

## Energy Distribution Analysis of Explosive Shockwave Pressure Based on Preferred Wavelet Packet and Wigner-Ville Distribution

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**Abstract** - Energy distribution is an important factor in evaluating damage effectiveness of explosive blast, including overpressure and impulse. It means that the higher the energy in certain frequency zone, the higher effective of damage to the target which has the same nature frequency zone. However, the energy extraction for explosive blast is quite difficult because it's steep rising, short duration and with a lot of conference signals. Therefore, a new energy analysis method based on preferred wavelet packet and Wigner-Ville distribution (WVD) is proposed. In this paper, firstly, we construct the wavelet packet to reduce the noise of signal, and then decompose the original signal, later calculate the energy spectra through the wavelet packet. Secondly, the WVD is introduced to extract the frequency and amplitude characteristics of each period of shockwave signal. Finally, three measured signals are taken as examples to verify the effectiveness and reliability of proposed method. Taking into account nature frequency of specific targets, this method can provide reference for achieving high efficiency damage of explosive blast.

**Keywords** - explosive shockwave; energy spectrum; preferred wavelet packet; Wigner-Ville distribution (WVD)

### I. INTRODUCTION

The estimating of explosion shockwave is of great importance, since this is concerned with protecting property and persons working on sites that handle, store or transport large quantities of flammable materials, whether the explosion occurs at a close or a distant location. In all kinds of warhead static, dynamic explosion experiments, explosion shockwave is an approval index in evaluating the warhead damage efficiency [1]. The evaluation criteria of the explosion shockwave include overpressure criteria, impulse criteria, and overpressure and impulse criterion. But a large number of experiments show that the larger shockwave overpressure and impulse don't always bring the more serious consequences [2]. In fact, the damage of target is also related to the energy distribution about frequency of shockwave [3]. Moreover, the higher the energy spectrum in a frequency band of explosion shockwave is, the strong the damage of target in the same frequency band is. However many of the current researches are focused on the analysis of wave signals in time domain or frequency domain and don't consider the feature of energy spectrum [4], and there is no universal energy attenuation model so far. In order to assess the damage of warheads accurately, it is essential to study energy spectrum and its attenuation model of explosion shockwave.

From the past years, the extraction of energy spectrum is mainly through FFT, STFT, and wavelet transform in many studies [5]. Traditional FFT signal always analysis signals in the frequency domain, but frequency of one point in time domain cannot be obtained in general; STFT usually is used to deal periodic stationary signal. Since the

window function of STFT is fixed, it cannot be used to process signals with mutation frequency. Preferred wavelet packet and the quadratic time-frequency transform which developed localized thought of STFT [6-7]. So it can choose corresponding frequency band adaptively to match the frequency spectrum of the signal phase according to the signal characteristics and analysis requirements to overcome the shortcoming that the window size does not vary with frequency [8]. Wavelet packet is used in underwater pressure signal analysis and then signal frequency range is received [9-10]. WVD is used in rock explosion wave signal to find the signal energy in different time periods [11]. The explosion shockwave signal is a typical transient non-stationary signal which is steep rising, and short lasting. Besides, because the result is influenced by many parasitic effects, so measured shockwave signal is with complex component. Consequently, so it's necessary to de-noise the measured signal before later analysis.

Therefore, a new preferred Wavelet packet combining with WVD method is proposed, which is based on the characteristics of the non - stationary and the complex components of the signal, aiming at extracting energy spectrum of blast shockwave pressure signal. An experiment is designed to study the energy spectrum of the signals, which are derived from TNT explosives at 3 different distances. In the paper, firstly, wavelet packet is used to pre-treatment signals, to separate signal and noise effectively; then wavelet packet is used to decompose the de-noised signals and extract energy spectra from different frequency; later, WVD is used to do time-frequency analysis and extract the value of the signal energy in a specific period of time. Designing suitable

explosive distance aiming at different target according to the natural frequencies of the target, this method can lay the foundation for the realization of the high efficiency damage to the target of the explosion shockwave.

