Topological Analysis on Degree of Freedom of Mechanism Based on Decomposition and Synthesis of Moving Element

Jianguo Luo[*,1], Jianyou Han[2]

[1]College of Mechanical & Electrical Engineering, North China Institute of Science & Technology, Beijing, 101601,China
[2]Beijing University of Science & Technology, School of Mechanical Engineering, Beijing, 100083,China

***Abstract* — Based on the analysis of sub moving elements in fixed coordinate system from pure translation and pure rotation moving elements, the equivalent space motion element is obtained by means of coordinate transformation, three dimensional decomposition dealt with each joints, equivalent main moving element set and equivalent vice moving element set and equivalent rotation moving element obtained then. Planar six-dimensional topological expressing graph discovered to to analysis the degree of freedom, which include three topological line on the left side adopted to express the corresponding translation moving element set on three dimensional coordinate axis respectively, still include another three topological line on the right side adopted to express the corresponding rotation moving element set on three dimensional coordinate axis respectively, horizontal line with different height and thickness used to display the section, different computation and analysis methods presented to find the construction and number of degree of freedom(DOF) of serial mechanism and parallel mechanism and hybrid mechanism.**

Key words — Mechanism，Moving element，Topological expression，degree of freedom

I. INTRODUCTION

Research on the mechanism is the process of human understanding of nature and the transformation of nature, it is the combination of theory and practice to boost the process of human development. The theory of graph theory has been introduced to mechanics by Freudenstein [1] and Dobrjanskyj more than hundred years, express the topological structure of kinematic chain with the graph theory[2] can realize the transformation from space to plane, which make the complex structure simple and intuitive. Wang et al.[3] introduced the topological theory, defines the topological space of parallel mechanism, analyzed the topological features of parallel mechanism, topological transformation method on parallel mechanism studied then. Luo et al.[4] proposed the topological optimization model of maximum output displacement as the target and minimum input terminal performance constraint of compliant mechanism. Niu et al.[5] adopted a new method to extract the topological graph of compliant mechanism, the CAD system can be used to identify the parameters of the model. At the same time, the topological graph can be expressed by matrix[6], such as the adjacency matrix, correlation matrix and other results. Analysis and processing of mechanism realized by means of mathematical tools , it provides an effective means for the synthesis and innovative design of the mechanism. More than 150 years history exist for the solution on DOF of mechanism, the traditional Kutzbach-Grübler formula[7] is suitable for most of the mechanisms, with the development of mechanics, a lot of new mechanism and typical mechanism not meet the above formula[8], Huang et al.[9] proposed to make use of the constraint spiral to solve the DOF of mechanism, it could be applied to all the known mechanisms.

However, existing methods can only analyze the DOF of the mechanism abstractly, consists and characteristics of elements in the spatial coordinate system can not be clearly expressed, and it can not be expressed by one-dimensional group sets for the three-dimensional spatial moving capability of mechanism. In this work, a new topological operation and analysis method based on the decomposition and synthesis of moving elements proposed, It can reflect the constitute elements of the DOF and its specific features. Which provide a very convenient way and basis for the follow-up analysis on working space and singularity of mechanism, still make a new contribution to the theory research and progress of mechanism.

II. SPATIAL EQUIVALENT MAPPING OF MOVING ELEMENT

The mechanism is formed by one or more kinematic branch chains, different mechanism of structure obtained by different combination of forms, such as the serial mechanism and parallel mechanism and hybrid mechanism, In either form, the basic unit is the joint. As the most basic unit of mechanism, the joint is decomposed into the translation moving element and rotation moving element, on the basis of the above, combination form of multiple translation or multiple rotation or superposition of translation and rotation can be

obtained from that. To analysis the DOF of mechanism equal to analysis the moving capability and constitution of mechanism, the moving capability of mechanism is integrated by the moving capability of each joints in it, after found the moving capability of each joints, corresponding synthetic methods and rules given as well, We will get the number and constitute of DOF.

