

## Impact of Risk Aversion on Retailer's Optimal Shelf Space Decision

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**Abstract** - In a Conditional Value-at-Risk framework, this paper deals with a risk-averse newsvendor model under the assumption that the random demand is shelf-space sensitive. An analytical solution is provided for finding the optimal shelf-space. The influences of system parameters on the optimal decisions are investigated as well. Numerical examples illustrate how the optimal expected profit changes with the system parameters.

**Keywords** - Conditional value-at-risk; Newsvendor model; Shelf space

### I. INTRODUCTION

In practice, the retailer can increase sales by increasing the shelf space or display inventory for a product. Inventory can promote demand for a variety of reasons. At least two types of stimulating effects of inventory on demand have been identified. The first one is that it enhances product visibility, stimulate potential demand, and signal a popular product. The second is that it provides consumers with an assurance of future availability. The sales for many kinds of products approve this phenomenon, such as toys, consumer electronic products, fresh fruits, etc. [1] explored an EOQ-type inventory problem where the demand rate is a function of the inventory level. [2] provided the first treatment of demand-stimulation effects within a stochastic inventory model by extending the classical newsvendor model to include endogenous, inventory-dependent demand (for a given price). They proposed a multiplicative demand structure that models actual demand as a deterministic, inventory-dependent multiple of a base random variable with a fixed (inventory-independent) probability distribution. For products having demand rates that increase with inventory levels, [3] analyzed the effect of stocking decisions on firm profitability to develop managerial insights regarding the structure of the optimal inventory policy, and to understand how this policy differs from traditional approaches. [4] and [5] considered the coordination of a supply chain in which a manufacturer or wholesaler who supplies some item to retailers facing demand rates that depend on the shelf or display space that is devoted to that product by themselves and their competitors. [6] studied the inventory management problem of dual channels operated by one vendor. Demands of dual channels are inventory-level-dependent. [7] proposed a more general demand-modeling framework to capture the influence of a product's inventory on the demand distribution; this approach subsumes both demand scaling

(multiplicative) and shifting (additive) as special cases. In order to overcome the limitations of traditional risk measure as mean-variance and VaR, [8] and [9] has evolved a new measure of risk and is named Conditional Value-at-Risk (CVaR). CVaR measure ignores the contributions of profit beyond the specific quantile, focuses on the average profits for the lower quantile, and has better computational characteristics. Under CVaR criterion, [10] studied the impact of shortage penalty cost and degree of risk aversion effect on the retailer's optimal ordering quantity. [11] considered a risk-averse newsvendor with stochastic price-dependent demand under CVaR criterion. For both additive and multiplicative demand models, They provided sufficient conditions for the uniqueness and existence of the optimal policy. They showed the monotonicity properties and other characteristics of the optimal pricing and ordering decisions by comparative statics. [12] considered a supply chain with one risk-neutral manufacturer and one risk-averse retailer with CVaR as her risk measure. They showed that there exists a Nash-bargaining equilibrium for the wholesale price and order quantity with equal and unequal bargaining power.

Although some papers have investigated the newsvendor model with shelf space (inventory), in operation management, a lot of retailers are risk-averse, and random demand is affected by the shelf space for a product. In this paper, we want to investigate the risk-averse newsvendor under CVaR criterion with shelf space dependent demand. The remainder of this paper is organized as follows. Section 2 proposes the risk-averse newsvendor model under CVaR criterion in which stochastic demand is affected by shelf space. Section 3 finds and gives optimal shelf space. Section 4 presents the numerical examples to illustrate the possible impact of system parameters on retailer's expected profit. Section 5 concludes the paper with future researches.

II. THE MODEL

We consider a risk-averse retailer who sells a single product over a single selling season to determine the shelf space  $Q$  to maximize the expected profit. Assume that demand for the product, denoted by  $D$ , has the following multiplicative functional form:

$$D(Q) = M(Q) \cdot \xi, \tag{1}$$

where  $M(Q)$  is a deterministic and increasing function in shelf space  $Q$ , and  $\xi$  is a random factor with CDF  $F(\cdot)$ , PDF  $f(\cdot)$ . We let  $M(Q)$  take the form of:

$$M(Q) = \alpha Q^\beta, \tag{2}$$

where  $\alpha > \max(1, \frac{1}{A})$ , and  $0 < \beta < 1$ .

