A Novel Method for Blind Extraction of Signals using Empirical Mode Decomposition of Chaotic Energy Accumulation Zone in Underdetermined Systems

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***Abstract* — Starting from wavelet analysis of chaotic signals' time-frequency characteristic, we propose a new harmonic signals extraction method using Empirical Mode Decomposition (EMD). We analyze features of chaotic signals using wavelet transform to determine the parameters of harmonic signals and interference signals. The harmonic signals as source signals are hidden within chaotic energy accumulation zone. We use analog Gaussian channel noise generator on the observed signals according to the system requirement. We make empirical mode analysis of observed signals to establish virtual accepting arrays which implement the desired signal extracted from the accumulation zone in underdetermined system. This will reduce the additive effects of the channel noise on the extracted signals. Simulations verified the validity and general application of the method.**

Keywords - intrinsic mode functions; virtual receiving arrays; underdetermined system; wavelet analysis; extraction; smooth

I. INTRODUCTION

In recent years, with the use and study of chaotic signal is getting hot, chaos has become a new subject. Extract harmonic signal from chaotic background has become a very hot topic in chaotic signal processing field. Typical applications include extracting target signal from marine radar clutter, extracting maternal fetal ECG in biomedical, detecting and extracting target signal from visual evoked brain wave signal and so on [1-3]. These applications are all target signal extracted from chaotic background and models in which the number of observation signals is not less than that of source signals consistent with the assumptions of many algorithms. However, in practical applications, many systems do not meet this condition of positive definite or over-determined. The number of observations less than the number of source signals is called underdetermined system in practical communication system. The study on blind extraction of chaotic mixed-signal is still in the minority. In China, the most representative is Cheng Xiefeng [4], who proposed empirical modal analysis (EMD) for chaotic signals. Add intrinsic mode function (IMF) obtained as a supplementary signal to the mixed signals, in order to achieve the extraction of the desired signal. Tan Beihai [5] proposed underdetermined blind separation algorithm based on the estimating the number of source signals when it is unknown.

The harmonic signal blind extraction algorithm proposed in this paper is based on independent component analysis (FastICA). Extract desired signal hidden in the chaotic energy band from underdetermined mixed signals including harmonic signal, chaotic signal and interference signal at the receiving end. If the number of received signals is less than the total number of source signals which means blind extraction of multi input and low output, it is a typical ill posed problem in blind source separation. Aimed at solving the ill conditioned problem of this kind of blind source separation, there exists some processing methods, among which what is more familiar is the use of prior knowledge (such as spectrum analysis) for piecewise processing the original signal. In the case of underdetermined, if the presence of noise signals or other signals drowned desired signals [6], the ordinary blind extraction algorithm will fail. Another solution to solve this problem is to assume that signals are sparse and do frequency spectrum analysis for sparse signals. Because desired signals based on chaotic background selected in this article are non-sparse, in addition, spectrum of additive white noise in channel is very similar to spectrum of chaotic signal resulting in spectrum of chaotic signal is submerged by additive noise spectrum, which means the probability of solving the problem in frequency domain is slim. In view of the above problems, a viable and effective solution is presented.

II. CHAOTIC SYSTEM

This paper aims at using representative three dimensional chaotic systems of Chen and Rossler to analysis and simulate. Rossler chaotic system [7] is one of the typical secure communication systems and its mathematical model is made up of three first-order differential equations, which evaluate to

$$\begin{cases} \dfrac{dx}{dt} = -y - z \\ \dfrac{dy}{dt} = x + ay \\ \dfrac{dz}{dt} = b + z(x - c) \end{cases} \tag{1}$$

Where a,b,c are parameters, when $a = b = 0.2, c = 5.7$, Rossler system is in the state of chaos, whose chaotic attractor phase diagram is shown in Fig.1.

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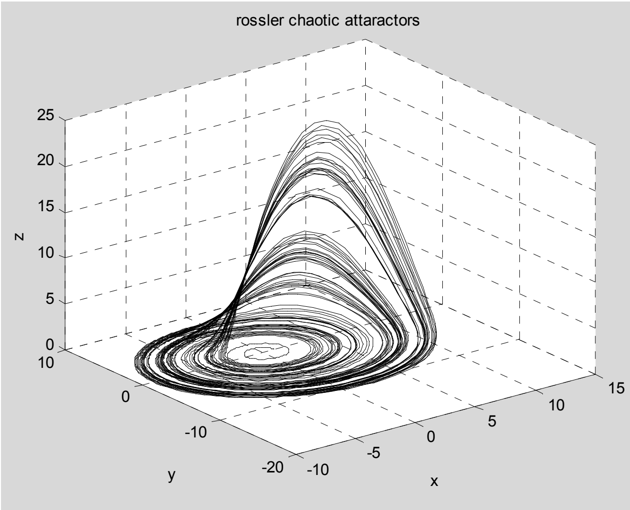

