

## Modelling Mutation Rupture Fracturing and Groups of Micro-Cracks Evolution in Coal and Rock Mass

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**Abstract** -- Coal and rock mass fracturing is an important technology on exploiting coalbed methane (CBM), because there are large numbers of natural cracks in coal and rock mass, the extension and evolution rule of cracks is very complicated. In order to describe the rule of cracks evolution accurately, different groups of cracks were considered in this paper, and base on mutation theory, the model of coal and rock mass fracturing and energy release are established. Hypothesis that the releasing energy is all transferred to crack propagation and evolution. According to energy conservation principle, calculation method for micro-cracks length and number in coal and rock mass mutation behavior is been established. Taking coal samples which were taken from right lane six, 3# layer, west block three, Zhangchen mine, Jixi, Heilongjiang as research objects, through calculation, the evolution parameters of coal and rock mass micro-cracks are obtained, and the rationality of the theoretical model presented in this paper is verified by laboratory fracturing experiments. After calculation, number error of micro-cracks is about 0.27%, and the energy error is about 5.42%. The new model provides a new method for further study of the evolution rules of micro-crack fracturing parameters of coal and rock mass.

**Keywords** -- Coal and Rock Mass Fracturing; Mutation Rupture; Micro-crack Evolution; Acoustic Emission

### I. INTRODUCTION

Coal and rock mass fracturing means the process of how micro-crack generates, expands and connects under water driving, with the purpose of forming a high-conductivity path in formation for oil and gas flow into the bottom hole. Describing the generation and distribution of hydrofracturing crack accurately is an important basis for optimizing fracturing construction parameters and improving fracturing effect. In past, researching on coal and rock mass fracturing is mostly concentrate on the evolution of a single crack. Testing shows that fracturing crack is a local destruction area which is composed of lots of micro-cracks by growing, expanding and merging. So we need to reveal the substantive characteristics of crack formation from the point of coal and rock mass damage evolution.

Zhang Qun summarizes the basic characteristics of coalbed methane in china based on years of coalfield geological exploration data[1]. MS Bruno and T Sitharam analysis the process of hydrofracturing rupture by coupling discrete element model (DEM) and particle model with seepage, respectively[2,3]. Shen Jin and Zhao Yangsheng establish the mathematical model for simulating crack expanding of low permeability coal and rock mass fracturing [4]. Echoviruses and Haimson [5,6,7,8] study the morphology of hydrofracturing crack, both in theoretical and experimental. Lv Youchang studies the rupture mechanism of coal and rock mass

under high-pressured water, and drive the formula of critical value of fracturing initiation pressure[9]. Su Peili uses computer to simulate dispersal behavior of fracturing fluid in crack network under different pressure [10]. Xie Heping uses fractal theory to study the evolution characteristics of concrete micro-cracks during damage process under

external force, and set up the correspondence relationship between damage variable of rock mass and fractal dimension, and then analysis the relationship between results of acoustic emission and fractal dimension under cyclic loading of rock mass[11,12]. Zhao Wanchun and Ai Chi establish the evolution model of fracturing matrix pores of rock mass and permeability tensor of micro-crack based on damage theory, and identify the internal relationship between volumetric strain, medium porosity and damage variable[13]. Cheng Yuanfang and Xu Taishuang study the morphological changes of the hydraulic fracturing cracks in coal, and obtained the judgment foundation for the transition between the horizontal and vertical cracks and complex cracks[14]. TM Roberts uses acoustic emission theory to observe the relationship between the crack expand rate and the load in the steel, in order to predict the service life of the steel[15]. B Zhang uses numerical simulation method to study the crack growth behavior of fracturing process[16].

Different structure units of coal bed have different geological characteristics, which have great differences on

coal bed methane, and the development of coal bed methane in China needs a higher level of fracturing technology, urgently. However, study the fracturing evolution of coal and rock mass can promote the development of new technology. Therefore, considering different groups of cracks in this paper, and base on mutation theory, establishing the model of coal and rock mass fracturing and energy release. Hypothesis that the releasing energy are all transferred to crack propagation and evolution, and according to energy conservation principle, calculation method for micro-cracks length and number in coal and rock mass mutation behavior is been established. The new model provides a new method for further study of the evolution rules of micro-crack fracturing parameters of coal and rock mass.

II. DEFINITION OF DAMAGE VARIABLE

Mutation means a change from a stable condition jump into another stable condition, or to say, continuous changes of some variables lead to abrupt changes in system evolution.