The above demand function form is one of the few models that have often been adopted in the literature for studying joint shelf space and price decisions: see [4], [6] and [7]. In this formulation,  $\alpha$  is the size of the market, and the parameter  $\beta$  is the shelf space elasticity of demand. The smaller the value of  $\beta$ , the more sensitive the demand is to change in shelf space. In order to obtain the unique optimal shelf space, we assume  $\alpha > \max(1, \frac{1}{A})$ .

The product is ordered at a constant cost of  $c > 0$  per unit. For simplicity, we assume that any unsold product at the end of the season bears no salvage value or disposal cost. Similarly, in the case of shortages, unsatisfied demand carries no additional penalty except for the loss of sales revenue. For seasonal or short life-cycle products, zero salvage value or holding cost and zero shortage penalty assumptions are appropriate reflections of reality. To maximize the expected channel profit, a retailer must simultaneously choose the shelf space and the retail price for the product. This decision has to be made before the retailer can observe the demand realization. Let  $\pi(Q)$  denotes the profit for any chosen shelf space  $Q$ . We have:

$$\pi(Q) = p \min(D, Q) - cQ. \tag{3}$$

The  $\eta$  quantile of a random variable  $Z$  is denoted as follows:

$$q_\eta(Z) = \inf\{z \mid P_r(Z \leq z) \geq \eta\}. \tag{4}$$

In this paper, we investigate how the risk-aversion influences the optimal shelf space and pricing decision under a CVaR type criterion. For the case of profit maximization, the  $\eta$ -CVaR is defined as

$$CVaR(\pi(Q)) = E[\pi(Q) \mid \pi(Q) \leq q_\eta(\pi(Q))]. \tag{5}$$

A more general definition of CVaR is:

$$CVaR(\pi(Q)) = \max_{v \in R} \{v + \frac{1}{\eta} E[\min(\pi(Q) - v, 0)]\}. \tag{6}$$

[7] and [8] have proved that these two definitions are equivalent, and furthermore, it is more convenient to use the latter in our analysis.  $\eta \in (0, 1]$  reflects the degree of risk aversion. Note that, the retailer is risk-neutral for  $\eta = 1$ .

III. OPTIMAL DECISION UNDER CVaR CRITERION

In this section, we investigate the optimal decisions on shelf space and pricing. First, we give the equations satisfied by the stocking factor  $z$  and sale price  $p$ . Then we compute the comparative statics to investigate the impacts of changing the unit ordering cost  $c$ , the size of the market  $\alpha$ , shelf space elasticity  $\beta$  is of demand, and the price elasticity  $\gamma$  on the optimal stocking factor and sale price.

**Proposition 1** Under the CVaR criterion, if  $p < \frac{c}{\beta}$ , the

unique optimal shelf space for the risk-averse newsvendor satisfies the following equation:

$$F\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) = \frac{(p - c)\eta - p\alpha\beta\tilde{Q}^{\beta-1}\Lambda\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right)}{p(1 - \beta)}, \tag{7}$$

Where:

$$\Lambda\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) = \int_A^{\frac{\tilde{Q}^{1-\beta}}{\alpha}} \left(\frac{\tilde{Q}^{1-\beta}}{\alpha} - x\right) dF(x) = \int_A^{\frac{\tilde{Q}^{1-\beta}}{\alpha}} F(x) dx.$$

**Proof.** The retailer's profit can be written as

$$\begin{aligned} \pi(Q) &= p \min(D(Q), Q) - cQ \\ &= p \min(\alpha Q^\beta \xi, Q) - cQ \\ &= (p - c)Q - p(Q - \alpha Q^\beta \xi)^+. \end{aligned}$$

From the definition of the CVaR, the optimal order quantity is:

$$\tilde{Q} = \arg \max_{Q \geq 0} \{ \max_{v \in R} g(Q, v) \},$$

Where

$$\begin{aligned} g(Q, v) &= v - \frac{1}{\eta} E[v - \pi(Q)]^+ \\ &= v - \frac{1}{\eta} E[v + p(Q - \alpha Q^\beta \xi)^+ - (p - c)Q]^+ \end{aligned}$$

$$\begin{aligned}
 &= v - \frac{1}{\eta} \int_A^\infty [v - (p - c)Q + p(Q - \alpha Q^\beta \xi)^+] \\
 &= v - \frac{1}{\eta} \int_A^{\frac{Q^{1-\beta}}{\alpha}} [v + cQ - p\alpha Q^\beta x]^+ dF(x) \\
 &\quad - \frac{1}{\eta} \int_{\frac{Q^{1-\beta}}{\alpha}}^\infty [v - (p - c)Q]^+ dF(x).
 \end{aligned}$$

For any fixed  $Q$ , we distinguish three cases in the following.