The end point of one single kinematic chain has a certain motion representation in each joint coordinate system, by means of homogeneous coordinate transformation, coordination of end point of kinematic chain can be transformed to coordination in the fixed base coordinate system, as shown in Fig. 1 and Expression (1), moving elements represented by points mapped from the space active coordinate system to the fixed coordinate system.

The homogeneous coordinate transformation, as shown in Fig. (1), it can solve many applications such as translation and rotation and scaling, through the addition of translation matrix, or through the multiplication of rotation matrix and scaling matrix, effective mapping of points from a coordinate system of two-dimensional or three-dimensional or even higher dimensional space to another coordinate system.

The equivalent mapping results of arbitrary line segment in three-dimension space is still a line segment or a point, the decomposition of the translation moving element in the three-dimensional fixed coordinate system realized then, as shown in Fig. (2) and Expression (2) to (4), the coordinates region is the mapping results of corresponding segment, the set of one-dimensional mapping contains only the coordinates region. Any circle can be seen as a rotation moving element of a rotation joint around determined axis, an ellipse or circle or line segment is obtained by projecting the circle to three planes of the three-dimensional coordinates, the ellipse can be divided into four segments of the arc again, as shown in Fig. (3) and Expression (5) to (6). In the three plane, the ellipse or circle or line segment has a corresponding projection line or point on the three axis, decomposition of arbitrary rotation moving element in three-dimensional fixed coordinate system realized as well, as shown in Fig. (4) and Expression (7), the rotation angle range of the corresponding plane curve (ellipse or circle) around three-dimensional Cartesian coordinate system is the mapping results, the coordinates region of the projection line segment or point in the corresponding plane of the plane curve (ellipse or circle) or line segment or point is the mapping results as well, the set of one-dimensional mapping include region of coordinates and region of angles.

$$P_0 = {}^1_0T \cdot {}^2_1T \cdot {}^3_2T \cdots\cdots {}^m_{m-1}T \cdot P_m \tag{1}$$

$$\{P_n\} \to \{P_x \cup P_y \cup P_z\}_{main} \tag{2}$$

$$\{\overline{P}\} \to \{\overline{P}_x \cup \overline{P}_y \cup \overline{P}_z\} \tag{3}$$

$$\{\overline{P'}\} \to \{\overline{P}'_x \cup \overline{P}'_y \cup \overline{P}'_z\} \tag{4}$$

$$\begin{aligned}\{R_O\}^e \to &\left\{R_{c_1}(\angle a_3c_1a_4)\right\}^c \cup \left\{R_{c_2}(\angle a_1c_2a_2)\right\}^c \\ &\cup \left\{R_{b_1}(\angle a_2b_1a_3)\right\}^c \cup \left\{R_{b_2}(\angle a_1b_2a_4)\right\}^c\end{aligned} \tag{5}$$

$$\{R_O\}^e \to \left\{R_{c_1}(\alpha_1)\right\}^c \cup \left\{R_{c_2}(\alpha_2)\right\}^c \cup \left\{R_{b_1}(\alpha_3)\right\}^c \cup \left\{R_{b_2}(\alpha_4)\right\}^c \tag{6}$$

$$\{R_n\} \to \{R_x \cup R_y \cup R_z\} \cup \{P_x \cup P_y \cup P_z\}_{vice} \tag{7}$$

Among that, Expression (2) to (4), P_x, P_y, P_z is the corresponding mapping results of decomposed sub moving element of arbitrary translation moving element P_n in three dimensional Cartesian coordinate system respectively, which defined as the main translation moving element, expressed with $\{\}_{main}$. $\overline{P}_x, \overline{P}_y, \overline{P}_z$ is the mapping vector of arbitrary vector $\overline{P}$ in three-dimensional Cartesian coordinate system, $\overline{P}'_x, \overline{P}'_y, \overline{P}'_z$ is the mapping vector of arbitrary vector $\overline{P'}$ in three-dimensional Cartesian coordinate system.