Figure 1. Chaotic attractors of Rossler system

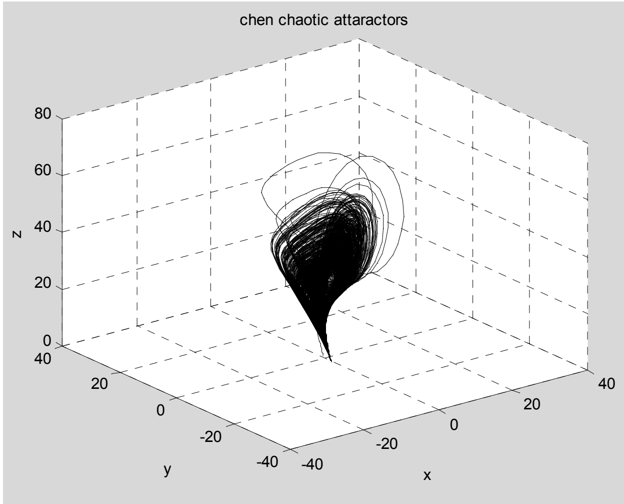

Figure 2. Chaotic attractors of Chen system.

Chen chaotic system [8] is common in practical application in secure communication and its mathematical model of expression is as follows:

$$\begin{cases} \dfrac{dx}{dt} = a\left(y-x\right) \\ \dfrac{dy}{dt} = \left(c-a\right)x - xz + cy \\ \dfrac{dz}{dt} = xy - bz \end{cases} \qquad (2)$$

Where $(x, y, z)^T \in R^3$ are the state of the system, a, b, c are system parameters. When $a = 35, b = 3, c = 28$, Chen system is in the state of chaos and chaotic attractor phase diagram is shown in Fig.2.

III. UNDERDETERMINED SYSTEM MODEL AND SOLUTIONS

Take underdetermined system, harmonic signals hidden and the additive noise interference into account, algorithm put forward in this paper can implement the blind extraction of harmonic signal embedded in chaotic signal based on an underdetermined system effectively. System model is shown in Fig.3.

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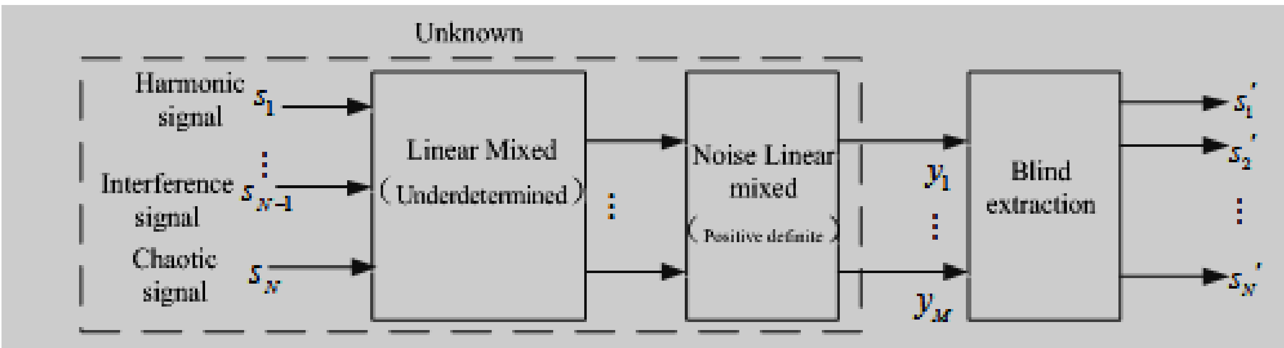

Figure 3. System mathematical model.

This model has the following characteristics:

First, systems with additive white Gaussian noise belong to underdetermined models. In blind source separation, when the number of observations is less than the number of source signals, it is an example of an underdetermined blind source separation. In simple terms, the number of known parameters is less than the number of unknown parameters which makes underdetermined blind separation unsolvable. In this model, the number of known parameter is set to be M (the number of observed signals at the receiving end), while the number of source signals at the sending end is N . In general, in order to solve underdetermined blind source

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separation problem, assume that the sources meet the sparseness, such as voice signals. Based on non-sparse, the extraction of the desired signal is blind extraction. Hybrid model for instantaneous blind source extraction under mixture models is:

$$Y(t) = A \times S(t) + N(t) \quad (3)$$

Where $A_{M\times N}$ is unknown channel mixing matrix $M < N$, randomly generated $S(t) = [s_1(t), s_2(t), \cdots, s_N(t)]^T$ represent N unknown source signals vector. $N(t) = [n_1(t), n_2(t), \cdots, n_N(t)]^T$ represents the additive white Gaussian noise vector. Separation model can be expressed as:

$$Y = W \times Y^{'} = W \times A \times S^{'} \quad (4)$$

$M_{M\times N}$ is separation matrix,

$Y(t) = [y_1(t), y_2(t), \cdots, y_M(t)]^T$

is observed signals vector. Blind extraction is designed to get the estimation of the desired signal from the observed signals.

Second, in spectral domain, frequency and amplitude of harmonic signal are hidden in the accumulation zone of chaotic signal, simultaneous, spectrum of chaotic signal is submerged by spectrum of additive noise completely in transmission channel. We know that solving this problem in the spectral domain is not feasible.