## II. PRINCIPLE OF PREFERRED WAVELET PACKET AND WVD

### A. Wavelet Packet Decomposition

The signal is decomposed into low frequency and high frequency by wavelet packet algorithm. In the next layer, frequency and the time analysis of the low frequency part and the high frequency part are analyzed at the same time, which improved the results of the time-frequency resolution [12-13].

Definitions of wavelet packet function, is:

$$u_{j,k}^n(t) = 2^{j/2} u(2^j t - k) \quad (1)$$

where  $n = 0, 1, 2, \dots$ , which means oscillating function  $j \in Z$  and  $k \in Z$ , which is scale parameter and translation parameters in the equation above.

Wavelet packet function should satisfy the following two-scale equations:

$$\begin{cases} u_{2n}(t) = \sqrt{2} \sum_{k \in Z} h(k) u_n(2t - k) \\ u_{2n+1}(t) = \sqrt{2} \sum_{k \in Z} g(k) u_n(2t - k) \end{cases} \quad (2)$$

When, the initial two wavelet packet functions are given by:

$$\begin{cases} u_0(t) = \sqrt{2} \sum_{k \in Z} h(k) u_0(2t - k) \\ u_1(t) = \sqrt{2} \sum_{k \in Z} g(k) u_0(2t - k) \end{cases} \quad (3)$$

Then define

$$u_0(t) = \varphi(t), \quad u_1(t) = \psi(t) \quad (4)$$

Where  $\varphi(t)$  and  $\psi(t)$  means orthogonal scaling function and orthogonal wavelet function.

As a result, the collection of functions  $\{u_n(t)\}, n = 0, 1, 2, \dots$ , according to equation, (2), is called the wavelet packet of the orthogonal scaling function. The original signals de-noising process with wavelet packet can be divided into three steps, wavelet packet decomposition of signal, threshold quantization of wavelet packet decomposition coefficient and

reconstruction of signals' wavelet packet. Assuming that  $x(t)$  represents the measured signals, then algorithm of wavelet packet's decomposition and reconstruction is calculated using

$$\begin{cases} d_{j+1}^{2n} = \sum_k h(k - 2t) d_j^n(k) \\ d_{j+1}^{2n+1} = \sum_k g(k - 2t) d_j^n(k) \end{cases} \quad (5)$$

$$d_j^n = 2 \left[ \sum_{\tau} h(k - 2\tau) d_{j+1}^{2n+1}(k) + \sum_{\tau} g(k - 2\tau) d_{j+1}^{2n+1}(k) \right] \quad (6)$$

where  $d_j^n(k)$  means the corresponding number  $k$  coefficients of nodes  $(j, n)$  after wavelet packet decomposition and  $(j, n)$  means the number  $n$  frequency band in the number  $j$  layer.

### B. Calculation of Energy Spectrum Based on Wavelet Packet

The measured signal is decomposed into  $j = 2^i$  sub-bands in the number  $i$  layer using the binary scale transform, based on wavelet packet decomposition algorithm. The band width of each layer of the reconstructed signal is  $f_s/2^i$  and  $f_s$  means the NYQUIST frequency of signals. Through analyzing the energy distribution of each frequency component of the original case, the energy distribution of each frequency component of the original signal and the position of the main vibration frequency are easy to obtain. The distribution of the frequency band node in two layer of wavelet packet decomposition is shown in Fig.1.

