Study on Optimization Algorithm of Underway Replenishment Routine Project for Naval Battle Group

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***Abstract* — To solve the problem of Underway Replenishment Routine Project for Naval Battle Group, the Underway Replenishment Routine Project and conventional Traveling Salesman Problem were compared. In terms of the similarity and difference between URRP and TSP, an improved TSP algorithm was applied to describe the URRP. According to the differences of the requirements to URRP in peacetime and wartime, the models of URRP and the objective functions were set up respectively. Ant Colony Algorithm was used to solve URRP problem. The URRP problem in peacetime was used as an example to validate the feasibility of the models and algorithms.**

Keywords — Underway replenishment routine project; Traveling salesman problem; Ant colony algorithm; Optimized routine

I. INTRODUCTION

In the process of naval battle group underway replenishment operations, supply time of vessels equals to the combination of combatant vessels or supply ships' arriving time and supply ships' working time. For naval battle group supply operation, the number of vessels in formation is large and there is long physical distance between vessels, and each ship usually has several supply position, also, the supply strategy varied, leading to supply problem is relatively complex. Even in the same strategy, for different supply routine choice, the supply time also exists a big difference; it will lead to unnecessary waste of time, thus affect whole combat effectiveness of the group.

At present, domestic research on problems of replenishment at sea focus on alimentative equipments [1-3]. The researches of URRP have not been found so far. In the foreign country, represented by USN, the researches mainly focus on expanding the TSP problem. This paper uses the principle of graph theory, refers to the improved ant colony algorithm, constructing these problem into URRP problem (different from TSP) for different supply strategy under different operational conditions. We used the function with minimum supply time and maximum operational effectiveness to optimize Underway Replenishment Routine Project and got the optimized routine.

II. THE RELATIONSHIP AND DIFFERENCES BETWEEN TSP AND URRP

The TSP and the URRP share common points, but there are many differences. Thus, while using TSP to describe URRP, on one hand, attention should be paid to adopt useful experiences in TSP and on the other hand, we should take the features of URRP into consideration. Besides, consideration in URRP, on some aspects, is much deeper than that of the common TSP. Therefore, the approach to solve URRP problem is to get the optimum solution of URRP on the common points of URRP and TSP, by taking the TSP description as basis and combining particular features of the URRP with concrete requirements.

Common points could be concluded as follows: TSP and URRP belongs to complex combinatorial optimization problems, which can be described by the graph theory and can use the vertices and empowerment to represent the distance between cities (or vessels). Like TSP problem, traveling salesman or supply ship will traverse each city (or vessel) only once at a time that will not form a loop then. And the ultimate goal of solving both problem will include that how to find the shortest distance (or minimum time).

Differences also could be described as follows:

1) The composition of figure. For URRP, the vertices that described by graph theory is not necessarily the ship's position in the formation, it also can be the supply position of the ship. Each node in the TSP represents a city, but each node in the URRP means a supply position (Each battle ship could have several supply positions), and only one supply position for each ship will be visited. If the two nodes are the same, the TSP and URRP can be described by one network diagram. In URRP's network diagram, the initial position is usually an independent node (which means that the initial position of supply ship differ from that of any other vessel). In the TSP, the initial position of a traveling salesman is usually a city. Beyond that, URRP usually does not need the supply ship get back to the initial position eventually.

2) The determination of constraint condition. For URRP, when solving the optimal routine problem, we

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need not only to meet the corresponding constraints in the TSP but also to satisfy the special requirement of URRP, for example, the required maximum time (supply time is less than the specified maximum time) and the number of supply ships that were allowed to leave their specific position, etc.

3) The choice of the objective function. The objective function and quantity of URRP differ under different operational environments. For example, in environment of battlefield, except the minimum supply time function, usually the largest group combat effectiveness would be chose as the second function.

4) In battlefield environment, supply time is limited. In order to ensure the effectiveness of battle group operational, some ships might not be supplied, that means the URRP may not go through all of the ships like the TSP.

III. THE MATHEMATICAL DESCRIPTION OF URRP PROBLEM

For URRP, its mainly problem is the determination of supply order and location. Ships' sealift involves multiple aspects, including different ways of supply, supply equipments, supply strategies, etc. In this paper, research is mainly based on lateral sealift of liquid cargo replenishment. To accurately build the model of URRP, the conditions of sealift were assumed as follows:

1) The materials in supply ship could be able to meet all the requirements of battle group;

2) Warships and supply ships are organized into one group and keep the same speed;

3) Supply time of a specific warship is a fixed value;

4) Supply ships could serve warships at any time when assigned, without thinking of the effect of environmental factors;

5) The replenishment time begins from the first supply ships leave their position to start replenishment to the supply ships finish the last vessel's supply and return to their initial position.