The system model is underdetermined with multiple antenna receiver. The source signals linear mixed contain harmonic signal, interference signal and chaotic signal are underdetermined, namely the number of mixed output antenna is less than the original signals. Mixed signals are transmitted to the receiving end by the multi-antenna and there exists Gauss white noise's positive additive in the channel and receiving end receives the initial signal sent by antennas.

In summary, algorithm proposed in this paper can simulate the practical use of underdetermined models. Blind extraction of underdetermined system is doing preprocessing for a certain route of initial signals, that is, using EMD to broke the selected initial signal down into a collection of intrinsic mode functions of IMF component signals and using IMF component signal as a separate supplementary to realize blind signal extraction which is based on FastICA algorithm. This method has proved its feasibility and effectiveness through simulation experiments and determined the similarity degree of extracted signals by similarity coefficient.

IV. DESCRIPTION OF BLIND EXTRACTION ALGORITHM

Algorithm in this paper is based on the FastICA algorithm to achieve blind extraction of the desired signal [9] which is under two dimensional chaotic systems. Basal algorithm FastICA is an algorithm that extracts each component successively and this component will be removed from the original data and continues to extract the rest of the data successively. Extended algorithm steps are as follows:

Step one: do wavelet transform for each chaotic system and analysis chaos energy accumulation zone, frequency and amplitude to obtain the range of its amplitude-frequency characteristics as a basis for subsequent simulations.

Step two: chaotic signal analysis determines the range of amplitude and frequency parameters for harmonic signals and interference signals which primarily ensure interference signal and harmonic signal will be completely hidden within the chaotic energy accumulation zone.

Step three: select channel mixing matrix randomly according to simulation system to determine the original mixed signals.

Step four: generate a corresponding number of Gaussian white noise according to the number of the original mixed-signal to simulate the effect of channel noise on the original mixed-signal, producing the observed signal clusters.

Step five: receiving end chooses one route from received signal as EMD decomposition IMF component group and the one is the first path component which reflects the noise signal best as a complementary component for received signal (or select two former IMF components sequentially). Add the selection component to the received signal matrix to obtain separate signal matrix.

Step six: set supplementary competed separation signal Y as input signal, to obtain the desired component $S^{'}$ by FastICA.

Step seven: smooth the extracted expected component $S^{'}$ to get the final desired signal.

There are two reasons why first path component of IMF is selected in step five (or select two former IMF components sequentially). First, the received initial signals are underdetermined (the number of received signal is less than the number of source signals), and supplemental IMF component can turn underdetermined extraction into positive definite extraction, which meets the requirement of FastICA blind extraction. Second, some document has proved that effect of additive channel noise on the extraction of desired signal is additive[10,11].Add the first path component of IMF that reflects channel noise best but not relevant with noise component ensures the effectiveness of isolation.

V. SIMULATION EXPERIMENTS AND RESULTS ANALYSIS

Since the algorithm is mainly aiming at the extraction of harmonic signals hidden in the chaotic energy accumulation zone, therefore before simulation, the first thing is to determine the chaotic energy accumulation zone. Because of the chaotic signals are non-stationary, so Fourier transform is not significant to chaotic signals. In this algorithm, the method used to determine the energy accumulation zone is wavelet transformation, which does not require whether the signal is stationary or not. Do wavelet transformation for the two selected chaotic systems. Three-way waveforms of the two chaotic systems are shown in Fig.4 and Fig.5. Fig.6 and Fig.7 is wavelet transform of corresponding waveform signal.

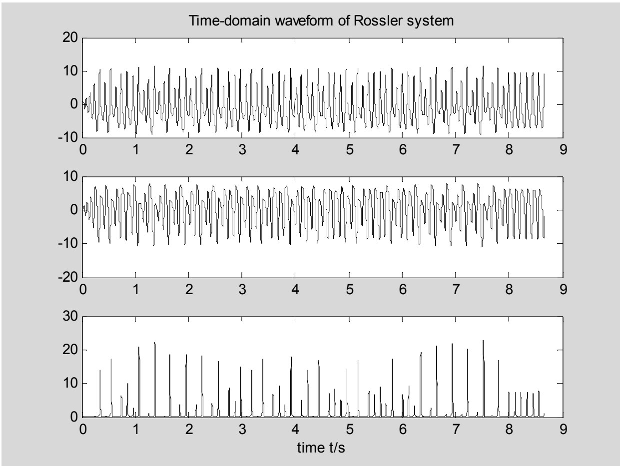

Figure 4. Three-way waveforms of Rossler Chaotic System.