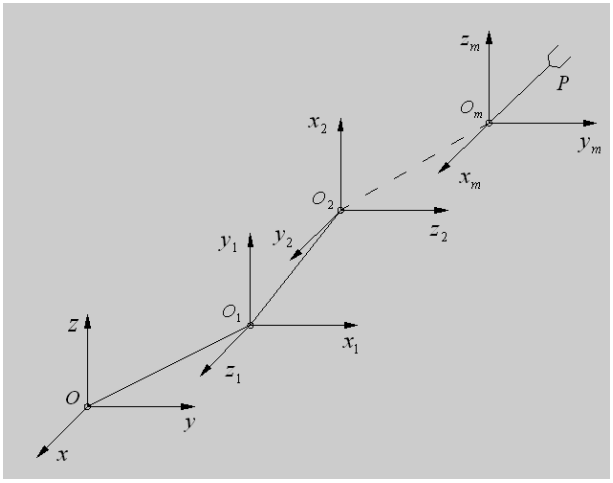


Fig.1 Homogeneous transformation mapping of moving element

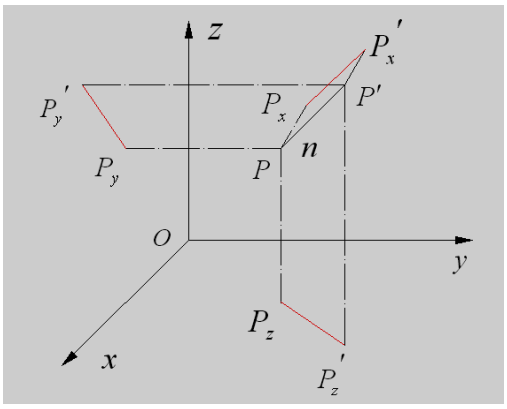


Fig.2 mapping of translation

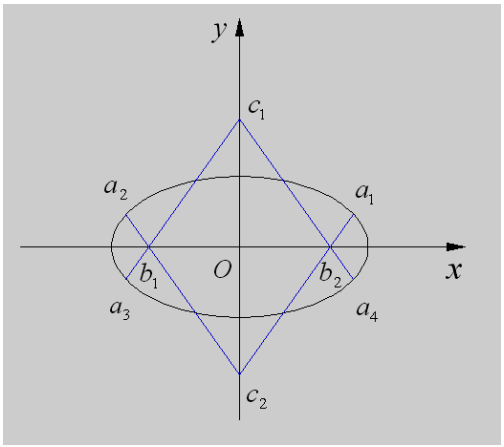
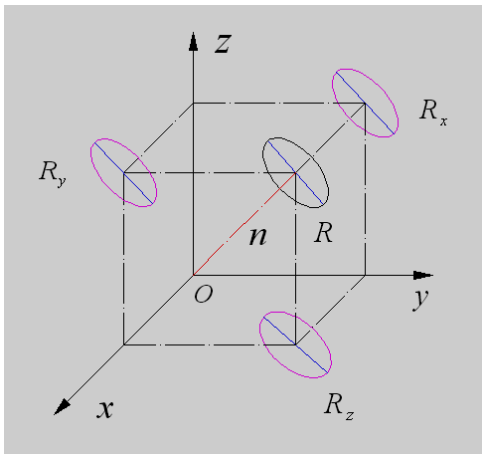
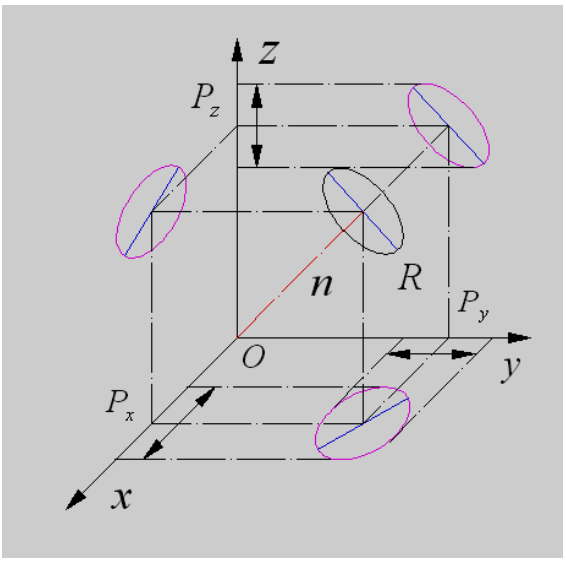


Fig.3 Mapping of decomposition of ellipse

In Expression (5) and Expression (6), $R_{c_1}(\angle a_3c_1a_4), R_{c_2}(\angle a_1c_2a_2), R_{b_1}(\angle a_2b_1a_3), R_{b_2}(\angle a_1b_2a_4)$ is the moving interval corresponding to the decomposed moving element of the three dimensional ellipse moving element R_O, $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ is the angle region corresponding to these moving interval.