Assume that battle group consists of n warships and 1 supply ship, and every warship has m supply positions. Figure 1 depicts a supply problem of a battle group with 2 warships and each warship has 2 supply positions.

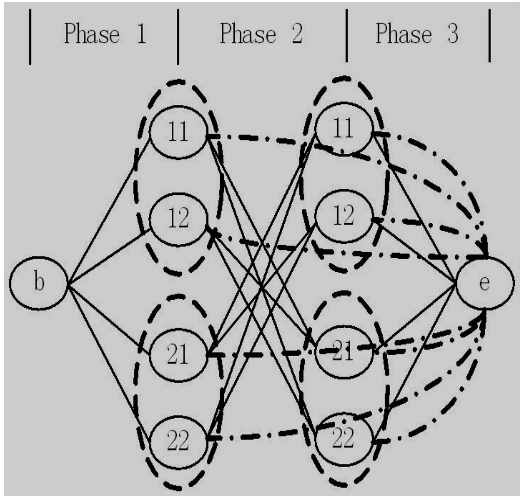

Fig.1 A Network Structure for Underway Replenishment of Navy Battle Groupc

In the figure: character b represents the starting position, e represents the ending position, 11, 12, 21 and 22 represent ships' supply position.

If you remove the dashed line, the figure displays peacetime network structure. Otherwise, it is the wartime network structure.

Among them, p, q represent ships' supply position; $d_{b(i,p)}$ represents voyage time from the starting position to supply position p for ship i; r_i represents the supply time for ship i; $d_{(i,p)(j,q)}$ represents the voyage time from supply position p of ship i to supply position q of ship j; $d_{(i,p)e}$ represents the voyage time from the last supply position to end position; $X^{\mathbf{1}}_{b(i,p)} = \mathbf{1}$ means that the first supply was conducted in position p and the supplied ship is ship i, others are zero; $X^{k}_{(i,p)(j,q)} = \mathbf{1}$ means that the k time of supply was conducted in position q and the supplied ship is ship j, the k-1 time of supply was conducted in position p and the supplied ship is ship i and other numbers are zero; $X^{(n+\mathbf{1})}_{(j,q)e} = \mathbf{1}$ means that, in the final supply stage, the position is q and the supplied ship is j, others are zero; ω_i represents the combat capability of ship i; h represents the available time for voyage.

(1) In the voyage process or in peacetime, the most effective and quick methods to complete the sealift are required, which means the shortest supply time. At this time, sealift problem can be described as follows:

The objective function is:

$$\min \sum_{i=1}^{n}\sum_{p=1}^{m}\left(d_{b(i,p)}+r_i\right)X_{b(i,p)}^{1}+$$
$$\sum_{k=2}^{n}\sum_{i=1}^{n}\sum_{p=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{m}\left(d_{(i,p)(j,q)}+r_j\right)X_{(i,p)(j,q)}^{k}+$$
$$\sum_{j=1}^{n}\sum_{q=1}^{m}d_{(i,p)e}X_{(j,q)e}^{(n+1)}$$

The constraint conditions [4] are:

$$\sum_{j=1}^{n}\sum_{q=1}^{m}X_{b(j,q)}^{1}=1 \quad (1)$$

$$X_{b(i,p)}^{1}=\sum_{j=1}^{n}\sum_{q=1}^{m}X_{(i,p)(j,q)}^{2}, \quad \forall i,p \quad (2)$$

$$\sum_{j=1}^{n}\sum_{q=1}^{m}X_{(i,p)(j,q)}^{(k-1)}=\sum_{j=1}^{n}\sum_{q=1}^{m}X_{(i,p)(j,q)}^{k}, \quad \forall k\geq 3,i,p \quad (3)$$

$$\sum_{j=1}^{n}\sum_{q=1}^{m}X_{(i,p)(j,q)}^{k}=X_{(i,p)e}^{(n+1)}, \quad \forall i,p \quad (4)$$

$$\sum_{i=1}^{n}\sum_{p=1}^{m}X_{(i,p)e}^{(n+1)}=1 \quad (5)$$

$$X_{b(i,p)}^{1}+\sum_{k=2}^{n}\sum_{j=1}^{n}\sum_{q=1}^{m}X_{(i,p)(j,q)}^{k}=V_{(i,p)}, \quad \forall i,p \quad (6)$$

$$\sum_{p=1}^{m}V_{(i,p)}=1, \quad \forall i \quad (7)$$

The constraint conditions mainly illustrate that 1) At every stage only one ship can be supplied and each ship can be supplied only once in one position; 2) After the k-1 stage of supply to a ship, another ship only can be supplied on the k stage.