Applied load to rock mass, internal pore structure isn't changed (which means the rock mass returns to the initial state after unloaded) during the elastic stage, when damage deformation happens of rock mass, rock mass volume and internal pore structure change, cracks are produced and expanded. For different types of rock mass, in order to show constitutive relationship of rock mass damage stages in fracturing, we define the damage variable of rock mass deformation is:

$$D_i = \frac{\phi_{0f}^i - \phi_f^i}{\phi_{0f}^i - \phi_D^i}, (i = 1, 2, \dots, n) \quad (1)$$

Where:  $D_i$  is the damage variable in micro-crack damage evolution;  $\phi_{0f}^i$  is the initial porosity of the micro-cracks;  $\phi_f^i$  is the micro-cracks porosity after damage deformation, which determine the damage degree of rock mass;  $\phi_D^i$  is the micro-cracks porosity when critical damage happens, generally .

Hypothesis that the damage deformation of rock mass under external load meets the stress equivalence, and cracks are extensional bursts, deducing the relationship of damage deformation porosity and spreading direction strain of the  $i$ th micro-cracks:

$$\varepsilon_i = \frac{\ln(1 - \phi_f^i) - \ln(1 - \phi_{0f}^i)}{1 + 2\mu(1 - \omega_i)}, (i = 1, 2, \dots, n) \quad (2)$$

Where:  $\mu$  is elastic Poisson ratio,  $\omega_i$  is the spreading direction strain of the micro-cracks under external load,

is the defined damage variable by relatively reduced of the rock loaded area.

According to deduction, during damage deformation stages, relationship of the micro-cracks and is:

$$\omega_i = \sqrt{1 - \ln \left[ 1 + \frac{D_i(\phi_{0f}^i - \phi_D^i)}{1 - \phi_{0f}^i} \right]}, (i = 1, 2, \dots, n) \quad (3)$$

III. ENERGY CONSERVATION EQUATION OF ROCK MASS RUPTURE IN COAL AND ROCK MASS FRACTURING

Ignore kinetic energy loss of fracturing fluid in rock mass during fracturing process, Based on energy conservation principle, work of the external force (fracturing pump) and gravitational potential energy of fracturing fluid before crack cracking translate into rock mass elastic strain energy and damage strain energy, which means

$$W_{bi} + U_{bi} - (E_{ei} - E_{ip}) = 0, (i = 1, 2, \dots, n) \quad (4)$$

where  $W_{bi}$  is the external force work on the  $i^{th}$  micro-cracks;  $U_{bi}$  is the gravitational potential energy of the  $i^{th}$  micro-cracks fracturing fluid;  $E_{ei}$  is the elastic strain energy of the  $i^{th}$  micro-cracks;  $E_{ip}$  is the damage strain energy of the  $i^{th}$  micro-cracks.

A. Work of the External Force (Fracturing Pump)

Work of the external force (fracturing pump)

During fracturing process, whole work of fracturing pump on rock mass is

$$W_b = \eta \cdot P \cdot t \quad (5)$$

Meanwhile

$$W_b = \sum_{i=1}^n W_{bi}, (i = 1, 2, \dots, n) \quad (6)$$

where:  $P$  is the power of fracturing pump;  $t$  is continuous working time of fracturing pump;  $\eta$  is pump efficiency,  $\eta = 0.6$  °.

B. Gravitational Potential Energy of Fracturing Fluid

Gravitational potential energy of fracturing fluid in wellbore is

$$U_h = \frac{1}{2} \rho_l g \int_0^h dV_l \quad (7)$$

Meanwhile

$$U_h = \sum_{i=1}^n U_{hi}, (i = 1, 2, \dots, n) \quad (8)$$

Where:  $\rho_l$  is the density of fracturing fluid;  $h$  is the vertical depth from head to fracturing interval;  $dV_l$  is infinitesimal volume of fracturing fluid.

### C. Elastic Deformation Strain Energy of Micro-crack

Hypothesis a unit volume of a rock mass, strain energy of the  $i$ th micro-cracks in elastic deformation is

$$E_{ei} = \int_0^{\varepsilon_{ei}} \sigma(\varepsilon_i) d\varepsilon_i \quad (i = 1, 2, \dots, n) \quad (9)$$

Where:  $E_{ei}$  is the  $i$ th micro-cracks strain at the end of elastic deformation;  $\sigma(\varepsilon_i)$  is the bearing stress of side of fracturing crack.

During elastic deformation period, constitutive relationship is

$$\varepsilon_i = \frac{\sigma(\varepsilon_i)}{E}, (i = 1, 2, \dots, n) \quad (10)$$

Where:  $E$  is rock mass elastic modulus.

