Research on NURBS-Circular Hybrid Interpolation Algorithm in High-speed Machining

Xiaojun Wu, Junyi Cai, Zhixue Tong, Lile He[1]

School of Mechanical and Electrical Engineering, Xi'an University of Architecture and Technology, Xi'an, Shaanxi, 710055, China
[1]Tel: (86)130-9699-0831, Email: hllnh2013@163.com

Abstract **— Aiming at path discontinuity, serious feed-rate fluctuation and low efficiency in machining consecutive micro line segments, a hybrid interpolation algorithm including NURBS curves, circular arcs and lines is proposed. Firstly, desired line segments are screened out according to the length and angle constraint and then fitted into cubic NURBS curves. Secondly, three kinds of circular arc transition models for sharp corners are conducted with full consideration of machine mechanical properties. Finally, a interpolation algorithm based on Newton-Raphson and linear acceleration/deceleration method is applied to the whole contour. Simulation results demonstrate that the proposed algorithm reduces the contour error and advances the machining efficiency by improving the continuity of contour.**

Keywords- Consecutive micro line segments; NURBS curve fitting; Circular arc transition; Newton-Raphson method

I. INTRODUCTION

Parametric curves including Bezier curve, B-spline and NURBS curve are widely used in Computer Aided Design (CAD) modeling system to present the surface of complex objects. The problem of improving the efficiency while ensuring the accuracy in parametric curve machining has become the hot point of research. Siemens not only integrates NURBS interpolator into SINUMERIK 840D CNC system, but also puts forward conception of compressor, which can be used to compress a large amount of micro line segments code to form spline curves [1]. But unfortunately, these technologies are strictly confidential and not open to us. As NC machining technology in China is rather behind that of other developed countries, almost all CNC systems are equipped with linear and circular interpolation only. When machined by these traditional systems, complex curve should be discretized into micro line segments firstly by CAD/CAM. This method of using micro line segments as tool path without any optimization mainly brings out two problems [2]: (1) the conflict between machining accuracy and data amount is inevitable, which means the data amount is becoming larger with increasing of requirements for accuracy. Too much data code places the communication burden on CNC system, and thus reduces the reliability of the whole system; (2) the machining accuracy of the workpiece is seriously influenced by feedrate discontinuity, as well as the service life of the machine tool.

To cope with the problems mentioned above, a great deal of research work has been done, which can be divided into three categories: direct transition method, local transition method and curve fitting method [3]. Wang et al [4] established a mathematical model of micro line segments and derived equations for calculating feedrate around the corners. Zhang et al [5] proposed an idea of imaginary circular arc to set constraints for feedrate of the connective points. Erkorkrnaz et al [6] put forward S-type acceleration and deceleration (acc/dec) method to minimize the vibration of the mechanical system. Those methods, which we called direct transition methods, aim to get the maximum feedrate at the corner between adjacent micro line segments according to some conditions. As the name suggests, it does not change the path of line segments, but the feedrate at the corner allowed is too low. What's worse, the processing quality is unstable on account of the difficulty to calculate corner point and velocity precisely.

In order to relieve the problems existing in the corner machining, a growing number of scholars adopted local transition method. He et al [7] improved the feedrate by blending Ferguson spline into sharp corners. Stephen et al [8] inserted a kind of special spline, whose curvature is continues, between the adjacent line segments to solve the drastic distortion of curvature. The local transition method enhanced the speed at the corner. However, the contour errors are difficult to control when the angles at the corner are small.

With the rising of computing power, another approach named curve fitting method receives more and more attention. Wang and Ren et al [9, 10] fitted micro line segments into NURBS curve or quintic spline curve. But the specificity of feedrates at the joints of line segments and spline curves should be taken into account, while they didn't.