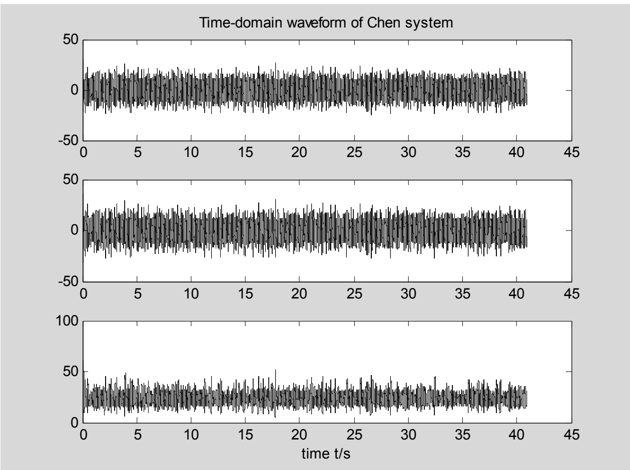

Figure 5. Three-way waveforms of Chen Chaotic System

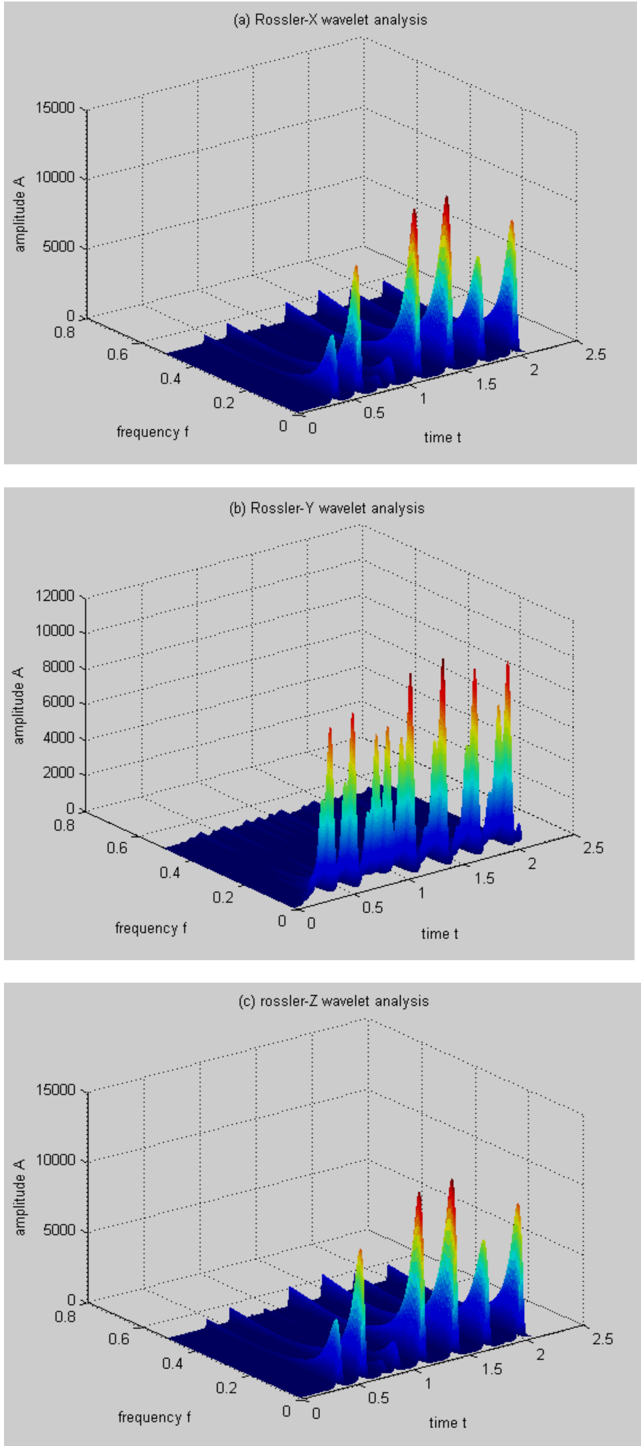

Figure 6. Wavelet transform of Rossler chaotic system.