(a) Mapping of ellipse



(b) Mapping of line segment

Fig.4 Mapping of rotation

In Expression (7), R_x, R_y, R_z is the corresponding mapping results of decomposed sub rotation moving element of arbitrary rotation moving element R_n in three dimensional Cartesian coordinate system respectively, P_x, P_y, P_z is the corresponding mapping results of decomposed sub translation moving element of arbitrary rotation moving element R_n in three dimensional Cartesian coordinate system respectively, which defined as the vice translation moving element, expressed with $\{\}_{vice}$.

Based on the analysis of decomposed moving element in a three-dimensional fixed coordinate system of the basic moving elements of the a single translation joint and a single rotation joint, homogeneous coordinate transformation adopted to each joints on kinematic chain, equivalent moving elements in fixed coordinate system obtained then, set of equivalent main translation moving elements and equivalent vice translation moving elements and equivalent rotation moving elements obtained then, as shown in Fig. 5, the same operation done to each joint of arbitrary serial branch, the whole set of equivalent moving elements found as well.

$$\{{}^0P_z\} \rightarrow \{{}^0P_z\}_{main} \tag{8}$$

$$\{{}^1R_y\} \rightarrow \{{}^0R_y\} \cup \{{}^0P_x({}^1R_y) \cup {}^0P_y({}^1R_y) \cup {}^0P_z({}^1R_y)\}_{vice} \tag{9}$$

$$\{{}^2R_x \cup {}^2R_z\} \rightarrow \{{}^0R_x \cup {}^0R_z\} \cup \{{}^0P_x({}^2R_x) \cup {}^0P_y({}^2R_x) \cup {}^0P_z({}^2R_x)\}_{vice} \cup \{{}^0P_x({}^2R_z) \cup {}^0P_y({}^2R_z) \cup {}^0P_z({}^2R_z)\}_{vice} \tag{10}$$

$$\{{}^mP_y\} \rightarrow \{{}^0P_y\} \tag{11}$$

$$\begin{aligned}&\{{}^0P_z\}\cup\{{}^1R_y\}\cup\{{}^2R_x\cup{}^2R_z\}\cup\{{}^mP_y\}\rightarrow\\&\{{}^0P_z\}\cup\{{}^0R_y\}\cup\{{}^0P_x({}^1R_y)\cup{}^0P_y({}^1R_y)\cup{}^0P_z({}^1R_y)\}\\&\cup\{{}^0R_x\cup{}^0R_z\}+\{{}^0P_x({}^2R_x)\cup{}^0P_y({}^2R_x)\cup{}^0P_z({}^2R_x)\}\\&\cup\{{}^0P_x({}^2R_z)\cup{}^0P_y({}^2R_z)\cup{}^0P_z({}^2R_z)\}\cup\{{}^0P_y\}\\&=\{{}^0R_x\cup{}^0R_y\cup{}^0R_z\}\cup\{{}^0P_y\cup{}^0P_z\}_{main}\cup\{{}^0P_x\cup{}^0P_y\cup{}^0P_z\}_{vice}\end{aligned}\tag{12}$$

Take the serial branch as the example as shown in Fig. (5), moving elements in each coordinate system mapping transformed to the fixed coordinate system, we get the equivalent moving elements set, Expression (8) indicate the equivalent mapping result of the translation moving element ${}^0P_z$ in the fixed coordinate system. Expression (9) indicate the equivalent mapping result of the rotation moving element ${}^1R_y$ around the axis O_1y_1, include equivalent mapping set of rotation moving element and vice translation moving element, ${}^0P_x({}^1R_y),{}^0P_y({}^1R_y),{}^0P_z({}^1R_y)$ is the equivalent vice translation moving element along the axis x,y,z of rotation moving element ${}^1R_y$ in the fixed coordinate system. As the same, we can get the equivalent mapping results of rotation moving element ${}^2R_x,{}^2R_y$ in the coordinate system $O_2x_2y_2z_2$, still the equivalent mapping results of translation moving element ${}^mP_y$ in the coordinate system $O_mx_my_mz_m$.