### D. Damage Deformation Strain Energy of Micro-crack

Hypothesis a unit volume of a rock mass, strain energy of the  $i$ th micro-cracks in damage deformation is:

$$E_{pi} = \int_{\varepsilon_{ei}}^{\varepsilon_{bi}} \sigma(\varepsilon_i) d\varepsilon_i, (i = 1, 2, \dots, n) \quad (11)$$

According to damage degradation theory of Ramberg-Osgood [17], damage stress-strain constitutive equation of the  $i$ th micro-cracks is

$$\varepsilon_i = \left[ \frac{\sigma(\varepsilon_i)}{(1-\omega_i)K} \right]^m, (i = 1, 2, \dots, n) \quad (12)$$

where:  $K, m$  are material parameters of coal and rock mass.

## IV. MUTATION RUPTURE MODEL

Combined formula (9) and formula (11), formula (4) can express as the differential of  $\varepsilon_i$ , divide  $d\varepsilon_i$

$$\frac{dW_{bi}}{d\varepsilon_i} + \frac{dU_{hi}}{d\varepsilon_i} - \frac{\sigma(\varepsilon_i)}{E} \frac{d\sigma(\varepsilon_i)}{d\varepsilon_i} - \sigma(\varepsilon_i) = 0 \quad (i = 1, 2, \dots, n) \quad (13)$$

Formula (13) is the differential form of energy balance equation for coal and rock mass work of fracturing pump and gravitational potential energy of fracturing fluid in wellbore before form a group of crack. Hypothesis is the inflection point in stress-strain curve of the micro-cracks in damage process. Using Taylor's formula to deduce formula (13), and abandon higher order term is:

$$\begin{aligned} \sigma(\varepsilon_i) \frac{d\sigma(\varepsilon_i)}{d\varepsilon_i} &= \sigma(\bar{\varepsilon}_i) \cdot \sigma'(\bar{\varepsilon}_i) \\ &+ [\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i) + [\sigma'(\bar{\varepsilon}_i)]^2] (\varepsilon_i - \bar{\varepsilon}_i) \\ &+ \frac{[2\sigma'(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i) + \sigma''(\bar{\varepsilon}_i) \cdot \sigma'(\bar{\varepsilon}_i) + \sigma(\bar{\varepsilon}_i) \cdot \sigma'''(\bar{\varepsilon}_i)]}{2} (\varepsilon_i - \bar{\varepsilon}_i)^2 \\ &+ O(\varepsilon_i - \bar{\varepsilon}_i)^3 \end{aligned} \quad (14)$$

$$\sigma(\varepsilon_i) = \left[ \sigma(\bar{\varepsilon}_i) + \sigma'(\bar{\varepsilon}_i) \cdot (\varepsilon_i - \bar{\varepsilon}_i) + \frac{\sigma''(\bar{\varepsilon}_i)}{2} (\varepsilon_i - \bar{\varepsilon}_i)^2 \right] + O(\varepsilon_i - \bar{\varepsilon}_i)^3 \quad (15)$$

According to formula (12) of coal and rock mass damage stress-strain constitutive relationship, at  $\bar{\varepsilon}_i$  point

$$\sigma(\bar{\varepsilon}_i) = [(1-\omega_i)K] \bar{\varepsilon}_i^{\frac{1}{m}} \quad (16)$$

$$\sigma'(\bar{\varepsilon}_i) = \frac{1}{m} (1-\omega_i) K \bar{\varepsilon}_i^{\frac{1}{m}-1} + \frac{d(1-\omega_i)}{d\bar{\varepsilon}_i} K \bar{\varepsilon}_i^{\frac{1}{m}} \quad (17)$$

$$\sigma''(\bar{\varepsilon}_i) = \frac{1-m}{m^2} \bar{\varepsilon}_i^{\frac{1}{m}-2} (1-\omega_i) K + \frac{2}{m} \frac{d(1-\omega_i)}{d\bar{\varepsilon}_i} K \bar{\varepsilon}_i^{\frac{1}{m}} \quad (18)$$

$$\begin{aligned} \sigma'''(\bar{\varepsilon}_i) &= \frac{(1-m)(1-2m)}{m^3} \bar{\varepsilon}_i^{\frac{1-3m}{m}} (1-\omega_i) K + \frac{3(1-m)}{m^2} \bar{\varepsilon}_i^{\frac{1-2m}{m}} \cdot \frac{d(1-\omega_i)}{d\bar{\varepsilon}_i} K \\ &+ \frac{3}{m} \bar{\varepsilon}_i^{\frac{1-m}{m}} \frac{d^2(1-\omega_i)}{d\bar{\varepsilon}_i^2} K + \frac{d^3(1-\omega_i)}{d\bar{\varepsilon}_i^3} \bar{\varepsilon}_i^{\frac{1}{m}} K \end{aligned} \quad (19)$$