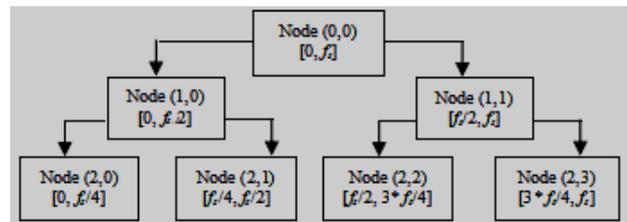


Fig.1 Frequency distribution tree of two layer wavelet packet

Assuming the energy corresponding to the reconstructed signal in the number  $i$  layer of the decomposition is  $E_{i,j}$ , which is defined by the relation

$$E_{i,j} = \int |w_{i,j}(t)|^2 dt = \sum_{k=1}^m |x_{j,k}|^2 \quad (7)$$

Where  $x_{j,k}$  means the discrete amplitudes of reconstructed signal  $w_{i,j}$ ;  $j = 0, 1, \dots, 2^i - 1$ ;  $k = 1, 2, \dots, m$ ;  $m$  is the number of signal discrete sampling points

Equation, (7), gives the energy of the signal at a given frequency. Assuming the total energy of the signal being analyzed is  $E$ , there is

$$E = \sum_{j=0}^{2^i-1} E_{i,j} \quad (8)$$

The energy proportion of each band accounting for the total energy of signal is

$$e_j = \frac{E_{i,j}}{E} \times 100\% \quad (9)$$

### C. Wigner-Ville distribution

The traditional Fourier transform converts the time domain signal to the frequency domain. However, this method does not have any time domain information, and it cannot give a specific time period for the occurrence of a certain frequency component. As a result, this paper chooses Wigner-Ville distribution to extract energy features of the shockwave pressure signal.

$$W_x(t, \omega) = \int x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right) e^{-j\omega\tau} d\tau \quad (10)$$

Wigner-Ville distribution was proposed by Wigner in 1932 and used in quantum mechanics, which was then introduced into the field of signal processing. It can be regarded as the distribution of signal energy in both time and frequency domains. Wigner-Ville distribution of a single signal is defined as its instant autocorrelation, which is FFT transform of:

$$r_x(t, \omega) = x\left(t + \frac{\omega}{2}\right) x^*\left(t - \frac{\omega}{2}\right).$$

So the Wigner-Ville distribution is:

$$W_x(t, \omega) = \int x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right) e^{-j\omega\tau} d\tau \quad (11)$$

Therefore, the time-frequency characteristics of the measured shockwave signal can be calculated by the equation, (11).

### III. EXPERIMENTAL SETUP AND RESULTS

In order to study the energy spectrum characteristics of explosion shockwave obtained from the same explosives in different blast center distance, it is necessary to have enough experimental data. A set of experiments are designed for pressure test of explosion shockwave.

The test consists of 12 groups explosive shockwave, including in 3m, 4m, and 6m. The shockwave pressure measuring system mainly consists of ICP-type shockwave pressure sensors, KISTLER's signal conditioner, and PXI data acquisition system, as shown in Fig.2. Fig. 3 shows the layout of three shockwave pressure sensors set on experiment. The vital setups of measuring system adopt the external trigger. In the experiment, the explosive source installation height is 3m, the sampling frequency of data acquisition system is 1MHz, and the whole sampling time is 25ms.