Synthesize the equivalent mapping results from Expression (8) to (11), we can found the the equivalent mapping results of all moving elements in the serial branch as shown in Fig. (5), which include the equivalent mapping set of main translation moving element $\{{}^0P_y\cup{}^0P_z\}_{main}$ and equivalent mapping set of vice translation moving element $\{{}^0P_x\cup{}^0P_y\cup{}^0P_z\}_{vice}$ and equivalent mapping set of the rotation moving element $\{{}^0R_x\cup{}^0R_y\cup{}^0R_z\}$.

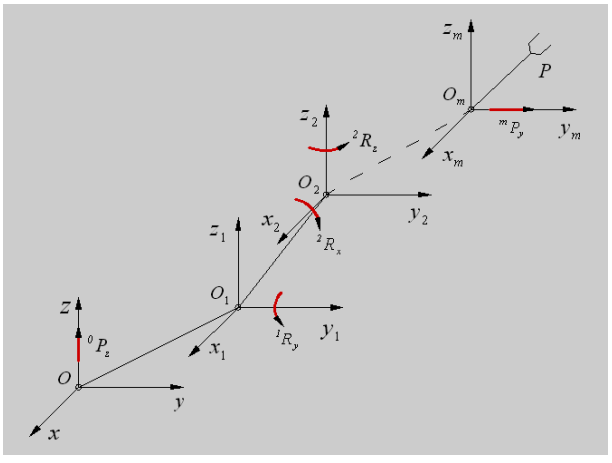


Fig.5 Moving element on serial branch

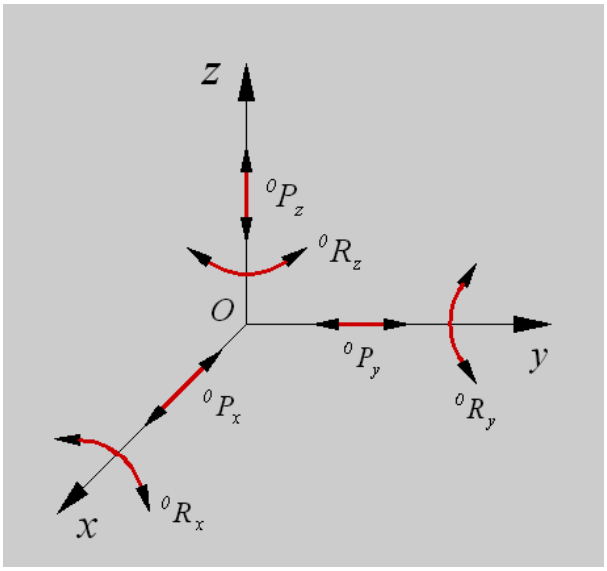


Fig.6 Equivalent mapping results of Fig.(5).

III. TOPOLOGICAL ANALYSIS ON DOF

Some geometric problems come from mathematical analysis are mainly studied by topology. So far, the main research of topology invariant properties and invariants with topological transformation in topological space. Graph theory is a branch of mathematics, its research objects is map, it is mathematical theory and method for studying the vertices and edges, research object of graph theory is equivalent to one-dimension topology. Based on the basic principle and method of graph theory and topology, three topological expressing line for translation moving element set corresponding to the axis in fixed coordinates system constructed, the equivalent mapping main moving elements on branch represented by low and thin horizontal region lines, as show in Fig. (7), the subscript in the graph indicate the interval coordinate. Three topological expressing line for rotation moving element set corresponding to the axis in fixed coordinates system constructed on the right of the above mentioned three lines, the equivalent mapping rotation moving elements on branch represented by low and thin horizontal region lines, as show in Fig. (8), the equivalent mapping vice moving elements on branch represented by high and thick horizontal region lines on the left three lines, as show in Fig. (8), the subscript in the graph is the interval coordinate, superscript in the graph indicate the rotation center and radius.

