, the whole program is ended.

In the fourth step of the algorithm, that K elites in P_t selected to part of the subspace V makes good information of solutions be fully utilized and promotes the algorithm to converge faster to the optimal solution. This method is particularly suitable for optimization of unimodal function. Numerical experiments show that convergence rate accelerates significantly if using elite preservation strategy. It is not the better if K is selected to the larger. It is true that the bigger of K can make better use of the solution information, but the larger K, the smaller the degree of freedom of the group in V. A smaller degree of freedom is easy to make the solution hover within a local region of the search space. For multimodal function, it is easy to fall into local optimal solution. The selection of K is related to the size of M. If M is larger, K should be selected to be larger. That selecting L individuals in V is to make more effectively use of the subspace information. For optimization problems, K is generally selected to $M/2$ or slightly smaller. $L = M$ where M is generally selected to one to three times of the number of variables.

Malab program about the above optimization method is written as *GT_GYZJ.m*.

C. Solving Optimization Model

The program of the non-equidistant weighting accumulated generating operator is written as *WRtAGO.m*, and the programs of the non-equidistant weighting inverse accumulated generating operator as *IWRtAGO*.m. Taking the above algorithm whose program is written as *NWGM11.m* as the objective function, main program is written as *main_NWGM11.m*. The known data is regarded as data input, and the target function *NWGM11*.m is called by using elite multi-parent crossover algorithm or other intelligent optimization method, and then the weighting function and the initial value of the correction amount C_1 can be obtained.

IV. EXAMPLE

P. G. Foleiss researched that there is the influence of the temperature on fatigue strength under the long life symmetry cycle of many materials. Table 1 shows the experimental data of the change relation of Ti alloy fatigue strength along with temperature [19], which is a sequence of non-equidistant sequence. $x^{(0)}(t_k)$ in Table 1 is Ti alloy fatigue strength σ_{-1} at t_k where $t = [t_1, t_2, \cdots, t_k, \cdots, t_m]$ and $\sigma_{-1} = [x^{(0)}(t_1), x^{(0)}(t_2), \cdots, x^{(0)}(t_k), \cdots, x^{(0)}(t_m)]$.

TABLE I. CHANGE RELATION OF TI ALLOY FATIGUE STRENGTH ALONG WITH TEMPERATURE (MPA)

t_k / °C		**100**	**130**	**170**	**210**	**240**	**270**	**310**	**340**	**380**
$x^{(0)}(t_k)$		560	557.54	536.10	516.10	505.60	486.1	467.4	453.8	436.4
Model I	$\hat{x}^{(0)}(t_k)$	560.001	557.534	538.373	517.302	499.496	484.698	468.041	451.931	436.400
	$e(t_k)$	-0.0002	0.00115	-0.4240	-0.2330	1.2072	0.2884	-0.1372	0.4116	0
Model II	$\hat{x}^{(0)}(t_k)$	560	557.307	538.436	517.634	500.078	485.516	469.076	453.167	437.823
	$e(t_k)$	0	0.0418	-0.4358	-0.2972	1.0922	0.1201	-0.3586	0.1395	-0.3260
Model III	$\hat{x}^{(0)}(t_k)$	560	556.780	537.929	517.148	499.610	485.064	468.640	452.748	437.418
	$e(t_k)$	0	0.1362	-0.3412	-0.2031	1.1847	0.2132	-0.2654	0.2319	-0.2334

V. CONCLUSION

The non-equidistant linear weighting grey model NWGM(1,1) and its optimization model is effective and the solution method is reasonable. When the relative average error is greater by using the traditional model, the proposed method in this paper can be used to improve modeling accuracy. The proposed method in this paper has stronger adaptability, so it is not only suitable for equidistant modeling, but also for non-equidistant modeling. It is worth widely applying in engineering and other related fields.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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