For achieving the purpose of high speed and high precision in micro line segments machining, a hybrid interpolation algorithm mixed NURBS and circular arc is proposed. First of all, micro line segments satisfied with the constraint conditions were screened out to be fitted into cubic NURBS curves based on the averaging technique (AVG). Next, circular arc was used for the transition between lines and fitted curves. This process can help to improve the continuity of the machining contours. Finally, all the interpolating elements, including micro lines, circular arcs and NURBS curves were interpolated based on Newton-Raphson method.

II. NURBS CURVE FITTING

A. Review of NURBS curve

NURBS is defined as the only mathematical method for describing product shape in STEP (Standard for the Exchange of Product Model Data). A *k*th-degree NURBS curve can be described as follows [11]:

$$\begin{cases} P(u)=\dfrac{\sum_{i=0}^{n}\omega_i d_i N_{i,k}(u)}{\sum_{i=0}^{n}\omega_i N_{i,k}(u)} \\ N_{i,0}(u)=\begin{cases}1 & u_i \le u < u_{i+1} \\ 0 & otherwise\end{cases} \\ N_{i,k}(u)=\dfrac{u-u_i}{u_{i+k}-u_i}N_{i,k-1}(u)+\dfrac{u_{i+k+1}-u}{u_{i+k+1}-u_{i+1}}N_{i+1,k-1}(u) \end{cases} \quad (1)$$

Where d_i (i=0, 1, ⋯, n) are the control points, ω_i (i=0, 1, ⋯, n) are the corresponding weights, $N_{i,k}(u)$ are the *k*th-degree basis functions of rational B-spline, U is the knot vector, which is defined as follows:

$$U=\{\underbrace{a,\cdots,a}_{k+1},u_{k+1},u_{k+2},\cdots,u_n,\underbrace{b,\cdots,b}_{k+1}\} \quad (2)$$

Defined by $\{d_i\}$ and $\{\omega_i\}$, a set of weighted points $d_i^{\omega_i}=(\omega_i x_i,\omega_1 y_i,\omega_i z_i,\omega_i)$ are got. A B-spline curve in four-dimensional space is presented as follows:

$$P^{\omega}(u)=\sum_{i=0}^{n}N_{i,k}(u)d_i^{\omega_i} \quad (3)$$

There is a mapping H can map $P^{\omega}(u)$ in four-dimensional space to $P(u)$ in three- dimensional space, which is given as:

$$P(u)=H\{P^{\omega}(u)\}=\sum_{i=0}^{n}R_{i,k}(u)d_i \quad (4)$$

The key to NURBS curve fitting lies in how to calculate control point d_i after getting data point q_i. Les Piegl and Tiller Wayne [12] indicated that AVG is a good choice to calculate knot vector U, which is expressed in Eq. 5.

$$\begin{cases} u_0=u_1=\cdots=u_k=0 \\ u_{k+j}=\dfrac{1}{k}\sum_{i=j}^{j+k-1}\tilde{u}_i, \qquad j=1,2,\cdots,n-k \\ u_{n+1}=u_{n+2}=\cdots=u_{n+k+1}=1 \end{cases} \quad (5)$$

B. Preprocess of curve fitting

With a view to reduce fitting error, it is an essential procedure to screen out desired micro line segments from NC code based on geometric characteristics before NURBS curve fitting. The screening conditions are discussed in detail now.

1) Length constraint

It is clear that long line segments are likely the original outline of workpiece, and do not need to be fitted into curve. Otherwise, the fitting error is so great that has a negative effect on high precision machining. So the desired micro line segments must meet the requirement of length constraint firstly. Near-circular arc approximation method [13] is applied to analysis the effect of line length on fitting error, which is shown in Fig.1.

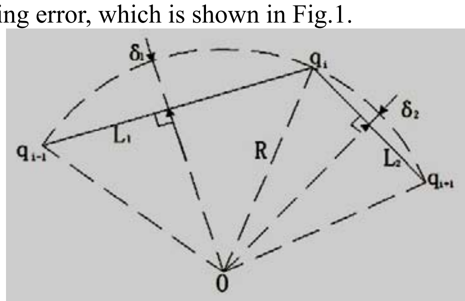

Fig.1 Diagram of length constraint

Where q_{i-1}, q_i and q_{i+1} are the data points, L_1 and L_2 are the line length between two adjacent points, R is the radius of near-circular arc, δ_1 and δ_2 are the chord errors, which are considered as major source of fitting errors.