(2) In wartime, battle groups may be engaged in combat suddenly so that not all of supply works can be finished in such a short time. In this case, only the maximum combat effectiveness of groups in a prescriptive time can be ensured. URRP, here, is a routine planning problem with time limitation. The supply time is a term in constraint conditions and not all ships is required to be supplied. Thus, wartime-sealift routine planning can be described as follows:

The objective function is:

$$\begin{cases}\min \sum_{i=1}^{n}\sum_{p=1}^{m}\left(d_{b(i,p)}+r_i\right)X_{b(i,p)}^{1}+\\ \sum_{k=2}^{n}\sum_{i=1}^{n}\sum_{p=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{m}\left(d_{(i,p)(j,q)}+r_j\right)X_{(i,p)(j,q)}^{k}V_{(i,p)(j,q)}^{k}\\ +\sum_{j=1}^{n}\sum_{q=1}^{m}d_{(i,p)e}X_{(j,q)e}^{(n+1)}\\ \max \sum_{i=1}^{n}\sum_{p=1}^{m}\omega_i V_{(i,p)}\end{cases}$$

The constraint conditions [4] are:

$$\sum_{j=1}^{n}\sum_{q=1}^{m}X_{b(j,q)}^{1}=1 \quad (1)$$

$$X_{b(i,p)}^{1}=\sum_{j=1}^{n}\sum_{q=1}^{m}X_{(i,p)(j,q)}^{2}+X_{(j,q)e}^{2}, \quad \forall i,p \quad (8)$$

$$\sum_{j=1}^{n}\sum_{q=1}^{m}X_{(j,q)(i,p)}^{(k-1)}=\sum_{j=1}^{n}\sum_{q=1}^{m}d_{(i,p)e}X_{(i,p)(j,q)}^{(k)}+X_{(i,p)e}^{k}, \quad \forall k\geq 3,i,p \quad (9)$$

$$\sum_{j=1}^{n}\sum_{q=1}^{m}X_{(j,q)(i,p)}^{n}=X_{(i,p)e}^{(n+1)}, \quad \forall i,p \quad (10)$$

$$\sum_{k=2}^{n+1}\sum_{i=1}^{n}\sum_{p=1}^{m}X_{(i,p)e}^{k}=1 \quad (11)$$

$$X_{b(i,p)}^{1}+\sum_{k=2}^{n}\sum_{j=1}^{n}\sum_{q=1}^{m}X_{(j,q)(i,p)}^{k}=V_{(i,p)}, \quad \forall i,p \quad (12)$$

$$\sum_{p=1}^{m}V_{(i,p)}=1, \quad \forall i \quad (13)$$

$$\sum_{i=1}^{n}\sum_{p=1}^{m}r_iV_{(i,p)}+\sum_{i=1}^{n}\sum_{p=1}^{m}d_{b(i,p)}X_{b(i,p)}^{1}+\sum_{k=2}^{n}\sum_{i=1}^{n}\sum_{p=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{m}d_{(i,p)(j,q)}X_{(i,p)(j,q)}^{k}+\sum_{k=2}^{n+1}\sum_{i=1}^{n}\sum_{p=1}^{m}d_{(i,p)e}X_{(i,p)e}^{k}\leq h \quad (14)$$

Constraint conditions not only include the limitation in peacetime but also show that the supply tasks, in wartime, could be ended because of certain factors at any stage. Also the time limitation is described.

IV. THE ALGORITHM DESIGN OF URRP PROBLEM

1) When battle groups carries out tasks in peacetime, the underway replenishment routine project can be solved with ant colony algorithm.