(a)  $v \leq -cQ$ . In this case,  $g(Q, v) = v$  and thus

$$\frac{\partial g(Q, v)}{\partial v} = 1 > 0.$$

(b)  $-cQ < v \leq (p - c)Q$ . In this case, we can derive that

$$g(Q, v) = v - \frac{1}{\eta} \int_A^{\frac{v+cQ}{p\alpha Q^\beta}} [v + cQ - p\alpha Q^\beta x] dF(x). \text{ Th}$$

en,

$$\frac{\partial g(Q, v)}{\partial v} = 1 - \frac{1}{\eta} F\left(\frac{v + cQ}{p\alpha Q^\beta}\right).$$

Thus,

$$\frac{\partial g(Q, v)}{\partial v} \Big|_{v=-cQ} = 1,$$

$$\frac{\partial g(Q, v)}{\partial v} \Big|_{v=(p-c)Q} = 1 - \frac{1}{\eta} F\left(\frac{Q^{1-\beta}}{\alpha}\right).$$

(c)  $v > (p - c)Q$ . In this case,  $v - (p - c)Q > 0$ , and

$$\begin{aligned}
 g(Q, v) &= v - \frac{1}{\eta} \int_A^{\frac{Q^{1-\beta}}{\alpha}} [v + cQ - p\alpha Q^\beta x] dF(x) \\
 &\quad - \frac{1}{\eta} [v - (p - c)Q] [1 - F\left(\frac{Q^{1-\beta}}{\alpha}\right)].
 \end{aligned}$$

Then,

$$\frac{\partial g(Q, v)}{\partial v} = 1 - \frac{1}{\eta} < 0.$$

Let  $v^*$  be the optimal solution of  $\max_{v \in R} g(Q, v)$  for fixed  $Q$ . It is clear that for a fixed  $Q$ ,  $g(Q, v)$  attains its maximum when  $-cQ < v \leq (p - c)Q$ .

If  $Q \geq [\alpha F^{-1}(\eta)]^{\frac{1}{1-\beta}}$ , then  $v^*$  satisfies

$$1 - \frac{1}{\eta} F\left(\frac{v + cQ}{p\alpha Q^\beta}\right) = 0,$$

thus

$$v^* = p\alpha Q^\beta F^{-1}(\eta) - cQ,$$

and

$$g(Q, v^*) = -cQ + \frac{1}{\eta} \int_A^{F^{-1}(\eta)} p\alpha Q^\beta x dF(x).$$

The derivative of  $g(Q, v^*)$  with respect to  $Q$  can be written as:

$$\frac{dg(Q, v^*)}{dQ} = -c + \frac{\alpha\beta Q^{\beta-1}}{\eta} \int_0^{F^{-1}(\eta)} x dF(x) \leq p\beta - c < 0,$$

where the inequality follows from  $0 < \beta < 1$ .

If  $Q < [\alpha F^{-1}(\eta)]^{\frac{1}{1-\beta}}$ , then  $v^* = (p - c)Q$ . We thus have:

$$g(Q, v^*) = (p - c)Q + \frac{1}{\eta} \int_A^{\frac{Q^{1-\beta}}{\alpha}} [pQ - p\alpha Q^\beta x] dF(x),$$

Then:

$$\frac{dg(Q, v^*)}{dQ} = (p - c) + \frac{1}{\eta} \int_A^{\frac{Q^{1-\beta}}{\alpha}} [p - p\alpha\beta Q^{\beta-1} x] dF(x).$$

From the first-order condition,  $\tilde{Q}$  satisfy:

$$p - c - \frac{p}{\eta} (1 - \beta) F\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) - \frac{p\alpha\beta\tilde{Q}^{\beta-1}}{\eta} \Lambda\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) = 0.$$