Abandon term  $\sigma'''(\bar{\varepsilon}_i) = 0$ , arrange and get

$$-\frac{1}{2} \left[ (\varepsilon_i - \bar{\varepsilon}_i) + \frac{[\sigma'(\bar{\varepsilon}_i)]^2}{\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i)} \right]^2 + \frac{[\sigma'(\bar{\varepsilon}_i)]^4}{[\sigma(\bar{\varepsilon}_i)]^2 \cdot [\sigma''(\bar{\varepsilon}_i)]^2} + \frac{E \left( \frac{dW_{bi}}{d\bar{\varepsilon}_i} + \frac{dU_{hi}}{d\bar{\varepsilon}_i} \right)}{\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i)} - \frac{\sigma(\bar{\varepsilon}_i)[\sigma'(\bar{\varepsilon}_i) + E]}{\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i)} = 0 \quad (20)$$

Formula (20) is the folding mutation model of the  $i^{\text{th}}$  micro-cracks in fracturing process of rock mass damage burst.

Hypothesis that:

$$y_i = (\varepsilon_i - \bar{\varepsilon}_i) + \frac{[\sigma'(\bar{\varepsilon}_i)]^2}{\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i)}, (i = 1, 2, \dots, n) \quad (21)$$

$$\beta_i = \frac{2[\sigma'(\bar{\varepsilon}_i)]^4}{[\sigma(\bar{\varepsilon}_i)]^2 [\sigma''(\bar{\varepsilon}_i)]^2} - \frac{2E \left( \frac{dW_{bi}}{d\bar{\varepsilon}_i} + \frac{dU_{hi}}{d\bar{\varepsilon}_i} \right)}{\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i)} + \frac{2\sigma(\bar{\varepsilon}_i)[\sigma'(\bar{\varepsilon}_i) + E]}{\sigma''(\bar{\varepsilon}_i)} (i = 1, 2, \dots, n) \quad (22)$$

Changing formula (20) to a standard form of folding mutation model is

$$y_i^2 + \beta_i = 0, (i = 1, 2, \dots, n) \quad (23)$$

According to this standard form, when  $\beta_i \geq 0$  equation has nonsense; when  $\beta_i \leq 0$ , with  $\beta_i$  changing,  $y_i$  have different values and make up  $i$  groups paracurves.

When  $\beta_i = 0$ , which means the original point of the paracurve (Fig.(1)), any symmetric solutions on the paracurve corresponding to the two points, which represent the 2 balanced state (before and after) of rock mass that burst into micro-cracks in fracturing process.

Observing the paracurve, finding that with  $|\beta_i|$  greater, the two point have father distance from the original point, which means the coal and rock mass is more stable. Discussed the  $i^{\text{th}}$  micro-cracks, drawing the mutation energy curve (Fig.(2))

When

$$y_i < 0, \text{ i.e. } (\varepsilon_i - \bar{\varepsilon}_i) < \frac{-[\sigma'(\bar{\varepsilon}_i)]^2}{\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i)},$$

which is the lower half point of the paracurve, which means rock mass balance state before burst into micro-cracks.

When  $y_i > 0$ , i.e.

$$(\varepsilon_i - \bar{\varepsilon}_i) > \frac{-[\sigma'(\bar{\varepsilon}_i)]^2}{\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i)}$$

which is the higher half point of the paracurve, which means rock mass balance state after burst into micro-cracks. . If  $y_i$  goes from below to above passing the original point smoothly, burst process won't have energy accumulation and release; If  $y_i$  from below jumps to above, mutation behavior happens, which has energy accumulation and release (shade part).