The chord error can be derived from Fig.1 as follows:

$$\begin{cases} \delta_1=R-\sqrt{R^2-(L_1/2)^2} \\ \delta_2=R-\sqrt{R^2-(L_2/2)^2} \end{cases} \quad (6)$$

It can be seen from Eq.6 that chord error δ is positively correlated with line length L. In addition, the ratio of δ_1 and δ_2 is also influenced by the ratio of L_1 and L_2. Therefore, the line length and the ratio of adjacent line lengths ought to be limited in a certain range. The length constraint is represented as follows:

$$\begin{cases} L_i \le L_m \\ \lambda=\dfrac{L_i}{L_{i+1}}\in[\dfrac{1}{\lambda_m},\lambda_m] \end{cases} \quad (7)$$

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Where L_m is the maximum length, λ_m is the tolerance value of the length ratio.

2) Angle constraint

Too large angle between adjacent micro line segments means large curvature, which leads to large fitting error. The angle analysis diagram is shown in Fig.2.

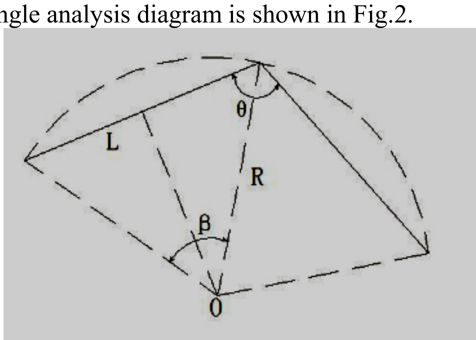

Fig.2 Diagram of angle constraint

Where θ is the angle between adjacent micro line segments, β is the central angle of corresponding line. For the sake of analysis, the lengths of adjacent line segments are assumed equal. The geometric relationships among these parameters can be expressed as follows:

$$\rho = \frac{1}{R} = \frac{\cos(\theta/2)}{L/2} \tag{8}$$

When L is a fixed value, curvature ρ decreases with the increasing of θ. Suppose θ_{min} is the minimum value of the angle between adjacent micro line segments, then the angle constraint can be expressed as Eq. 9:

$$\theta \geq \theta_{min} \tag{9}$$

Only the micro line segments meet the constraints of Eq.7 and Eq.9, can they be fitted into a NURBS curve.

The whole NURBS curve fitting process [12] can be illustrated as following flow chart:

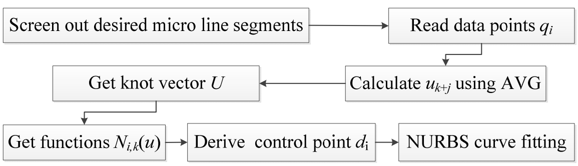

Fig.3 NURBS curve fitting flow chart (k=3)

III. MODELING FOR CIRCULAR ARC TRANSITION

After desired line segments are fitted into NURBS curves, there are many sharp corners lie at the junctions of curves and lines. How to deal with these corners becomes the key to improving machining efficiency and stability. There are two main methods: one is that the tool processes the corner directly at a low speed based on optimized acc/dec strategies. The other one is that an appropriate curve is adopted to smooth the corner on the premise of guaranteeing machining precision. The previous method has the advantage of higher precision and shorter transition time. However, it is also easily appear the problems of instability and poor efficiency. The paper takes the idea of the latter method which improves the stability and efficiency at the cost of certain precision.

The circular arc whose interpolation is embedded in CNC system is applied for corner transition.

A. Smoothing the corner between adjacent micro line segments

The model of two consecutive micro line segments blended with a circular arc is shown in Fig. 4.