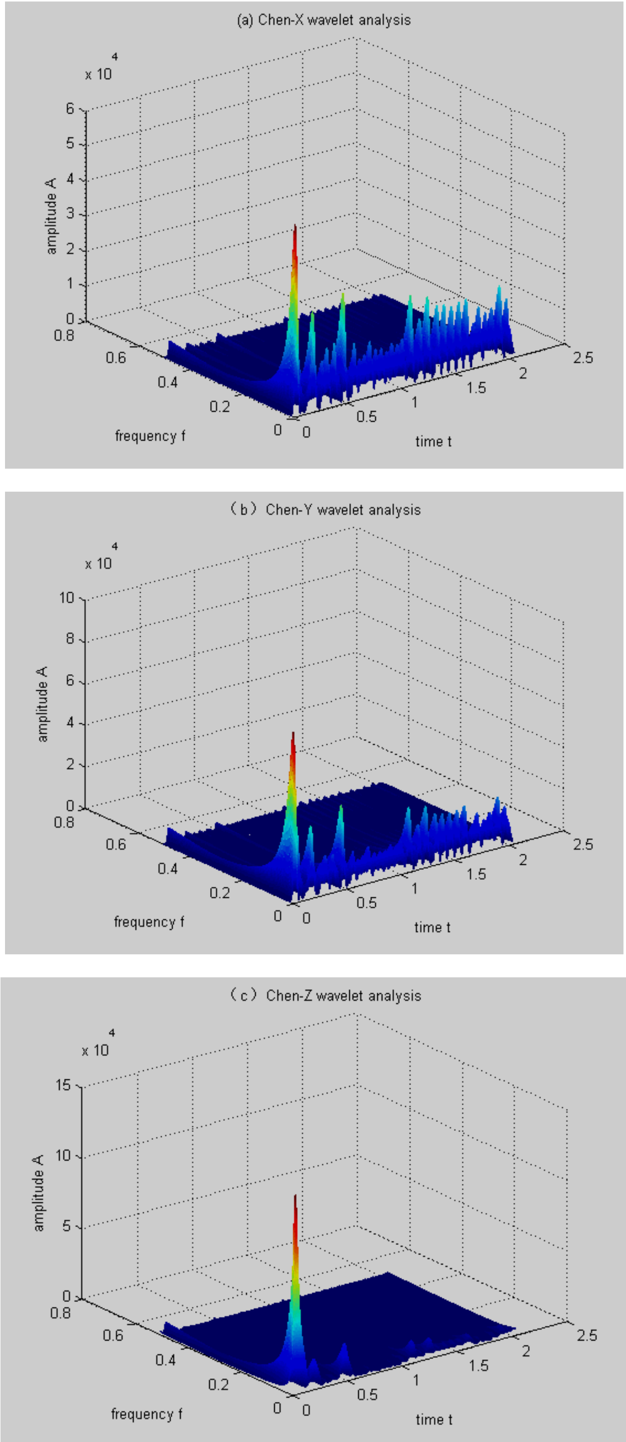

Figure 7. Wavelet transform of and Chen chaotic system

This paper only takes the x route chaotic signal as chaotic background. For x signal, by wavelet transform and calculation, we can get energy accumulation zone ranges for Rossler chaotic system and Chen chaotic system, they are 0~50Hz and 0~70Hz. Wavelet transform graphics determine frequencies of Rossler and Chen system are focused on 10Hz and 20Hz. Through the wavelet transform analysis of chaotic systems, frequency and amplitude of harmonic signals are chosen within the accumulation zone in order to achieve the effect of harmonic signals to be completely hidden. The frequency and the amplitude of the interference signal settings are focused on the energy accumulation zone, achieving jamming effect on extraction of harmonic signals. Channel mixing matrix has little effect on the performance of blind separation algorithm, so it can be arbitrary full row rank ($rank(A) = M$) matrix that randomly generated. Where M represents the number of required transmit antennas, M is no less than the number of desired signals. Using the extended algorithm proposed in this paper on the Rossler chaotic system and Chen chaotic system to verify the feasibility and effectiveness of the algorithm. Simulate communication simulation system when $N = 4, M = 3$ and $N = 4, M = 2$ respectively.

A *Rossler Chaotic System*

(1) System simulation when $N = 4, M = 3$

Analyze the spectrum of the Rossler system and set frequencies of harmonic signal are $f_0 = 10Hz$ and $f_1 = 15Hz$, both amplitudes are $A_0 = 40$. Mathematical expression is shown as follows:

$$S_{\sin} = A_0 \sin(2\pi f_0 t + \varphi) \tag{5}$$

Carrier frequency of interference signal BPSK is $\omega_c = 10Hz$, sampling frequency is 50 times of carrier frequency. The amplitude is $A = 50$, mathematical expression for interference (BPSK) signal is:

$$S_{BPSK}(t) = A\left[\sum_n a_n g(t - nT_s)\right]\cos\omega_c t \tag{6}$$

Take $SNR = 13dB$ for example (the range of speech communication SNR is $10-20dB$), because the performance of blind separation algorithm is not under the influence of channel mixing matrix, so channel matrix ($N = 4, M = 3$) can be generated randomly, A is shown as follows:

$$A = \begin{bmatrix} 0.7963 & 0.1338 & 0.1794 & 0.3909 \\ 0.1297 & 0.7446 & 0.1078 & 0.2951 \\ 0.2461 & 0.3006 & 0.6106 & 0.3560 \end{bmatrix}$$

After setting data parameters and channel conditions, simulation diagrams for underdetermined models based on the Rossler system are shown as below. Fig.8 and Fig.9 are spectral analysis of harmonic signal and the interference signal. Fig.10 and Fig.11 are waveform and spectrum analysis of two harmonic signals obtained by blind extraction.