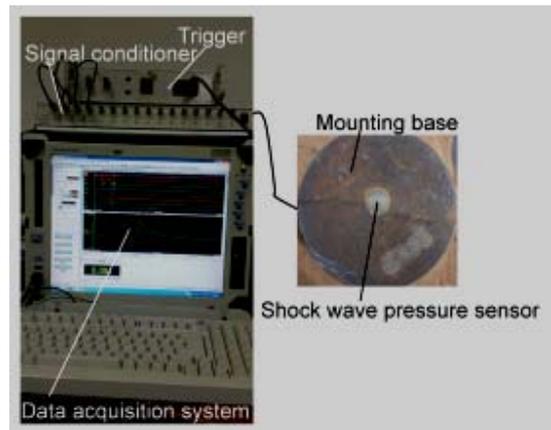


Fig.2 Explosion shockwave pressure measuring system

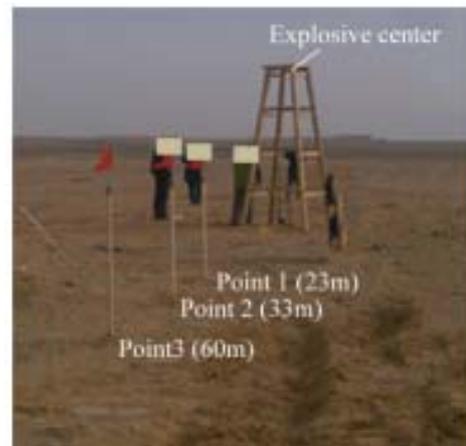
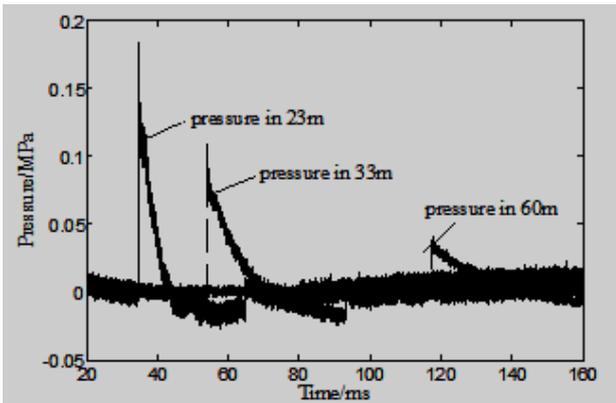
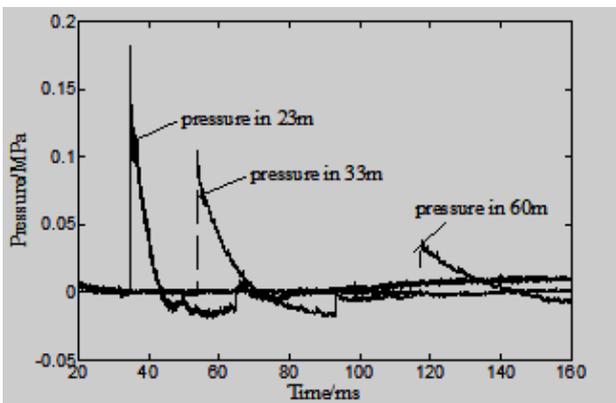


Fig.3 The layout scheme of shockwave pressure sensors

The experiment obtains 3 groups explosion shockwave pressure signals totally. Fig.4 (a) shows original explosion shockwave pressure signals. Through analysis of Fig.4 (a), the explosive shockwave signal is a typical transient non-stationary signal which has steep rising edge, fast mutation and short duration. However, as affected by electromagnetic interference, high heat and mechanical shock, the shockwave pressure signal contains serious noise. Therefore, it is necessary to discover an effective method to extract the characteristics of explosion shockwave energy spectrum.



-a. Measured shockwave pressure signals



-b. Explosion shockwave pressure signals after filtering

Fig.4 Explosion shockwave pressure signal before and after filtering

IV. ENERGY SPECTRUM ANALYSES

A. Wavelet packet analysis

Through the analysis of the commonly used wavelet packet, db8 wavelet packet is selected to decompose the data to 15 layers. The result is sub-bands and the frequency resolution of each sub-band is . To reduces noise, the front 655 sub-bands, which bandwidth is about 0~10 kHz, are reconstructed by wavelet packet. And finally, the filtered signal was obtained, as shown in Fig.4 (b). By comparison with Fig.4 (a), the result shows the wavelet packet filter reduces the noise the original signal, while retains the non-stationary, fast mutation, and short duration properties of explosion shockwave pressure signal.