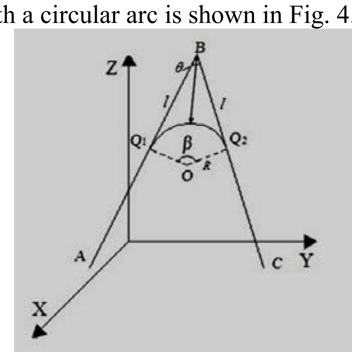

Fig.4 Sketch map of transition for two consecutive micro line segments

Where line AB and BC are the lines do not meet the constraints, and θ is their included angle. Arc Q_1Q_2 is the circular arc for transition, Q_1 and Q_2 are the tangency points, R is the radius of arc, β is the central angle, l is the length of line Q_1B or Q_2B.

According to the geometrical relationship of these parameters, Eq.10 is deduced.

$$\begin{cases} l = \dfrac{\sin\frac{\beta}{2}}{1-\cos\frac{\beta}{2}}e \\ R = \dfrac{\cos\frac{\beta}{2}}{1-\cos\frac{\beta}{2}}e \\ \theta = \pi - \beta \end{cases} \tag{10}$$

Where e is the shortest distance from point B to the arc, and it is also the maximum deviation between tool path and original contour. In order to ensure the deviation is in the allowable range, l and R should be limited by maximum contour deviation E_{max} allowed by the system. Substituting E_{max} into Eq.10 yields:

$$l_c = \frac{\sin\frac{\beta}{2}}{1-\cos\frac{\beta}{2}}E_{\max} \tag{11}$$

Another factor should be taken into account that one line segment not only need to be transited with prior one, but also with the following line or curve. The transition distance limited by the lengths of consecutive line segments l_s is expressed below:

$$l_s = \min\left(\frac{|AB|}{2}, \frac{|BC|}{2}\right) \tag{12}$$

Therefore, l and R are defined by the contour and the length of micro line, and their values can be calculated by Eq.13 and Eq.14:

$$l = \min(l_c, l_s) \tag{13}$$

$$R = \begin{cases} \dfrac{\cos\frac{\beta}{2}}{1-\cos\frac{\beta}{2}} E_{\max} & l_c \le l_s \\ \dfrac{\cos\frac{\beta}{2}}{\sin\frac{\beta}{2}} l_s & \text{otherwise} \end{cases} \tag{14}$$

B. Smoothing the corner between a line segment and a curve

Fig.5 illustrates the process of smoothing the corner between a line segment and a NURBS curve.

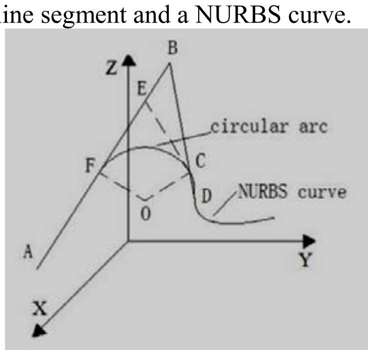

Fig.5 Sketch map of transition for a line segment and a NURBS curve

As shown in Fig.5, line BD is the line satisfied the constraints, while line AB is the one does not. If make circular arc transition for these two lines, the curve segment whose chord length equals to l has to be calculated firstly. This treatment will also change the path of fitted curve and aggravate the computational burden on CNC system. What's worse, the contour smoothness is destroyed because the chord is not tangent to the NURBS curve.

In order to ensure the tangent vectors of the circular arc is equal to that of NURBS curve at common tangent point, a certain part of line BD is left for arc transition in the preprocessing stage, shown as line BC in the Fig.5. The length of line BC is determined by Eq.11. Then make a tangent to the curve at point C. The tangent meets the line AB at point E. Finally, the circular arc transition for smoothing the corner between line AE and EC can be easily made according to previous chapter.

C. Smoothing the corner between adjacent curves

The way to smooth the corner between adjacent NURBS curves is described as Fig.6. The process is similar to the above mentioned and not be repeated here.