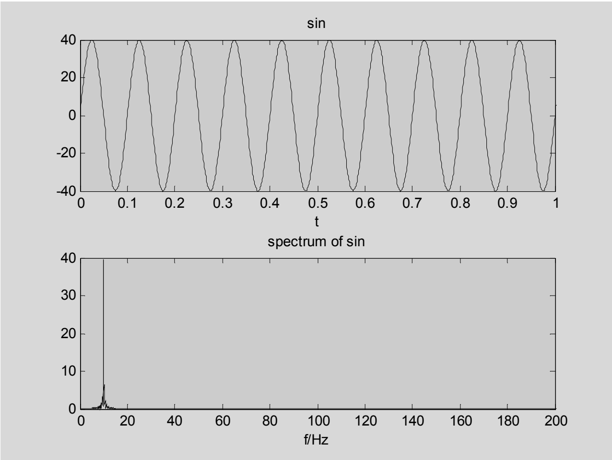

Figure 8. Waveform and spectrum of harmonic signal

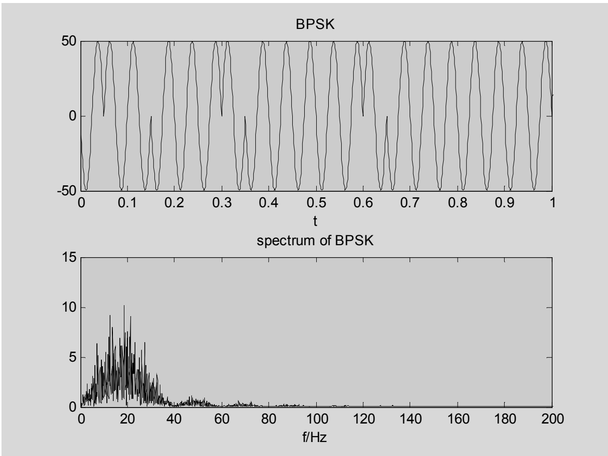

Figure 9. Waveform and spectrum of interference signal

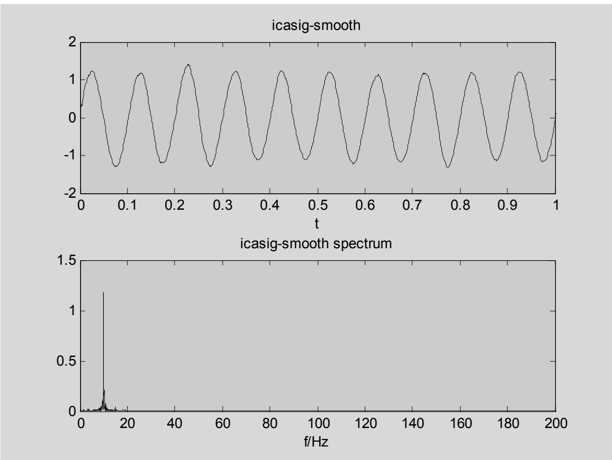

Figure 10. Waveform and spectrum of first extracted harmonic signal

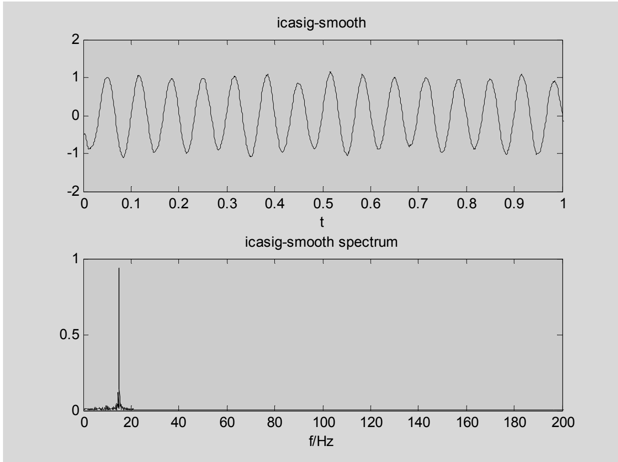

Figure 11. Waveform and spectrum of second extracted harmonic signal

Compare to the source signals, extracted signals have consistent frequency and the difference between amplitudes is more obvious. Similarity coefficient for two signals obtained by simulation are 0.9951 and 0.9935. Extracted graphics and similarity coefficient can prove the extended algorithm is very effective to solve blind extraction for desired signals in underdetermined systems and has strong execution.

(2) System simulation when $N = 4, M = 2$

The background is Rossler chaotic system, in the simulation we reduced the number of receiving antennas to verify the validity of the extended algorithm under underdetermined models and judge impact of the antennas' number on the extraction effect through comparison of similarity coefficients. Channel matrix is generated randomly, it is:

$$A = \begin{bmatrix} 0.8661 & 0.3262 & 0.1333 & 0.3422 \\ 0.3690 & 0.8925 & 0.1240 & 0.4333 \end{bmatrix}$$

There exists two harmonic signals, interference signal (BPSK) and x path component of chaotic signal in simulation. Two harmonic signals are $S_1(t) = A\sin(2\pi f_1)$ and $S_2(t) = A\sin(2\pi f_2)$, in addition, $f_2 = f_1 + \Delta f$, parameters are set the same as in the previous simulation. Take $SNR = 13dB$ for example, doing multiple simulations but only changing Δf , following data table are obtained:

TABLE 1 CORRELATION DATA BETWEEN SIMILARITY COEFFICIENT AND FREQUENCY INTERVAL IN ROSSLER CHAOTIC SYSTEM

frequency interval $\Delta f\,/Hz$	f_1 before smoothing	f_1 after smoothing	f_2 before smoothing	f_2 after smoothing
5	0.9706	0.9897	0.9470	0.9877
2	0.9712	0.9888	0.9486	0.9909
1	0.9732	0.9902	0.9497	0.9917

The data table not only proves the validity of this extended algorithm in MIMO case, but also the importance of smoothing steps which improves similarity degree of desired signal and extraction efficiency. In addition, in the case of minimum frequency interval, the two extracted signals can effectively and accurately reflect the characteristics of the desired signals with high stability.