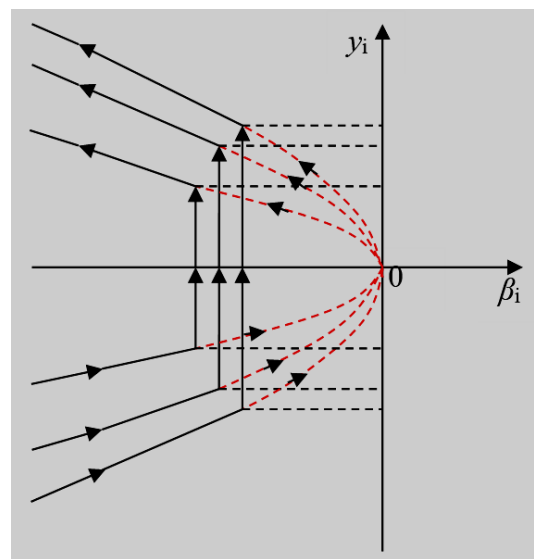


Fig.(1). Sketch of Groups Micro-Cracks Mutation Fracturing of Rock Mass

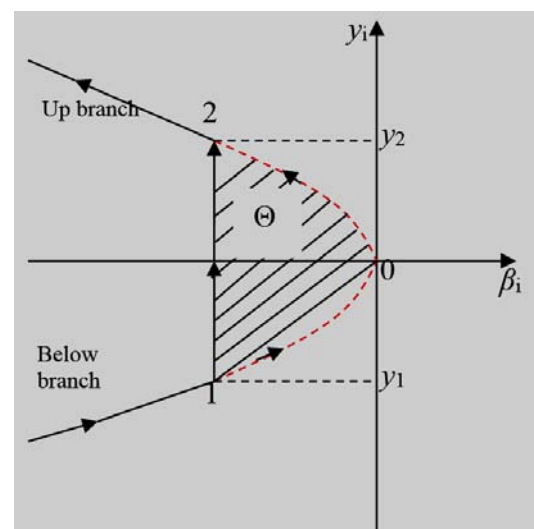


Fig.(2). Sketch of the  $i^{\text{th}}$  Group Micro-Cracks Mutation Fracturing of Rock Mass

According to the standard form of folding mutation model, two roots  $y_1 = y_i (< 0)$ ,  $y_2 = -y_i$  of any balance state are

$$y_1 = (\varepsilon_i - \bar{\varepsilon}_i) + \frac{[\sigma'(\bar{\varepsilon}_i)]^2}{\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i)}$$

$$= -\sqrt{-2 \left[ \frac{[\sigma'(\bar{\varepsilon}_i)]^4}{[\sigma(\bar{\varepsilon}_i)]^2 \cdot [\sigma''(\bar{\varepsilon}_i)]^2} + \frac{E \left( \frac{dW_{bi}}{d\bar{\varepsilon}_i} + \frac{dU_{hi}}{d\bar{\varepsilon}_i} \right) \sigma(\bar{\varepsilon}_i) [\sigma'(\bar{\varepsilon}_i) + E]}{\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i)} - \frac{\sigma(\bar{\varepsilon}_i) [\sigma'(\bar{\varepsilon}_i) + E]}{\sigma''(\bar{\varepsilon}_i)} \right]} \quad (24)$$

$$y_2 = -y_1 = \sqrt{-2 \left[ \frac{[\sigma'(\bar{\varepsilon}_i)]^4}{[\sigma(\bar{\varepsilon}_i)]^2 \cdot [\sigma''(\bar{\varepsilon}_i)]^2} + \frac{E \left( \frac{dW_{bi}}{d\bar{\varepsilon}_i} + \frac{dU_{hi}}{d\bar{\varepsilon}_i} \right) \sigma(\bar{\varepsilon}_i) [\sigma'(\bar{\varepsilon}_i) + E]}{\sigma(\bar{\varepsilon}_i) \cdot \sigma''(\bar{\varepsilon}_i)} - \frac{\sigma(\bar{\varepsilon}_i) [\sigma'(\bar{\varepsilon}_i) + E]}{\sigma''(\bar{\varepsilon}_i)} \right]} \quad (i = 1, 2, \dots, n) \quad (25)$$

When mutation happens,  $\frac{dW_{bi}}{d\bar{\varepsilon}_i} + \frac{dU_{hi}}{d\bar{\varepsilon}_i} = 0$ . So

$$y_1 = -\sqrt{-2 \left[ \frac{[\sigma'(\bar{\varepsilon}_i)]^4}{[\sigma(\bar{\varepsilon}_i)]^2 \cdot [\sigma''(\bar{\varepsilon}_i)]^2} - \frac{\sigma(\bar{\varepsilon}_i) [\sigma'(\bar{\varepsilon}_i) + E]}{\sigma''(\bar{\varepsilon}_i)} \right]} \quad (26)$$

$$y_2 = \sqrt{-2 \left[ \frac{[\sigma'(\bar{\varepsilon}_i)]^4}{[\sigma(\bar{\varepsilon}_i)]^2 \cdot [\sigma''(\bar{\varepsilon}_i)]^2} - \frac{\sigma(\bar{\varepsilon}_i) [\sigma'(\bar{\varepsilon}_i) + E]}{\sigma''(\bar{\varepsilon}_i)} \right]} \quad (i = 1, 2, \dots, n) \quad (27)$$