The solving process could be described as follows:

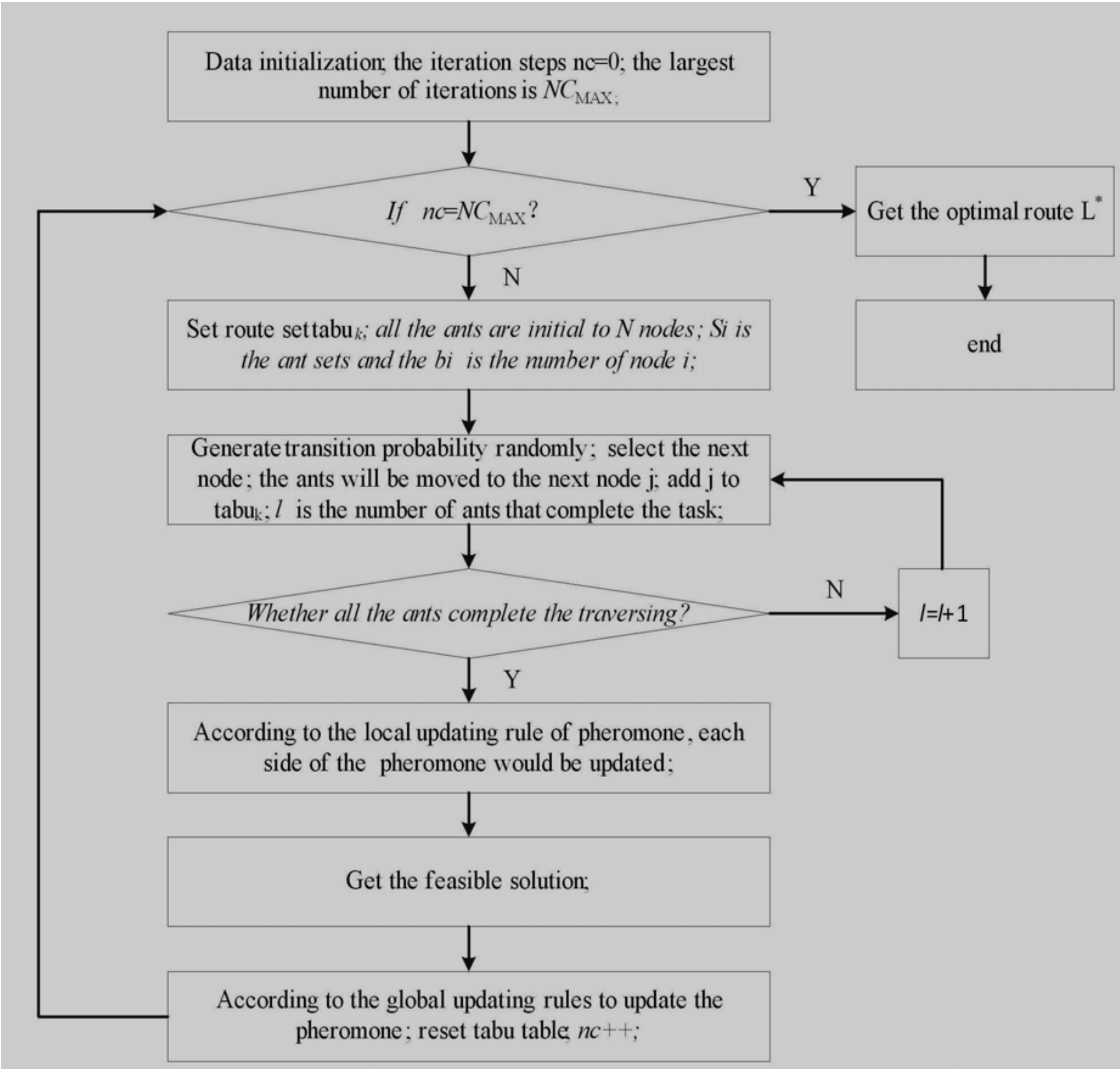

Fig. 2 Solve Flow Chart of Ant Colony Algorithm

2) When the warship group performs operational tasks, the underway replenishment routine project is a typical multi-objective programming problem. This problem can be converted into “ideal point” method to solve.

The solving method can be described as follows:

$$\left.\begin{array}{l}\min\sum_{i=1}^{n}\sum_{p=1}^{m}\left(d_{b(i,p)}+r_i\right)X_{b(i,p)}^{1}+\\ \sum_{k=2}^{n}\sum_{i=1}^{n}\sum_{p=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{m}\left(d_{(i,p)(j,q)}+r_j\right)X_{(i,p)(j,q)}^{k}V_{(i,p)(j,q)}^{k}\\ +\sum_{j=1}^{n}\sum_{q=1}^{m}d_{(i,p)e}X_{(j,q)e}^{(n+1)}\\ \text{constraintcondition}\end{array}\right\}$$

$$\left.\begin{array}{l}\max\sum_{i=1}^{n}\sum_{p=1}^{m}\omega_i V_{(i,p)}\\ constra\ \text{int}\ condition\end{array}\right\}$$