Let

$$G(Q) = p - c - \frac{p}{\eta} (1 - \beta) F\left(\frac{Q^{1-\beta}}{\alpha}\right) - \frac{p\alpha\beta Q^{\beta-1}}{\eta} \Lambda\left(\frac{Q^{1-\beta}}{\alpha}\right),$$

then,

$$\begin{aligned}
 G(A) &= p - c - \frac{p}{\eta} (1 - \beta) F\left(\frac{A^{1-\beta}}{\alpha}\right) - \frac{p\alpha\beta A^{\beta-1}}{\eta} \Lambda\left(\frac{A^{1-\beta}}{\alpha}\right) \\
 &> p - c - \frac{p}{\eta} F\left(\frac{A^{1-\beta}}{\alpha}\right) > 0,
 \end{aligned}$$

where the last inequality follows from  $\alpha > \max(1, \frac{1}{A})$ ,

and

$$\begin{aligned}
 G([\alpha F^{-1}(\eta)]^{\frac{1}{1-\beta}}) &= -c + p\beta - \frac{p\beta}{\eta F^{-1}(\eta)} \int_A^{F^{-1}(\eta)} F(x) dx \\
 &< -c + p\beta < 0
 \end{aligned}$$

where the last inequality follows from  $p < \frac{c}{\beta}$ .

Taking the derivative of  $G(Q)$  gives

$$\frac{dG(Q)}{dQ} = -\frac{p(1-\beta)}{\eta} \left[ \frac{(1-\beta)}{\alpha Q^\beta} f\left(\frac{Q^{1-\beta}}{\alpha}\right) + \beta Q^{1-2\beta} F\left(\frac{Q^{1-\beta}}{\alpha}\right) + \alpha\beta Q^{-\beta} \Lambda\left(\frac{Q^{1-\beta}}{\alpha}\right) \right].$$

Notice that:

$$\alpha > \max\left(1, \frac{1}{A}\right) \text{ and } 0 < \beta < 1, \text{ thus } \frac{dG(Q)}{dQ} < 0,$$

and  $G(Q)$  is monotonic in interval:

$$(A, [\alpha F^{-1}(\eta)]^{\frac{1}{1-\beta}}).$$

From the intermediate value theorem, we have

$$G(Q) = 0 \text{ has unique solution in } (A, [\alpha F^{-1}(\eta)]^{\frac{1}{1-\beta}}).$$

This completes the proof.

The following proposition describes how the optimal decisions  $\tilde{Q}$  changes with system parameters.

**Proposition 2** If sale price  $p < c/\beta$ , then

- (1)  $\tilde{Q}$  is increasing in the size of the market  $\alpha$  ;
- (2)  $\tilde{Q}$  is increasing in the degree of risk-aversion  $\eta$  ;
- (3)  $\tilde{Q}$  is decreasing in the order cost  $c$ ;
- (4)  $\tilde{Q}$  is increasing in the sale price  $p$ .

**Proof.** From Proposition 1, we have  $\tilde{Q}$  is the unique solution for

$$G(Q) = p - c - \frac{p}{\eta} (1-\beta) F\left(\frac{Q^{1-\beta}}{\alpha}\right) - \frac{p\alpha\beta Q^{\beta-1}}{\eta} \Lambda\left(\frac{Q^{1-\beta}}{\alpha}\right) = 0$$

And  $\frac{\partial G(\tilde{Q})}{\partial \tilde{Q}} < 0$ .

(1) By the implicit function rule, we have

$$\frac{d\tilde{Q}}{d\alpha} = -\frac{\partial G(\tilde{Q})/\partial \alpha}{\partial G(\tilde{Q})/\partial \tilde{Q}},$$

where

$$\frac{\partial G(\tilde{Q})}{\partial \alpha} = \frac{p\tilde{Q}^{1-\beta}}{\eta} \left[ \frac{1-\beta}{\alpha^2} f\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) + \frac{\beta\tilde{Q}^{1-\beta}}{\alpha} F\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) - \beta\Lambda\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) \right] > 0.$$

Thus,  $\frac{d\tilde{Q}}{d\alpha} > 0$ , that is,  $\tilde{Q}$  is increasing in  $\alpha$ .