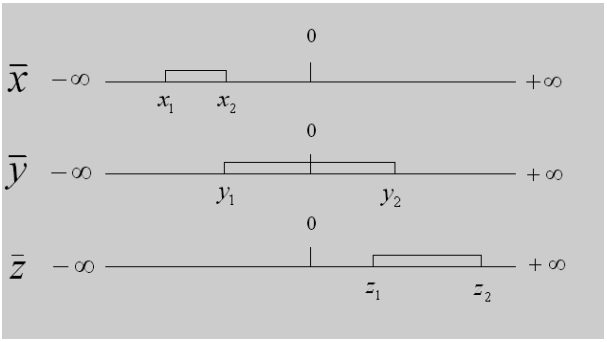


Fig.7 Topological express of equivalent mapping main moving elements

The moving capability of serial branch is the overlying of that of each joint, union operation carried out with set region on six topological expressing line respectively, it is clearly whether a non empty set or non single point set exist on the six topological expressing line respectively, the number of topological expressing line with non empty set or non single point set is the number of DOF of this serial branch, as well as its constitute can be seen clearly. $\{{}^0R_x \cup {}^0R_y \cup {}^0R_z\} + \{{}^0P_y \cup {}^0P_z\}_{main}$ is the definite or main DOF as shown in Fig. 5, $\{{}^0P_x \cup {}^0P_y \cup {}^0P_z\}_{vice}$ is the indefinite or following DOF, thus found the DOF of this mechanism is five, when the mechanism locate in special configuration, that is $\{{}^0P_x\}_{vice} \neq \Phi$, the indefinite DOF is three, there will be six for the DOF, as shown in Fig. (9).

Formula for DOF can be obtained as following

$$F = F(\bar{x}) + F(\bar{y}) + F(\bar{z}) + F(\hat{x}) + F(\hat{y}) + F(\hat{z}) \quad (13)$$

Among that, $F, F(\bar{x}), F(\bar{y}), F(\bar{z}), F(\hat{x}), F(\hat{y}), F(\hat{z})$ is the DOF and index along the axis x, y, z and index around the axis x, y, z.

When $\{\bar{x}\} = \{\overline{x_1 x_2}\} + \{\overline{x_3 x_6}\} \neq \Phi$ and it is non single point set, $F(\bar{x})$=1, else $F(\bar{x})$=0.

When $\{\bar{y}\} = \{\overline{y_1 y_2}\} + \{\overline{y_3 y_4}\} \neq \Phi$ and it is non single point set, $F(\bar{y})$=1, else $F(\bar{y})$=0.

When $\{\bar{z}\} = \{\overline{z_1 z_4}\} \neq \Phi$ and it is non single point set, $F(\bar{z})$=1, else $F(\bar{z})$=0.

When $\{\hat{x}\} = \{\overline{\alpha_1 \alpha_2}\} + \{\overline{\alpha_3 \alpha_4}\} \neq \Phi$ and it is non single point set, $F(\hat{x})$=1, else $F(\hat{x})$=0.

When $\{\hat{y}\} = \{\overline{\beta_1 \beta_2}\} \neq \Phi$ and it is non single point set, $F(\hat{y})$=1, else $F(\hat{y})$=0.

When $\{\hat{z}\} = \{\overline{\gamma_1 \gamma_2}\} \neq \Phi$ and it is non single point set, $F(\hat{z})$=1, else $F(\hat{z})$=0.