Doing integral for formula (23), and the total potential energy function of the  $i^{th}$  micro-cracks folding mutation model of coal and rock mass fracturing when burst happens is

$$\Theta_i = \frac{1}{3} y_i^3 + \beta_i y_i, \quad (i = 1, 2, \dots, n) \quad (28)$$

Before mutation happens, the balance state of the  $i^{th}$  micro-cracks ( $y_1 = y_i (< 0)$ ) satisfied

$$\Theta_1 = \frac{1}{3} y_1^3 + \beta_1 y_1 \quad (29)$$

After mutation happens, the balance state of the  $i^{th}$  micro-cracks ( $y_2 = -y_i$ ) satisfied

$$\Theta_2 = \frac{1}{3} y_2^3 + \beta_2 y_2 \quad (30)$$

Where:  $\beta_1 = \beta_2$ . When the balance state  $y_1$  jumps to  $y_2$ , coal and rock mass burst and generate  $i$  group micro-cracks, releasing energy of rock mass for every group is

$$\Delta\Theta_i = \Theta_2 - \Theta_1 = \frac{1}{3} (y_2^3 - y_1^3) + (\beta_2 y_2 - \beta_1 y_1) \quad (31)$$

Putting  $y_1 = y_i (< 0)$ ,  $y_2 = -y_i$  into formula (31), and getting the  $i^{th}$  micro-cracks release energy of mutation behavior is

$$\Delta\Theta_i = \frac{4}{3} y_i^3 = -\frac{4}{3} \left[ 2 \left[ \frac{\sigma(\bar{\varepsilon}_i) [\sigma'(\bar{\varepsilon}_i) + E]}{\sigma''(\bar{\varepsilon}_i)} - \frac{[\sigma'(\bar{\varepsilon}_i)]^4}{[\sigma(\bar{\varepsilon}_i)]^2 [\sigma''(\bar{\varepsilon}_i)]^2} \right] \right]^{\frac{3}{2}}, \quad (i = 1, 2, \dots, n) \quad (32)$$

## V. MODEL OF MICRO-CRACKS EVOLUTION

### A. Length Evolution Model of the Micro-cracks

During the evolution of the  $i^{th}$  micro-cracks, increasing strain energy of each micro-crack is:

$$U_{di} = 2 \int_0^{\alpha_i} G_i d\Omega_i = 2 \frac{1 - \mu^2}{E} \int_0^{\alpha_i} \sigma_i \sqrt{\pi \alpha d \alpha} \quad (33)$$

where:  $\sigma_i$  is the effective stress of each micro-crack;  $\alpha_i$  is the length of each micro-crack.

Computed the integral for formula (33), the increasing strain energy of the  $i^{th}$  micro-cracks is:

$$\Theta_i = \frac{1 - \mu^2}{E} \pi \alpha_i^2 \sigma_i^2 \quad (34)$$

So, total energy for length evolution of the  $i^{th}$  micro-cracks is

$$\Theta_{Ltotal} = \sum_{i=1}^n \Theta_i, \quad (i = 1, 2, \dots, n) \quad (35)$$

### B. Number Evolution Model of the Micro-cracks

Assuming that micro-cracks evolution follows the logistic regression, at the  $S^{th}$  period of fracturing, the total number of the  $i^{th}$  micro-cracks is:

$$n_{is} = F_1 \cdot n_{i(s-1)} + \frac{1}{1 - D_i} \frac{\ln \sigma_{is}}{\ln \sigma_c} \cdot (N - n_{i(s-1)}) \cdot \frac{n_{i(s-1)}}{N} \quad s = (1, 2, \dots, n) \quad (36)$$

where:  $N$  is the total number of micro-cracks evolution in effective growth zone;  $F_1$  is the survival rate of micro-cracks, 0.8-0.84;  $\sigma_c$  is uniaxial compressive strength of rock mass.

Total evolution energy of these  $i$  groups micro-cracks is

$$\Theta_{total} = \sum_{i=1}^n \Theta_{Ltotal} n_{is} \quad (37)$$

From formula (32) and (37), it can be seen that if ignore other energy during micro-cracks evolution, then the releasing energy of mutation rupture is all transferred to the energy for micro-cracks evolution:

$$\Delta\Theta_i = \Theta_{total} \quad (38)$$

Formula (38) is the total balance equation of micro-cracks evolution.

