Gravity Inversion Computations for Mass Anomaly Distribution of Regional Crust Based on Virtual Sphere Theory

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Abstract — **Based on the earth's geophysical structure, the point mass method is used to compute the inverse of the regional crust mass distribution. The earth is divided into two parts: the main body that is composed of several homogeneous concentric layers, and the abnormal mass in the regional crust. We calculate the gravity anomalies observations by the earth's gravity model with high accuracy and order. Then the point-mass model is used based on virtual sphere theory to simulate the experiments. In addition, mass depth is set to within 20km and the inversion region is less than observation area. Numerical inversion results show that there is strong relationship between the mass anomaly distribution of regional crust and the gravity anomaly on the surface.**

Keywords - mass anomaly distribution, virtual sphere theory, gravity inversion, point mass model.

I. INTRODUCTION

Earth's internal structure has been a difficult research in geophysical field for a long time. For dealing with this issue, there are many techniques such as seismic waves, high-pressure test, earth drilling and so on[1]. However, gravity inversion is usually ignored. Despite the gravity method can simulate internal structure of the earth [2], the study in this field is not enough, and we also need new theories and methods to solve these problems. In last century, virtual sphere theory was proposed to construct the regional gravity field model [3-5], and its theoretical foundation is Molodesky boundary value problem of which is equivalently converted into a simple virtual sphere boundary value problem. In numerical experiments, the theory of virtual sphere is evolved into point mass method. Chinese scholars improved this method by using the finite element method to approximate the local gravity field model[6]. The advantage of point-mass model is that the kernel function is simple [7]. Therefore, according to the superposition of the point masses, the model can be formatted by different frequency of the earth [7-9]. Based on the earth's interior structure [10], the earth's gravitational source is divided into two parts: abnormal masses in the regional crust and the main part that is composed of the mean concentric sphere. Virtual sphere theory is used to inverse the abnormal masses. A typical region is chosen to invers the masses distribution of the earth.

II. MEAN SPHERE THEORY

In physical geodesy, let us set a homogeneous density spherical layer of which the mass is M. Then the layer will generate a gravitational field. According to the Newton's law of universal gravitation, the external gravitational potential of the layer can be expressed as:

$$\mathrm{V} = \frac{fM}{r} \tag{1}$$

Where f is a constant value [11] which is set as $6.67 \bullet 10^{-11}$, r is the distance from the attracted point to the center of the sphere. It can be seen from the formula that the external gravitational field is equivalent the source field produced by the centroid of which the mass is also M. We suppose that the earth is composed of many homogeneous sphere layers. Despite the densities of these spherical layers are different, the external gravitational potential expressions are consistent with the one that all of the masses are concentrated into the center point of the sphere. Therefore, it is difficult to make the earth to be separated. According to the traditional division, the earth's interior structures are shown in Tab.1. It can be seen from the table that the density of the Earth's inner core and the transition layer is constant just like the same material. Another phenomenon is that the closer to the surface, the density of the layer is smaller. The reason for this case is that the pressure on the surface is less than the one at the depth of the earth. There is liquid material in both of mantle layer and outer core, hence we can assume that the density of these layers is also a constant. This is the basic theory that we divide the earth into two parts we will talk about it later.

TABLE 1. THE EARTH'S INNER CIRCLE STRUCTURE

The structure	Depth（km）	Density（g/cm^3）	Material status
Crust	0—33	2.6—2.9	Solid
Upper mantle	33—980	3.2—3.6	Part molten
Lower mantle	980—2900	5.1—5.6	Liquid & solid
Outer core	2900—4700	10.0—11.4	Liquid
Transition layer	4700—5100	12.3	Liquid & solid
Inner core	5100—6371	12.5	Solid

According to Tab.1, we can estimate the proportion of the mass between different layers. The density d in different layers is set in Tab.2. In geodetic, the mass and the gravitational constant are always combined into one parameter because the parameter can be more precisely obtained by satellite geodesy technique. So, according to the geometry, the mass equation is written as:

$$fM = fd\frac{4\pi}{3}(r_1^3 - r_2^3) \tag{2}$$

Here, r_1 and r_2 are the layer's long radius and short radius correspondingly.