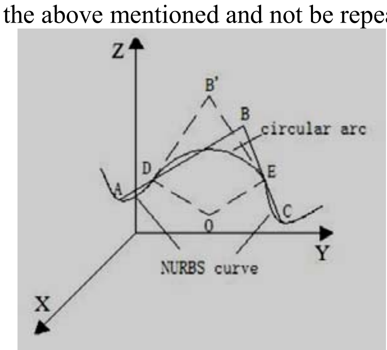

Fig.6 Sketch map of transition for adjacent NURBS curves

IV. INTERPOLATION ALGORITHM

A. Principle of interpolation

The whole contour is got after NURBS curve fitting and circular arc transition. Next step is to take post-processing of the contour and convert them to NC code. In another words, the tool path need to be planned by determining the interpolate points and the feedrates at these points. With the full consideration that all traditional CNC systems are equipped with linear and circular interpolator, how to get the accurate points of NURBS curve becomes the main work of the paper.

The principle of NURBS curve interpolation algorithm is illustrated in Fig.7.

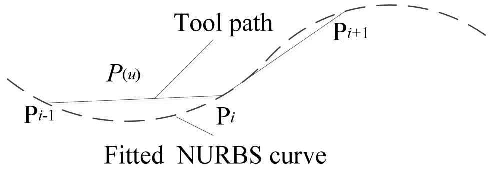

Fig.7 Principle of curve interpolation algorithm

Where $P(u)$ is the expression for NURBS curve, P_{i-1}, P_i and P_{i+1} are the interpolated points. L is the length of chord P_iP_{i+1}, which is related to machine tool. Their relationship can be calculated as follows:

$$L = V_{temp}T \approx |P_iP_{i+1}| = |P(u_{i+1}) - P(u_i)| \tag{15}$$

$$\eta = \left(1 - \frac{|P(u_{i+1}) - P(u_i)|}{V_{temp}T}\right) \times 100\% \tag{16}$$

Where V_{temp} is the temporary feedrate at point P_{i+1}, T is the interpolation period, η is feedrate fluctuation ratio. The task is to find a parameter u_{i+1} to keep η as small as

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possible. However, for $P(u)$ is a high-order equation, it is not easily to calculate the accurate u_{i+1} , but instead of approximate value u'_{i+1}. In fact, the nature of the problem is to solve nonlinear function. Newton-Raphson method is used to cope with this kind problem.

B. Newton-Raphson method

A function is constructed as flows:

$$F(\xi) = V_{temp}T - \left|P(\xi) - P(u_i)\right| \quad (17)$$

The accurate parameter ξ^* who satisfies the condition of $F(\xi^*)=0$ is the one we want. The standard format of Newton-Raphson formula is given as:

$$\xi_{k+1} = \xi_k - \frac{F(\xi_k)}{F'(\xi_k)} \quad (18)$$

Where ξ_{k+1} and ξ_k are the values of k+1 and k times of iterative computation respectively, $F'(\xi_k)$ is the first-order derivative of $F(\xi_k)$ at $\xi = \xi_k$. The initial iteration value ξ_0 is got by first order Taylor expansion:

$$\xi_0 = u_i + \frac{V_{temp}T}{\left|P'(u_i)\right|} \quad (19)$$

The feedrate fluctuation ratio and iterations are chosen to be the stopping criterion for iteration, namely:

$$\eta \le \eta_{\text{tol}} \quad \text{and} \quad k = n \quad (20)$$

Where η_{tol} is the maximum value set by user, n is the maximum iterations.