The DOF of mechanism corresponding to Fig. (9) can be found

$$\begin{aligned} F &= F(\bar{x}) + F(\bar{y}) + F(\bar{z}) + F(\hat{x}) + F(\hat{y}) + F(\hat{z}) \\ &= 1+1+1+1+1+1 \\ &= 6 \end{aligned} \quad (14)$$

The parallel mechanism is composed of a series of serial branch, according to the general theory and method of analysis on spatial moving capability of mechanism, intersection adopted to the spatial moving capability of each branch in parallel mechanism, thus we get the DOF. Make use of the analysis results on DOF obtained from the serial branch, intersection adopted to the moving element set on corresponding six topological expressing line between serial branches in parallel mechanism, the DOF obtained then. Take a parallel mechanism with three serial branches as an example, topological analysis of DOF of each branch as shown in Fig. (10) to Fig. (12), intersection carried out between the same moving element set on six topological expressing line respectively, we can get the analysis result of DOF as shown in Fig. (13).

The DOF of mechanism corresponding to Fig. (13) can be found

$$\begin{aligned} F &= F(\bar{x}) + F(\bar{y}) + F(\bar{z}) + F(\hat{x}) + F(\hat{y}) + F(\hat{z}) \\ &= 1+1+0+0+1+0 \\ &= 3 \end{aligned} \quad (15)$$

The hybrid mechanism is composed of serial branch and parallel mechanism, it can be seen as the union of moving capability of two serial branches, in order to found the DOF of hybrid mechanism, the first step is to found six topological expressing line of moving elements set of parallel mechanism, the second step is to found six topological expressing line of moving elements set of serial branch, the third step is to adopt union operation between the corresponding six topological expressing line of moving elements set of the aforementioned results in last two step, from the result of third step, we can found the number and constitute of DOF easily.

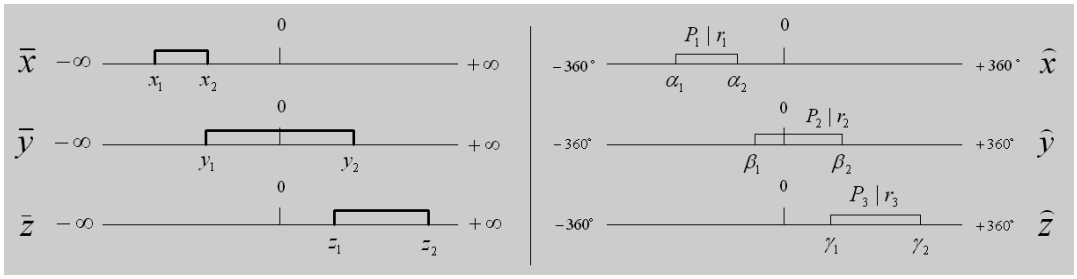


Fig.8 opological express of equivalent mapping vice moving elements and rotation moving elements

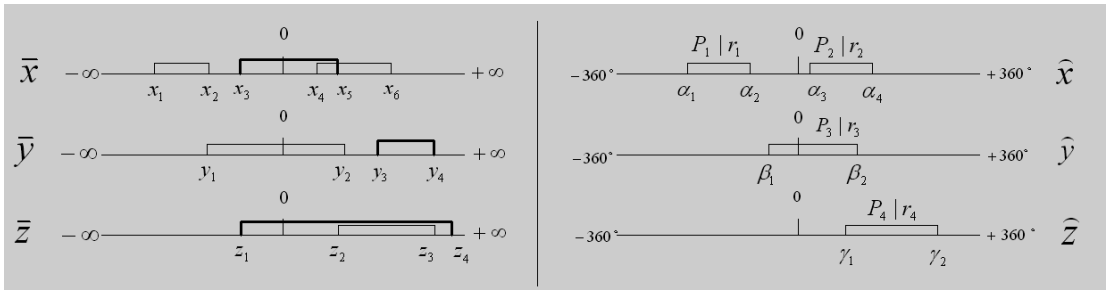


Fig.9 Topological analysis of DOF of mechanism of Fig. (5).