Let the earth is a plurality of concentric sphere homogeneous layers. In fact, we take this earth as the normal earth always expressed by a reference ellipsoid in geodesy field. The thickness data of the layers is shown in Tab.2. Based on equation (2), the material results calculated by the density and material multiplied with gravity are shown in Tab.2. The total mass value is $3.9769\cdot10^{14}$, almost equal to the geodetic constant parameter $fM = 3.986005\cdot10^{14}$. Hence, our hypothesis is reasonable for the total mass of the earth. It can be seen that the mass of the earth's crust is the smallest part in these layers, the outer core and mantle layers contain most of the earth's mass, up to 95.15%. Comprehensive the data of Tab.1 and Tab.2, we can find that the density of inner core is a fixed value, other layers except for crust contain liquid material. Therefore, we take these layers into homogenous mass sphere. So we divide the earth's mass into two parts: the main body is a homogenous mass sphere, and the abnormal mass is in the regional crust.

TABLE 2 THE PROPORTION OF THE MASS IN VARIOUS LAYERS

The structure	Density（g/cm^3）	Mass（fm:10^{14}m^{3}s^{-2}）	Proportion（%）
Crust	2.8	0.0313	0.79
Upper mantle	3.4	0.9302	23.39
Lower mantle	5.4	1.7329	43.57
Outer core	10.8	1.1210	28.19
Transition layer	12.3	0.0898	2.26
Inner core	12.5	0.0717	1.80
Total		3.9769	100

III. VIRTUAL SPHERE THEORY

We suppose that there is a virtual sphere of which the radius is R_B in the earth shown in Fig.1. The density of the layer is μ . According to the Newton's law, the gravitational potential can be written as.

$$V_p = f\int_{\sigma}\frac{\mu}{l}d\sigma \tag{3}$$

where l is the distance between point P and the moving point $d\sigma$ which is on the virtual sphere， and the calculation can be expressed as:

$$l = \sqrt{R_B^2 + R_P^2 - 2R_B R_P\cos\psi} \tag{4}$$

In this formula， R_P is the distance which is from point P to the center of the earth, ψ is the polar angle between point P and $d\sigma$ shown in Fig.1. ψ satisfies the triangle relation[12]:

$$\cos\psi = \sin(\varphi_p)\sin(\varphi_\sigma) + \cos(\varphi_p)\cos(\varphi_\sigma)\cos(\lambda_p - \lambda_\sigma) \tag{5}$$

where φ and λ are geocentric latitude and longitude correspondingly.

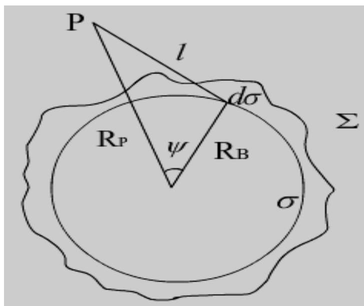

Figure.1 Virtual sphere.

To simplify the mathematics, we decompose the Earth's gravity field into the sum of the normal gravity field and the anomalous gravity field. The former gravity field, a first approximation of the actual gravity field, is generated by an ellipsoid of revolution with its center at the geo-center, called the referenced ellipsoid which of which the surface is an equi-potential surface. Since the normal gravity field can be directly evaluated from simple closed formulas, the problems are converted to the determination of the disturbing potential.

The theory of virtual is feasibility supported by the Keldysh-Lavrentiev theorem which is: If the Earth's surface $\sum$ is continuously differentiable, then any function f, harmonic and regular outside $\sum$ and continuous outside and on $\sum$, may be uniformly approximated by functions y, harmonic and regular outside an arbitrarily given sphere σ inside the Earth, in this sense that for any given ε >0, the relation |f-y|< ε 。 We assume that the anomalous potential T ' caused by virtual sphere is consistent with the one T caused by the real earth. Let us set that the virtual point masses are $\{m_i\}(i=1,2,\cdots,n)$, according to the gravitational potential feature, T' can be expressed as follow,

$$T_P^{'} = f\sum_{i=1}^{n} m_i l_{Pi}^{-1} = \sum_{i=1}^{n} M_i l_{Pi}^{-1} \qquad (6)$$

where the virtual masses $M_i = fm_i$, l is expressed in eq.(5). We get the observation data: $\{\Delta g_j\}(j=1,2,\cdots,N)$, and the boundary condition is given as follows

$$\frac{\partial T_j^{'}}{\partial r_j} + \frac{2T_j^{'}}{r_j} = -\Delta g_j \qquad (7)$$

Inserting Eq. (6) into Eq. (7), we can get

$$\Delta g_j = \sum_{i=1}^{n}[(r_j - R_B\cos\psi_{ji})l_{ji}^{-3} - 2r_j^{-1}l_{ji}^{-1}]M_i \qquad (8)$$

This equation is the observation equation, and it is a linear model. Thus we can get the unknown parameters M_i according to Eq. (8).