According to Fig.4 (b), the peak value of shockwave pressure at 23m is 0.182MPa, and the positive pressure duration is about 30.5ms; the peak value of shockwave pressure at 33m is 0.105MPa, and the positive pressure duration is about 40.2ms; and the peak value of shockwave pressure at 60m is 0.036MPa, and the positive

pressure duration is about 53.8ms. The shockwave pressure quickly arrive the peak, and then begin to decay rapidly. In the initial stage, the shockwave pressure is almost exponentially decreased, when dropped to a certain extent, the waveform is widening, and the rate of decline is slow. The results show that, in the case of an explosion, the peak value of shockwave pressure decreases with the increase of the distance of the measuring points.

According to the equation, (7), the energy spectrum of the signal is extracted by the wavelet packet decomposition, and the energy spectrum of the three kinds of explosion shockwave is obtained, as shown in Fig.5. To be more compact, the results are only in the range of 300Hz. After normalized the energy of three distance of shockwave pressure, the percentage of energy spectrum is achieved, as shown in Table 1.

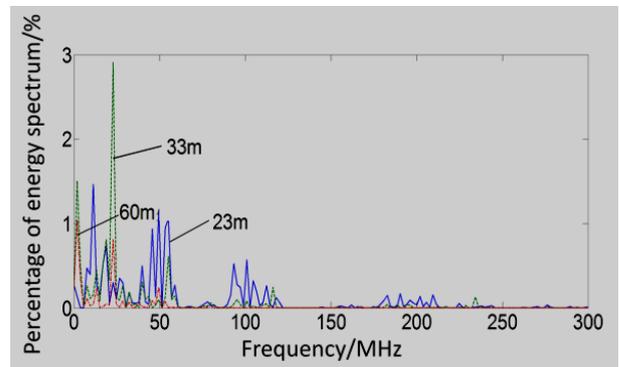


Fig.5 Energy distributions of three measured signals

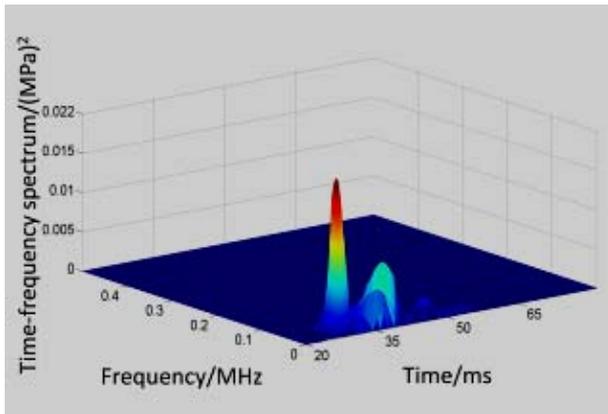
TABLE 1 ENERGY PERCENTAGE DISTRIBUTIONS OF SIGNALS IN DIFFERENT FREQUENCY BANDS

f/kHz	Energy percentage distribution/%		
	23m	33m	60m
0~200	69.84	85.25	93.95
200~500	8.61	6.72	1.87
500~1000	5.46	1.01	1.14
1000~2000	4.84	4.67	1.41
2000~3000	3.78	1.06	1.03
>3000	7.47	1.29	0.6

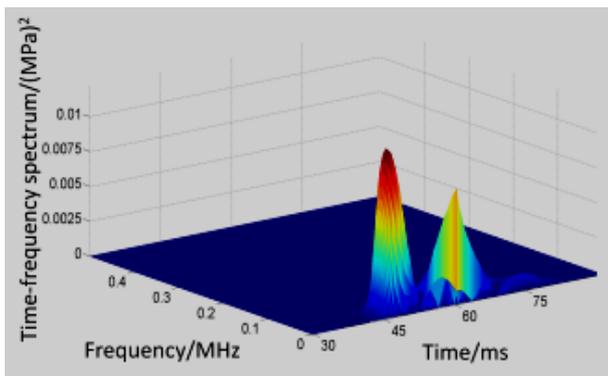
As can be seen, in fact, the energy distribution of explosive shockwave is in a wide range, and the energy spectrum at low frequency is larger. At 23m, the impact energy of the shockwave in the range of 0-500Hz is 78.45%. At 33m is 91.97%, and at 60m is 95.82%. The farther the distance is, the higher the percentage of low-frequency shockwave energy spectrum is, and on the contrary, the smaller percentage of high-frequency energy spectrum. Because when the frequency of the shockwave signal is equal to or close to the natural frequency of the target, the resonance response can be produced, this may damage the target more easily.