B Chen chaotic system

Under the chaotic background of Chen chaotic system, repeat the previous two parts to validate its versatility. Because of the spectrum of Chen chaotic system is mainly concentrated in 20Hz, set the frequencies of harmonic signals to be $f_0 = 20Hz$ and $f_1 = 25Hz$, both amplitudes are $A_0 = 40$. Carrier frequency of interference signal (BPSK) is $\omega_c = 20Hz$, sampling frequency is 50 times of carrier frequency, the amplitude of carrier is $A = 50$. Data parameters and channel are consistent with Rossler's underdetermined system when $N = 4, M = 3$. Fig.12 and Fig.13 are spectrum analysis diagrams of one route of harmonic signal and interference signal. Fig.14 and Fig.15 are waveform and spectrum analysis charts of two harmonic signals obtained by blind extraction.

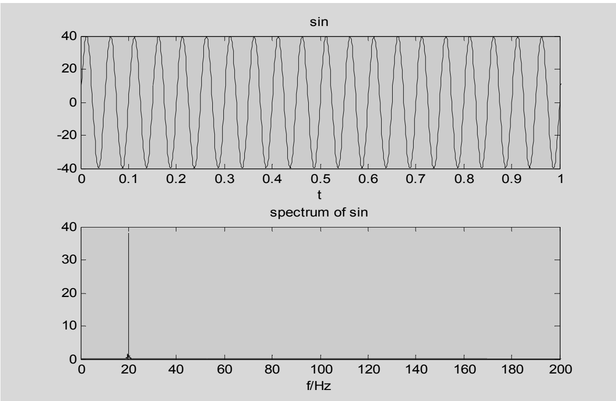

Figure 12. Waveform and spectrum of harmonic signal.

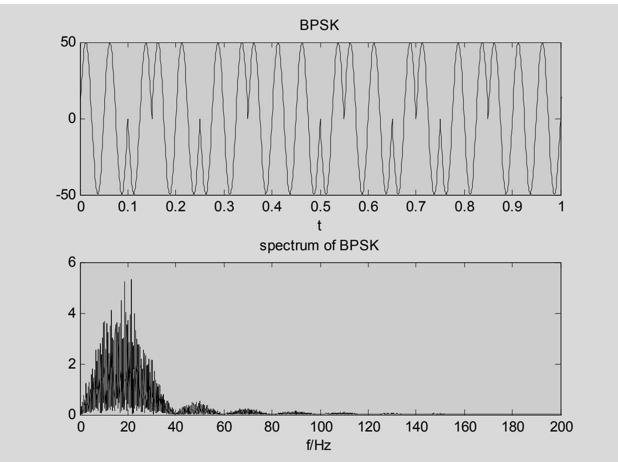

Figure 13. Waveform and spectrum of interference signal

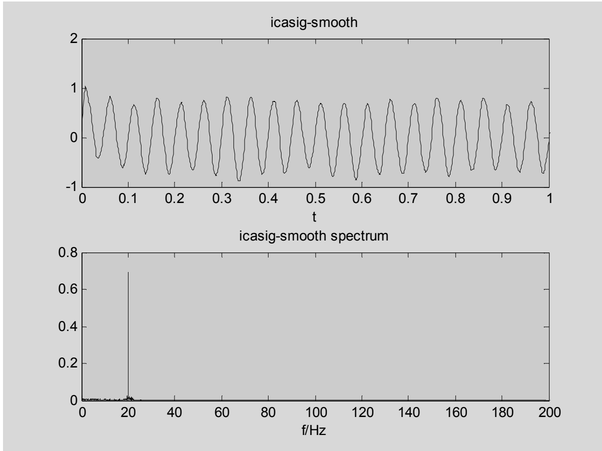

Figure 14. Waveform and spectrum of first extracted harmonic signal.

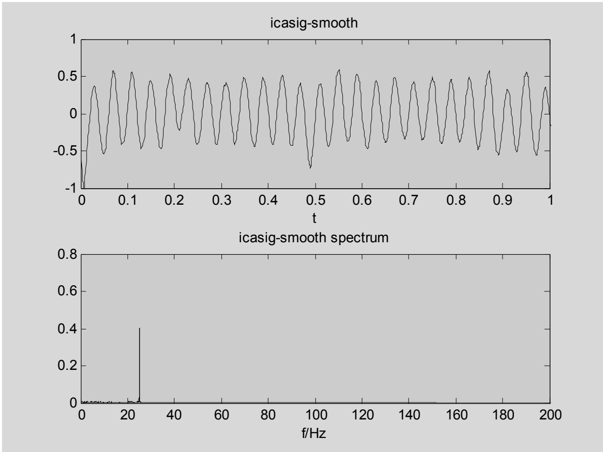

Figure 15. Waveform and spectrum of second extracted harmonic signal.

The verification of Chen chaotic system has proved the algorithm's validity. Similarity coefficients are 0.9864and 0.9735 imply that the algorithm also has a strong sense of stability in Chen chaotic system.