IV. NUMERICAL EXPERIMENTS

Inversion region is larger than the observation area, and the distance between the two edges is 1°, about 110km. The distance is determined by the Stokes integral experiment. In fact, we do not know what is the edge effect, but we can simulate the affection by gravity extension. Many experiments have been done according to Stokes integral, and one of numerical results is show in Fig.2. In this figure, the direction of gravity extension is up continues and the gravity unit is gravity disturbing in radius direction. We can find that, the maximum value in this figure is less than 8mGal. This means that, the affection of the gravity anomaly is very small when the distance is up to 50km. Therefore, in gravity extension, the edge should be set greater than 50km. Then in our experiments, we set the edge distance is 110km which is much greater than 50km because we intend to eliminate these effects.

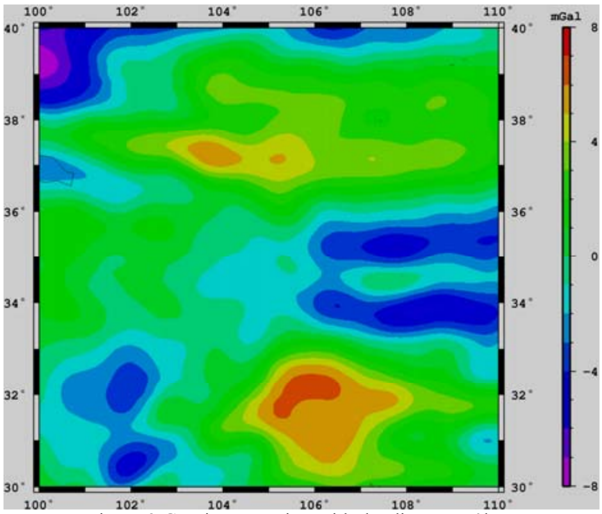

Figure.2 Gravity extension with the distance 50km.

Experiments region of observation data is $81^\circ E - 119^\circ$ E , $21^\circ E - 59^\circ$ E , shown in Fig.3. in this region, the gravity information is very abundant, and the structure of the crest is researched by many scholars. Gravity anomaly data is as the observation calculated gravity field model EGM2008[13-14]. It can be seen from the figure that the range of gravity information is -200 ~ 400mGal. Qinghai Tibet Plateau can be easily found according to the gravity anomaly profile. In the plateau, the gravity anomalies change greatly, while in other place, the gravity information undulation is gentle.