*B. Wigner-Ville distribution of shockwave*

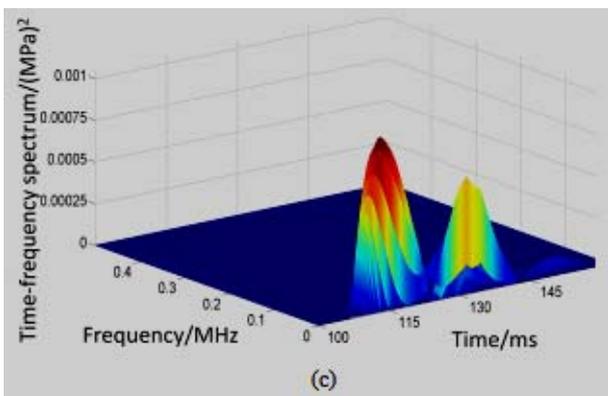
In fact, the energy spectrum characteristics of different frequency components and different time periods cannot be obtained according to Fig.5 in the same time. So the shockwave signal is analyzed by WVD, to obtain the three-dimensional relationship of time, frequency and energy of the signal.



a. WVD of explosive blast shockwave at 23m



b. WVD of explosive blast shockwave at 33m



c. WVD of explosive blast shockwave at 60m

Fig.6 Wigner-Ville distribution of explosive blast shockwave

The Wigner-Ville distribution time-frequency distribution at 23m, 33m and, 60m is calculated by the equation, (4), and the results are shown in Figs.6 (a), 6(b), and 6(c). Fig.6 shows: (1) Three-dimensional time-frequency distribution directly locates the time, frequency and energy of explosive shockwave; (2) By comparison Fig.6(a) with Fig.4, the peak time of energy spectrum is the same as the peak time of shockwave pressure; (3) Shockwave energy spectrum distributes over a wide range, The shockwave energy of 23m is located at 24.5-55ms, the main energy is concentrated in 500Hz, and there is a peak value, which corresponds to the measured wave in Fig.4. The shockwave energy of 33m is located at 43.9-84.1ms, the main energy is concentrated in 500Hz, and the energy peak is less than 23m. The impact energy of 60m is located at 107.1-160.9ms, the main energy is concentrated in 500Hz, and the energy peak is less than that of 33m. In short, energy spectrum amplitude is larger in arrange 200Hz; the damage effect of target in the range of 200Hz is enhanced.

IV. CONCLUSIONS

In summary, we an improved HHT method has been introduced to study the energy spectrum characteristics. However, the existing shockwave evaluation criteria are not related to the energy spectrum. Therefore, in this paper, a method for damage effectiveness evaluating the blast wave energy spectrum and natural frequency of target is presented. In this paper, the energy spectrum of the explosive shockwave pressure is analyzed by wavelet packet decomposition and Wigner-Ville distribution. According to the above analysis, the energy spectrum of TNT explosive at different distance is extracted effectively. The result shows that the energy distribution of explosive shockwave is in a wide range, and the energy spectrum is distributed at 0-200Hz low frequency. Therefore, in order to achieve the goal of efficient damage, the charge and burst distance can be adjusted for large energy spectrum in the same frequency with target, according to analysis of the natural frequency of specific target. Subsequent work will be carried out in depth study of the energy spectrum characteristics of different explosives.