### VI. EXAMPLE VERIFICATION

Coal sample shown in Fig.(3) is got from right lane six, 3# layer, west block three, zhangchen mine, jixi, Heilongjiang. Dip angle of coal bed is 16°, coal rock buried depth is from 904m to 910m, size of the coal sample is larger than 500×400×400mm, elasticity modulus is 48.6MPa, rock poisson ratio is 0.254, rock material parameter  $m=3$ ,  $K=16300$ , maximum horizontal stress is 30.2MPa, and minimum horizontal stress is 27.64MPa. Observing the coal sample, sectional coal layers are parallel to each other, color of different coal layers are different, which is associated with coal growth degree, layers have obvious levels. Coal and rock mass surface is out of flatness, which has a lot of cross sections and obvious edge angle characteristics. Different layers have grown some paralleled natural cracks, which are short and discontinuous distribution. Calculating the numbers of the natural cracks, the length is 3-6cm, and the growth density is 11-12/10cm (Fig. 4).

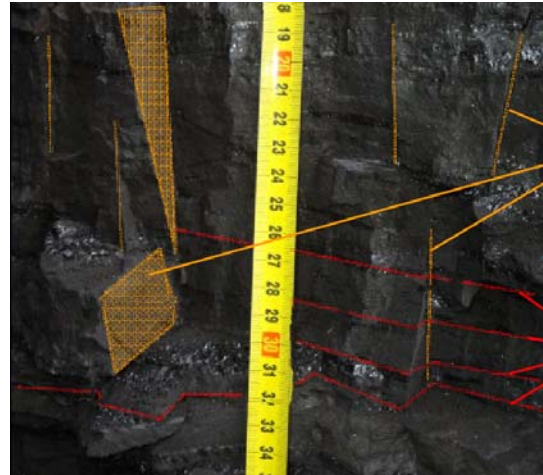


Fig.(4). Natural Crack Distribution Diagram of Coal Sample

Coal sample for testing is cut from raw coal, and the size is 300×300×300mm. Measuring the mechanical parameters of rock mass, and then calculating the number and length of micro-cracks after mutation behavior by theory, as well as energy release of the mutation behavior (Table.1). Assuming that micro-cracks are on closed state before fracturing test and net pressure is 0. Because of the instability of coal and rock mass structure, coating a layer of cement on coal sample can be reinforced. Sanding the surface of coal sample, and then conducting fracturing test on coal sample, confining pressure is 20MPa. When the rock mass rupture, test end. There are three groups' natural micro-cracks on the rock sample. Signing group 1, group 2, group 3, initial numbers of the 3 group micro-cracks are recorded on Table 1.



Fig.(3). Exterior Characteristics of Coal Sample in West Block 3



Fig.(5) Crack Morphology before Fracturing

TABLE 1. MECHANICAL PARAMETERS OF ROCK MASS

| Number  | Damage variable | Peak stress (MPa) | Rock mass strain | Releasing energy of mutation (J) | Micro-crack initial lengths (cm) | Microcrack lengths after mutation (cm) | Micro-crack initial numbers | Micro-crack numbers after mutation |
|---------|-----------------|-------------------|------------------|----------------------------------|----------------------------------|--|-----------------------------|------------------------------------|
| Group 1 | 0.912           | 59.751            | 0.005588         | -45.91546253                     | 3.1                              | 12.8                                   | 15                          | 112                                |
| Group 2 | 0.988           | 60.546            | 0.004850         | -58.20122844                     | 4.3                              | 14.1                                   | 18                          | 131                                |
| Group 3 | 0.967           | 57.385            | 0.004814         | -54.46379365                     | 3.9                              | 13.7                                   | 16                          | 124                                |