(2) By the implicit function rule, we have

$$\frac{d\tilde{Q}}{d\eta} = -\frac{\partial G(\tilde{Q})/\partial \eta}{\partial G(\tilde{Q})/\partial \tilde{Q}},$$

Where:

$$\frac{\partial G(\tilde{Q})}{\partial \eta} = \frac{p}{\eta^2} (1-\beta) F\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) + \frac{p\alpha\beta\tilde{Q}^{1-\beta}}{\eta^2} \Lambda\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) > 0.$$

Thus,  $\frac{d\tilde{Q}}{d\eta} > 0$ , that is,  $\tilde{Q}$  is increasing in  $\eta$ .

(3) By the implicit function rule, we have

$$\frac{d\tilde{Q}}{dc} = -\frac{\partial G(\tilde{Q})/\partial c}{\partial G(\tilde{Q})/\partial \tilde{Q}} = \frac{1}{\partial G(\tilde{Q})/\partial \tilde{Q}} < 0.$$

Thus,  $\tilde{Q}$  is decreasing in  $c$ .

(4) By the implicit function rule, we have

$$\frac{d\tilde{Q}}{dp} = -\frac{\partial G(\tilde{Q})/\partial p}{\partial G(\tilde{Q})/\partial \tilde{Q}},$$

where

$$\frac{\partial G(\tilde{Q})}{\partial p} = 1 - \frac{1-\beta}{\eta} F\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) - \frac{\alpha\beta\tilde{Q}^{\beta-1}}{\eta} \Lambda\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) > 0,$$

Thus,  $\frac{d\tilde{Q}}{dp} > 0$ , that is,  $\tilde{Q}$  is increasing in  $p$ .

This completes the proof.

Substituting (7) into (3), and taking expectation, we can write the optimal system expected profit as:

$$E[\pi(\tilde{Q})] = E[p\tilde{Q} - c\tilde{Q} - p(\tilde{Q} - \alpha\tilde{Q}^\beta \xi)^+]$$

$$\begin{aligned}
 &= p\alpha\tilde{Q}^\beta \int_A^{\tilde{Q}^{1-\beta}} x dF(x) - p\tilde{Q}F\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) + (p-c)\tilde{Q} \\
 &= \left(\frac{p\tilde{Q}}{\beta} - p\tilde{Q}\right)F\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) + \left(\frac{\eta\tilde{Q}}{\beta} - \tilde{Q}\right)(c-p) . \quad (8)
 \end{aligned}$$

From the above equation, we have the way by which  $E[\pi(\tilde{Q})]$  depends on  $\beta$ ,  $\gamma$  and  $\eta$  is more complex in general. In the following section, we explore this problem through a specific numerical example to investigate the possible effect of system parameters on expected profit.

#### IV. NUMERICAL EXAMPLES

As an example we consider the case where  $\xi$  follows a uniform distribution on  $[0, B]$ . Then, we have:

$$f(x) = \frac{1}{B}, F(x) = \frac{x}{B},$$

And

$$\Lambda\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) = \frac{\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right)^2}{2B}.$$

This assumption about stochastic factor in demand is used in a lot of papers, such as [2], [5], [11] and [12]. The optimal expected profit is given by:

$$E[\pi(\tilde{Q})] = (p-c)\tilde{Q} - \frac{p\tilde{Q}^{2-\beta}}{2\alpha B}, \quad (9)$$

where  $\tilde{Q}$  satisfy

$$p-c - \frac{p}{\eta}(1-\beta)F\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) - \frac{p\alpha\beta\tilde{Q}^{\beta-1}}{\eta}\Lambda\left(\frac{\tilde{Q}^{1-\beta}}{\alpha}\right) = 0. \quad (10)$$

The impacts of risk-averse level  $\eta$ , the sale price  $p$ , order cost  $c$ , and the shelf space elasticity  $\beta$  on the expected profit are presented in figure 1-4.

In figure 1, we let  $\alpha = 1, \beta = 0.5, B = 1, c = 3$ , and  $p = 3$  in order to investigate the impacts of risk-averse level  $\eta$  on the expected profit.

In figure 2, we let  $\alpha = 1, \beta = 0.5, B = 1, c = 3$ , and  $\eta = 0.5$  in order to investigate the impacts of sale price  $p$  on the expected profit.