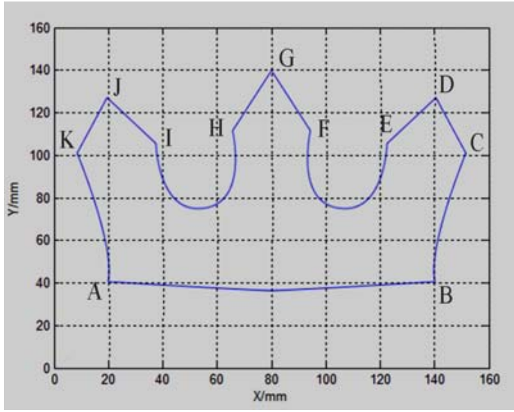
It should not be ignored that the feedrates of interpolated points are also affected by the machine features and contour error, which can be derived by following equation:

$$V_i = \min(V_{\max}, \sqrt{a_{\max} R_i}, \frac{2}{T}\sqrt{\delta_{\max}(2R_i - \delta_{\max})}) \quad (21)$$

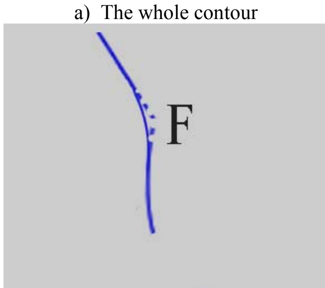
Where $V_{\max}$ is the allowed maximum feedrate of CNC system, and $a_{\max}$ is the allowed maximum acceleration. If we pursue constant feedrate machining blindly, the feedrate of the whole process has to be set a low value, which does not meet the requirements of high speed machining. Therefore, the chord length L should be modified in real time according to Eq.21.

V. PERFORMANCE ANALYSIS AND SIMULATION RESULTS

To evaluate the performance of the proposed hybrid interpolation algorithm, the paper takes a two-dimensional view of a crown as the study subject, and gives simulations on a PC using C++. Set the parameter of maximum contour deviation E_{max} is 0.002mm. The final contour after being fitted and smoothed is shown in Fig.8.



a) The whole contour



b))Local amplification

Fig.8 Machining contour of the crown

Fig.8a is the whole contour of the crown, where Point A is the start point of the curve, as well as the end point. Other points (from B to K) are the corner points of adjacent micro line segments. These regions are treated by circular arc transition, which can be seen in Fig.8b. After these treatments, the contour is much smoother and it is more beneficial to enhance machining speed.

For the sake of further analysis on the superiority of the fitted curve, the feedrate curve under Newton-Raphson method are planned out, as shown in Fig.9. The parameters are set as follows: interpolation period T = 2 ms, the acceleration of the machine tool a = 500 mm/s^2, the maximum feedrate $v_{\max}$ = 50 mm/s, maximum feedrate fluctuation ratio η_{tol} = 0.1%, the maximum iterations n = 2.

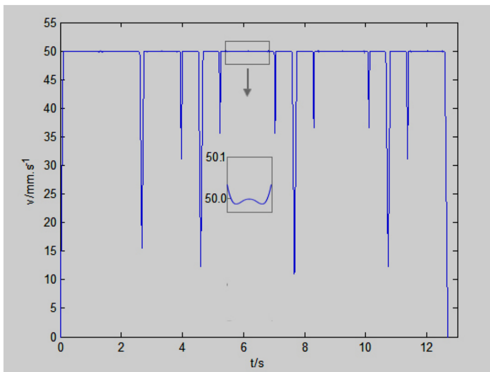


Fig.9 The curve of feedrates

From Fig.9, it can be seen that the feedrates keep steady in a large range and the ones around the sharp corners are low because of the limitations of the mechanical properties. The results fully satisfy high speed and high accuracy machining.

VI. CONCLUSIONS

Aiming at the shortcomings of complex curve machining in traditional CNC system, a NURBS-circular hybrid interpolation algorithm is proposed. The original contour is optimized by NURBS curve fitting and circular arc transition. Then an interpolation algorithm based on Newton-Raphson is adopted to reduce feedrate fluctuation with full consideration of mechanical characteristics. The proposed algorithm is simulated on the PC using C++. The results verify the scientific rationality and high performance of the hybrid algorithm.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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