The optimal and worst solutions of two formulas are $\overline{\lambda}$, $\underline{\lambda}$ and $\overline{\varphi}$, $\underline{\varphi}$ can be obtained (we can get them from a revision to the ant colony algorithm). As well as the proximity degree between non-inferiority routine L and positive ideal R_L and negative ideal point r_L , so:

$$R_L=\theta_1\frac{D(\overline{\lambda})}{D(L)}+\theta_2\frac{\omega(L)}{\omega(\overline{\varphi})},\quad r_L=\theta_1\frac{D(L)}{D(\underline{\lambda})}+\theta_2\frac{\omega(\underline{\varphi})}{\omega(L)}$$

θ_1 and θ_2 are the weight of supply time and the combat effectiveness. $\theta_1+\theta_1=1$. Their value can be determined by experts according to the battlefield situation and the battle process, etc.

Relative proximity degree between each non-inferiority routine and ideal points is:

$$\varepsilon_L=R_L/(R_L+r_L),\quad 0\le\varepsilon_L\le 1$$

The largest supply routine of ε_L will be the best routine.

V. THE SIMULATION EXPERIMENT AND ANALYSIS

Assume that a battle group consists of 1 supply ship and 6 warships, and different importance is attached to different ships. The supply time required for each ship is not identical and every warship has a number of supply points (To supply on ships, only one point could be chose). The initial coordinate of supply ship is (0, 0). All the relative position between the ship supply points is showed in Fig.3. The requirements are to arrange reasonable replenishment routine for the shortest supply time. (Assume that the speed of supply ships is fixed, so it means total voyage distance is the shortest).

The simulation platform is Matlab 6.5, the initial parameters are as follows: the number of ants is m=60; heuristic factor is α=3; expected heuristic factor is β=1; pheromones evaporation coefficient is ρ=0.85; the largest number of iterations is $NC_{\max}$=200; the pheromone evaporation strength coefficient is Q=100. After simulation, the optimal routine of URRP will be obtained: b→11→42→31→22→52→61, the shortest distance is 56.26nm.

The experiment shows that the optimization algorithm on replenishment routine project is suitable to the characteristics of replenishment at sea and the actual needs. The ant colony algorithm is an effective method to get the optimized solution of URRP problems.

VI. CONCLUSION

The Underway Replenishment Routine Project and conventional Traveling Salesman Problem have similarities as well as differences. By analyzing those similarities and differences, this paper treats underway replenishment routine project as an improved TSP problem. And according to the different requirements and characteristics for replenishment at sea in wartime and peacetime, different optimization model is established, as well as the URRP objective function and constraint conditions.

Taking the sealift optimization project in peacetime for examples, building ant colony algorithm model and using ant colony algorithm, the rationality and feasibility is validated and the underway replenishment routine project has been solved.

The results can be used to provide the basis for underway replenishment routine project for naval battle group. Results of this study would be significant in both theory and practice for the decision of underway replenishment routine project problem.

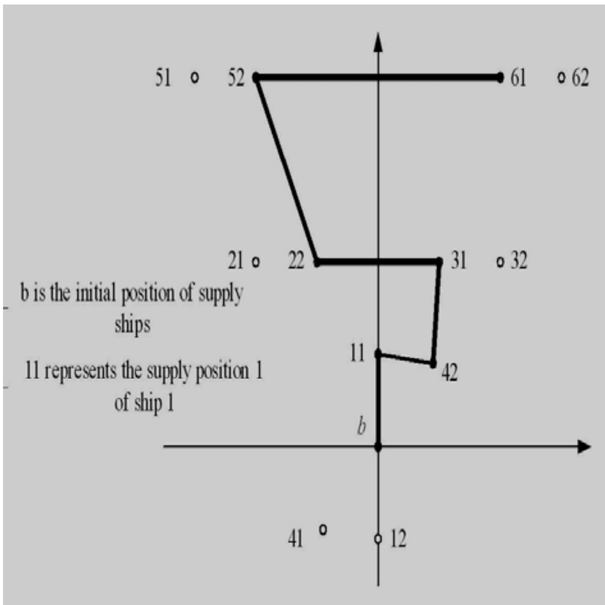

Fig.3 the Relative Rendezvous Locations and the Optimized Routine

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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