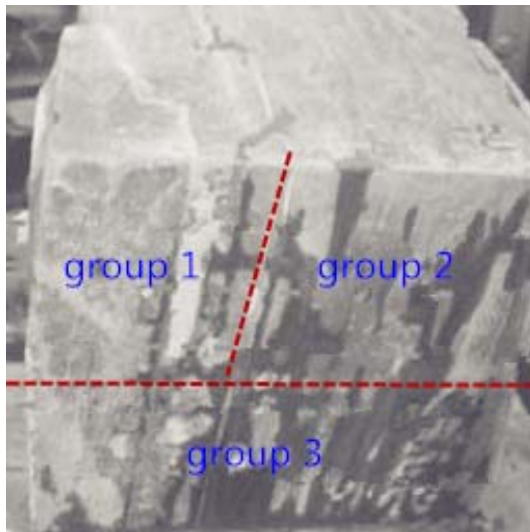


Fig.(6). Crack Morphology after Fracturing

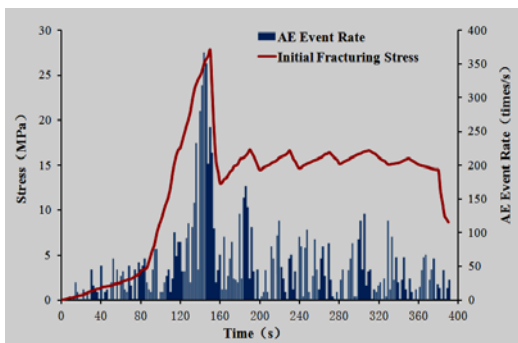


Fig.(7). Fracturing Curve by Acoustic Emission

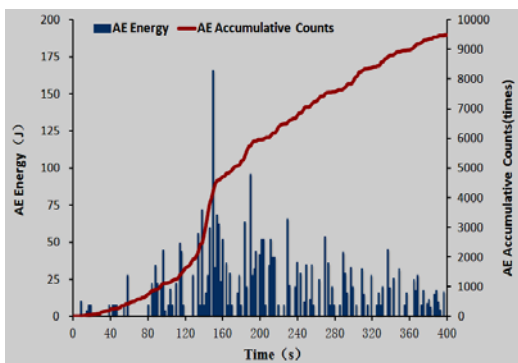


Fig.(8). AE Energy Changed with Time

From Fig.(7) we can see that at initial loading stage, rock mass stays in a compaction state and micro-cracks are closed. A small amount of AE event happens, which means the state has almost no crack initiation or expansion, which lasting 80 seconds. With the increasing of injection pressure, the acoustic emission counts show a gradual increasing, while the rock mass is still stays at the elastic deformation state, large amounts of micro-cracks evolution haven't occurred, which lasting 50 seconds. When the injection pressure reaches to the peak strength of rock mass, mutation rupture happens, where the value of the injection pressure is 27.8MPa, external work and gravitational potential energy of fracturing fluid are all transformed to mutation releasing energy of rock mass, acoustic emission signals has increased dramatically in a short period and the maximum ringing numbers reaches to 366. Micro-cracks are expanded by the releasing energy and then merged to form large cracks. Continuing to put pressure on coal samples, the number of AE events is increasing but still staying at a low level, which indicates that while the stress reach to the peak value, coal samples have obvious deformation and develop toward the stage of failure, which lead to a sharp adjustment of internal structure and a severe damage deformation of rock mass. After forming a number of cracks, the fracturing curve gradually tends to stable. Stopping pressure, the fracturing curve quickly decreased.

From Fig.(8) we can see that the acoustic emission energy remains at a low level before the injection pressure reaches to the peak strength of rock, which indicates that the micro crack propagation in coal rock is in a relatively stable state, and less of the plastic damage; when the injection pressure is about to reach to the peak strength, which means the test piece is near the damage, the acoustic emission energy increases rapidly, as well as the number of AE events, which means the process of deformation and damage have energy accumulation. When the injection pressure reach the peak value, mutation happens and a lot of energy accompany with the expansion and evolution of the crack as well as the energy of the acoustic emission reaches to the maximum value at the same time.

As the environment of indoor simulation test and the field measured data using similar criteria, the results of the test (Fig. 5) are similar with the theoretical calculation

data (Table.1). The measured with the theoretical number error of micro-cracks is about 0.27%, and the energy error is about 5.42%.

## VII. CONCLUSION

(1) The model of energy release of mutation rupture in coal and rock mass fracturing and the model of micro-cracks evolution are established, and study the micro-cracks expansion rules of coal and rock mass under mutation rupture, by contrast analysis of Theoretical calculation and actual data, the essential rules of rock mass rupture and micro-cracks evolution are revealed accurately, based on mutation theory.

(2)Hydraulic fracturing test of Coal bed shows that, cracks morphology after fracturing are affected by the natural crack system in coal bed, which are easy to form complex crack network system. After the first mutation rupture (main rupture), with injection pressure increasing, the fracturing curve waving many times, which means the micro-cracks mixing together and forming cracks, but this process has no obvious mutation behavior happens.

(3)Theoretical calculation and actual data have some errors, which are mainly come from the surface roughness by observing coal samples after fracturing. This will be the subsequent problem which needs to discuss next.

## CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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