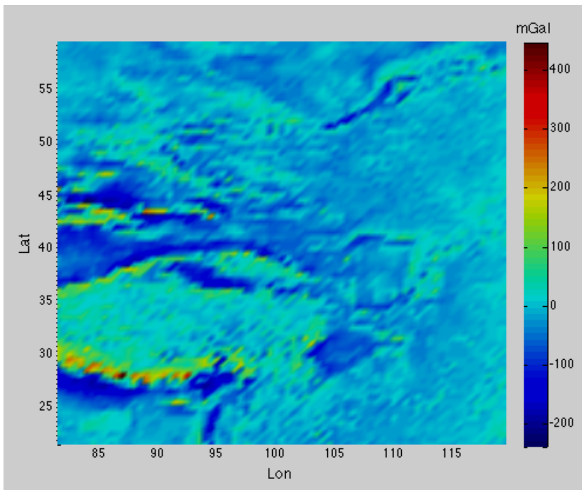

Figure.3 The gravity anomaly for the observation data

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Based on our assuming and point mass technique, anomaly mass in crust is shown in Fig.4. We have to note that the version numerical homogeneous sphere is 4.6135E9. Comparing the mass of Earth gravity value with the product 3.986005E14, the result is a small play value. This means that the experiment is more realistic. Compared with Fig.3 and Fig.4, we can find that the inversion results with observations have some similarities to a certain extent. Therefore, we calculated the correlation between the mass anomalies and the gravity anomalies, and the results are $\begin{matrix} 1.0000 & 0.0108 \\ 0.0108 & 1.0000 \end{matrix}$. The results express that the gravity anomalies and the mass anomalies are high correlated. However, the accuracy of the concrete results still needs to be verified through other methods. If we can combine thickness information of the crust, we can further calculate the area density anomalies. From the figure, it can also be found that the edge effect is obvious, showing a ribbon. The reason for this phenomenon is that the gravity anomalies are not enough. However, in order to obtain the mass anomalies of a region by gravity inversion, it is necessary to extend the inversion region. Because we should ensure that the field source of gravity anomalies must be derived from these masses.

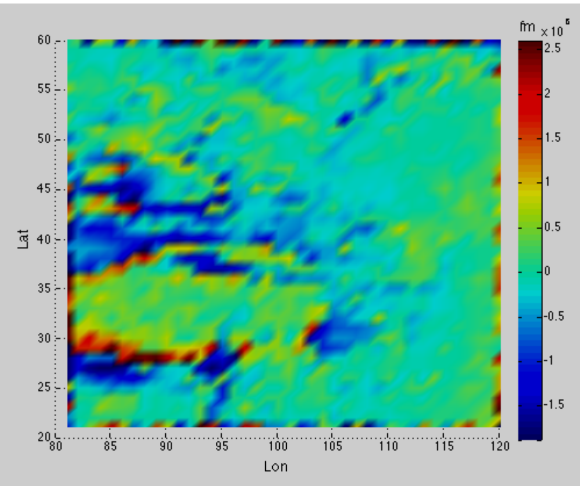

Figure. 4 Inversion results for abnormal mass in the regional crust

According to the crust model[15], the thickness information of the crust in simulated area are calculated by crust1.0 of which the resolution is 1 degree. And the results are shown in Fig.5, it can be seen that the crust thickness undulation is from 20km to 80km of which the amplitude is vary large. Compared with Fig.3 and Fig.5, the correlation between the abnormal mass and the thickness are not strong. So if we want to find more physical information in the crust, we should combine the density of the crust. We have to note that if our theory were right and the thickness is conformed, then the crust density can be obtained by gravity inversion. In addition, gravity data can be easily gotten because we can go anywhere outside the earth, while the measurement of seismic wave is difficult because it depends on the earthquake. That is to say, compared other physical methods, gravity inversion is better on the point of data acquisition.

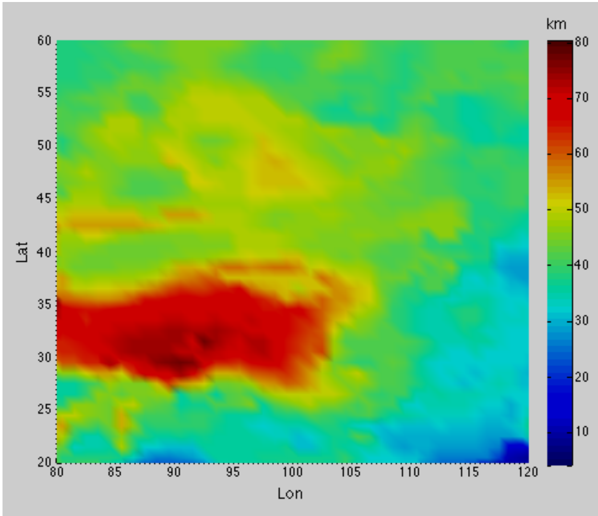

Figure. 5 Thickness of the crust.

V. CONCLUSION

According to the theory and the numerical experiments, we can conclude that gravity method could be used to inverse the mass anomaly in the crust. It is reasonable that the earth is divided into homogeneous spherical layer material and regional crustal anomaly combined with the geophysical and gravity potential theory. Numerical experiments are shown that gravity inversion for regional crustal anomaly mass is some unstable state of the use of point-mass model method, the relevant theoretical results need further validation studies. Note that there is a superior that the gravity data can be easy obtained. In practice, it is a good idea that combined with geophysical results of the crust, and then crustal abnormal density can be calculated.

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