When $N = 4, M = 2$, simulation set in Chen chaotic system and parameters of chaotic system are the same as Rossler when $N = 4, M = 2$. Take $SNR = 13dB$ for example, doing multiple simulations but only changing Δf , following data table are obtained:

TABLE 2 CORRELATION DATA BETWEEN SIMILARITY COEFFICIENT AND FREQUENCY INTERVAL IN CHEN CHAOTIC SYSTEM

frequency interval $\Delta f\ /Hz$	f_1 before smoothing	f_1 after smoothing	f_2 before smoothing	f_2 after smoothing
5	0.9671	0.9735	0.9445	0.9490
2	0.9665	0.9745	0.9451	0.9702
1	0.9581	0.9747	0.9442	0.9751

It can be seen from above that algorithm is suitable for Chen chaotic system and performance of blind separation is stable under different frequency interval which means the effect of frequency interval on separation is almost negligible. Results obtained are the same as Rossler chaotic system.

In the view of the above two systems, when $N = 4, M = 3$, source signals are two harmonic signals, interference signal and chaotic signal, after many simulations, we can get graphs of different SNR on the similarity coefficient, it is shown as Fig.16.

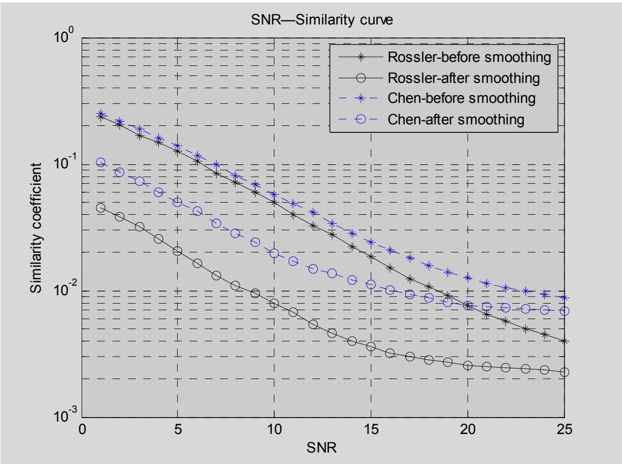

Figure 16. The desired signal SNR-Similarity curve of Chen and Rossler chaotic system

It can be seen apparently, similarity coefficient of desired signals in Chen chaotic system is much better than that of Rossler whether they are smoothed or not and separation effect of Chen chaotic systems is stronger than the Rossler system. In addition, graph clearly supports the effectiveness and feasibility of proposed algorithms in this article. The algorithm is efficient and accurate implementation of the expected signal extraction in Chen chaotic system or Rossler chaotic system.

VI. CONCLUSION

Complete extraction of harmonic signals is one of the objective realization of chaotic secure communication and it can also reflect the safety and efficiency of secure communication system. The algorithm proposed in this paper is based on experience modal analysis and wavelet analysis, we can introduce virtual accept arrays into underdetermined system for efficient extraction of the desired signal and smooth the signal to improve the algorithm's anti-noise robustness.

Further research work is to produce more multiplex IMFs signals by EMD decomposition signal. The different selection of virtual receiving array IMFs will have different effect on the algorithm or not and effectiveness of the algorithm has requirements for the harmonic signal's frequency, amplitude and channel noise remain to be verified.

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REFERENCES

[1] H. Leung, "Experimental modeling of electromagnetic scattering from an ocean surface using chaos theory," Chaos Solitons & Fractals, vol. 2, pp. 25-43, 1992.
[2] T. Schreiber and D. K. Kaplan, "Signal separation by nonlinear projections: The fetal electrocardiogram," Physical Review E, vol. 53, pp. R4326-4329, 1996.
[3] J. P. Pijn, N. J. Van and A. Noest, "Chaos or noise in EEG signals: dependence on state and brain site," Electroencephalography & Clinical Neurophysiology, vol. 79, pp. 371-381, 1991.
[4] X. C. Xie, L. Xu and R. Y. Yan, "Research of chaotic mixed-signal underdetermined blind source separation method," Journal of Nanjing University of posts and telecommunications, vol. 30, pp. 29-34, 2011.
[5] B. H. Tan and S. L. Xie, "Underdetermined blind source separation Based on estimation of source number," Journal of electronics and information, vol. 30, pp. 863-867, 2008.
[6] A. Taleb and C. Jutten, "Source separation in post-nonlinear mixture," IEEE Transactions on Signal Processing, vol. 47, pp. 2807-2820, 1999.
[7] O. E. Rössler, "An equation for continuous chaos," Physics Letters A, vol. 57, pp. 397-398, 1976.
[8] G. R. Chen and T. S. Ueta, "Yet another chaotic attractor," Int. J. Bifurcation Chaos, vol. 9, pp. 465-469, 1999.
[9] M. Zhu, C. Ji and Y. Yu, "The classification and status of blind signal separation," Chemical automation and Instrumentation, vol. 36, pp. 7-11, 2009.
[10] W. D. Jiao, S. X. Yang and S. T. Wu, "Study of noise removal technique based on independent component analysis," Journal of Zhejiang University, vol. 38, pp. 872-876, 2004.
[11] Q. Xiang, C. S. Lin and J. F. Cheng, "Algorithm of blind source separation in noisy backgrounds," Data acquisition and processing, vol. 21, pp. 42-45, 2006