REFERENCE

- [1] ZHANG Y P, CHI J CH, GONG Y Q, et al. Measure technology of blast shockwave pressure and disposal method of complicated signal [J]. Chinese Journal of Scientific Instrument, 2007, 28(8) (Suppl): 324-327.
- [2] CAO F X. Research on comprehensive damage effect from explosion [D]. Nanjing: Nanjing University of Science and Technology, 2008.
- [3] ZHANG Y F, DU H M, ZU J. Research on post treatment method for shockwave signals [J]. Engineering & Test, 2010, 50(4): 15-18.

- [4] SSCHET I, GARDEBAS D, CALDERARA S, et al. Blast wave parameters for spherical explosives detonation in free air [J]. *Open Journal of Safety Science and Technology*, 2011, 1-31.
- [5] OMANG M, CHRISTENSEN S.O, BORVE S, et al. Height of Burst Explosions: A comparative study of numerical and experimental results [J]. *Journal Shockwaves*, 2009, 19(2): 135-143.
- [6] JEREMIE R, JIE Z. An approach to determining the TNT equivalent of high explosives [J]. *Scientific Technical Review*, 2006, 56(1): 58-62.
- [7] KONG D R, LI Y X, ZHU M W, et al. The Analysis of Error Factors on the Measure Precision of Piezoelectricity. Pressure Measuring System and Precaution [J]. *Chinese Journal of Scientific Instrument*, 2002, (23): 160-162.
- [8] LI S Y, LIN J. ECG signal de-noising using a combined wavelet transform algorithm [J]. *Chinese Journal of Scientific Instrument*, 2009, 30(4): 689-693.
- [9] LI W, ZHANG Z H, LI Q M, et al. Research on features of time-frequency of underwater target by underwater explosion [J]. *Journal of Ship Mechanics*, 2013, 17(7): 800-806.
- [10] WEN H B, ZHANG J, YIN Q. Time- frequency characteristic analysis of underwater explosion pressure based on wavelet transform [J]. *Journal of Vibration, Measurement & Diagnosis*, 2008, 28(2): 155-158.
- [11] KONG L, SU J J, LI Z R, et al. Energy spectrum analysis of several kinds of explosive blast [J]. *Chinese Journal of Explosives & Propellants*, 2010, 33(6): 76-79.
- [12] FREDERIK J S, DANDO B D E, RICHARD M A. Automatic detection and rapid determination of earthquake magnitude by wavelet multi-scale analysis of the primary arrival[J]. *Earth and Planetary Science Letters*, 2006, 250(1/2): 214-223.
- [13] ZHAO J P, LIN H. Energy Distribution and separation of blast wave based on time-frequency energy analysis technology [J]. *Chinese Journal of Rock Mechanics and Engineering*, 2012, 31(A01): 3278-3285.
- [14] DAUBECHIES I. The wavelet transforms, time-frequency localization and signal analysis [J].*IEEE Trans Inf Theory*, 1990, 36(5):961-1005.
- [15] LI W, ZHANG Z J, CHEN C H, et al. Feature of energy distribution of underwater target by underwater Explosion [J]. *Chinese Journal of high Pressure Physics*, 2012, 26(5): 537-544.
- [16] CAI K. Semi-blind fetal electrocardiogram extraction by suppressing the cross-terms of the Wigner-Ville representations [J]. *Chinese Journal of Scientific Instrument*, 2012, 33(4): 781-787.
- [17] WOOLFSONMS, BIGANBC, CROWEAJ A, et al. Method to separate sparse components from signal mixtures [J]. *Digital Signal Processing*, 2008, 18(6): 985-1012.
- [18] JIAO C J, DING J M. Analysis of natural frequency of pre-stressed composite beam with external tendons [J]. *Engineering Mechanics*, 2011, 28(2): 193-197.
- [19] SHEN Y, GE AH H. Analysis on frequency of free vibration on pedestrian over cross [J]. *Unique construction*, 2004, 21(1): 53-55.