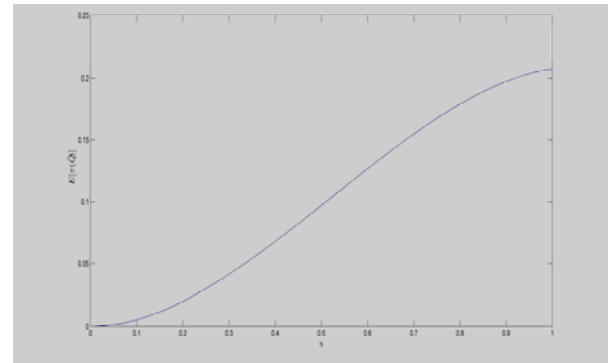


Fig. (1). Influence of risk averse level on expected profit

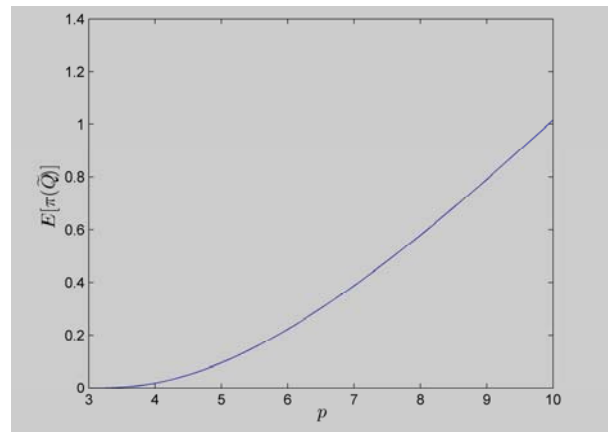


Fig. (2). Influence of sale price on expected profit

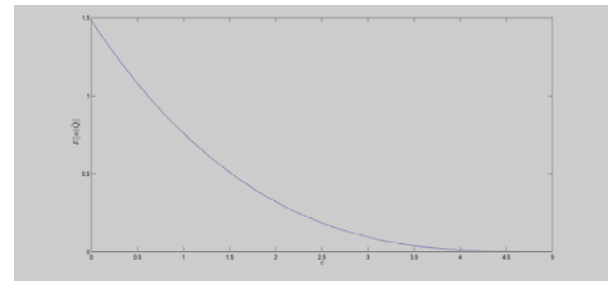


Fig. (3). Effect of order cost on expected profit

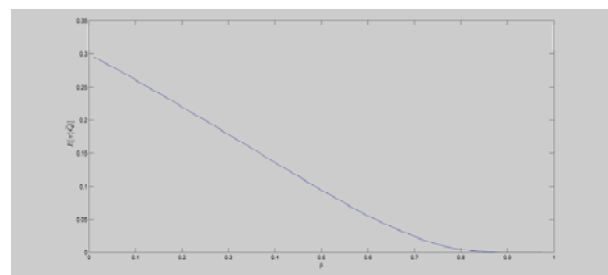


Fig. (4). Effect of shelf space elasticity on expected profit

In figure 3, we let  $\alpha = 1, \beta = 0.5, B = 1, p = 5$ , and  $\eta = 0.5$  in order to investigate the impacts of order cost  $c$  on the expected profit.

In figure 4, we let  $\alpha = 1, c = 3, B = 1, p = 5$ , and

$\eta = 0.5$  in order to investigate the impacts of shelf space elasticity  $\beta$  on the expected profit.

From figure 1, we find the expected profit is increasing in  $\eta$ , that is to say the more risk averse the retailer is, the smaller  $E[\pi(\tilde{Q})]$  is. From figure 2, we have the expected profit is increasing in sale price  $p$ . From figure 3, we have the expected profit is decreasing in order cost  $c$ . From figure 4, we see the expected profit is decreasing in shelf space elasticity  $\beta$ , but the decreasing speed of  $E[\pi(\tilde{Q})]$  is decreasing in  $\beta$ , which imply the more price elasticity is, the smaller the impact of  $\beta$  on  $E[\pi(\tilde{Q})]$  is.

## V. CONCLUSIONS

We have explored the shelf space decision for a risk-averse newsvendor. We use the CVaR criterion as the performance measure. We give the optimal shelf space decision for multiplicative demand models. Under the CVaR criterion, we perform the comparatics to investigate the impacts of ordering cost, shelf space elasticity, sale price, and degree of risk-aversion on the optimal shelf space. In addition, we presents the numerical examples to illustrate the possible impact of system parameters on retailer's expected profit. Our analysis complements the past work in this area and offers managerial insights to the practitioners or managers in similar decision-making settings. We think it is interesting to investigate the shelf space decision under other risk-averse measure in the future.

## CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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