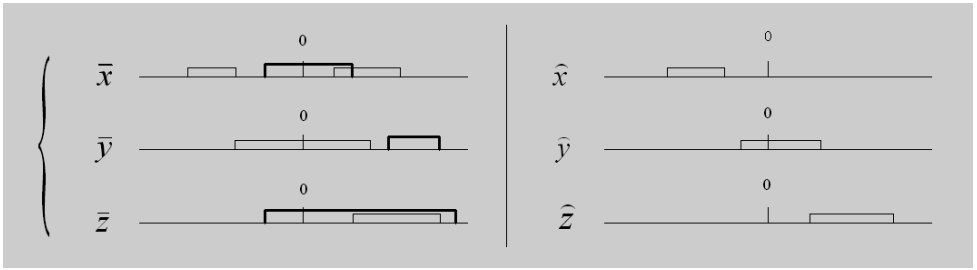


Fig.10 Topological analysis of DOF of the first branch in imaginary parallel mechanism

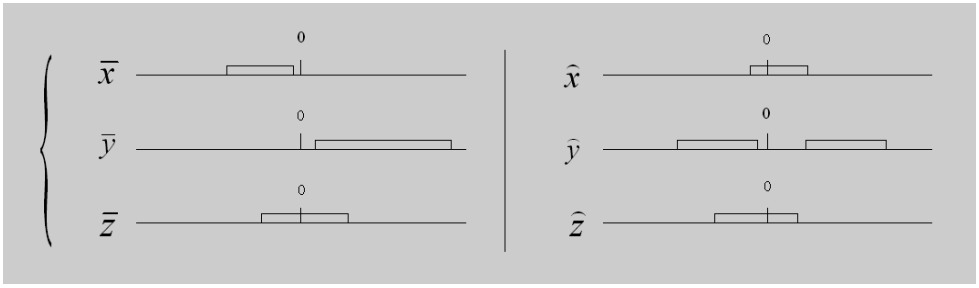


Fig.11 Topological analysis of DOF of the second branch in imaginary parallel mechanism

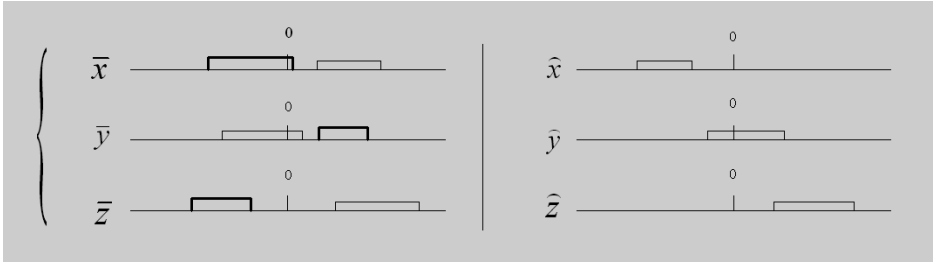


Fig.12 Topological analysis of DOF of the third branch in imaginary parallel mechanism

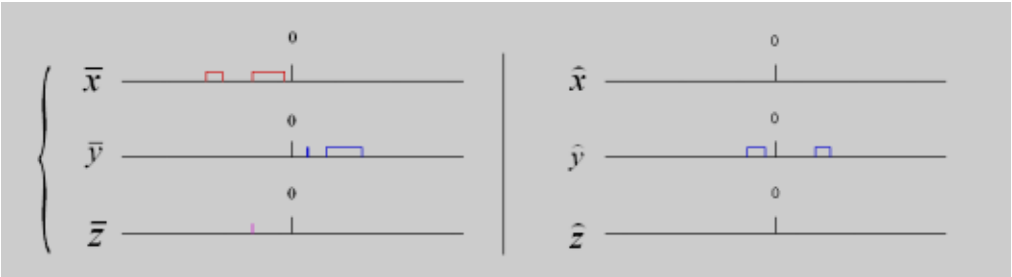


Fig.13 Topological analysis of DOF of imaginary parallel mechanism

IV. CONCLUSION

(1) The moving element of any joint can be equivalent mapped to corresponding moving elements of fixed three-dimensional coordinate system by homogeneous transformation, the translation moving element corresponding to equivalent main translation moving element set only, the rotation moving element corresponding to equivalent rotation moving element set and equivalent vice translation moving element set.

(2) Equivalent main translation moving element set,equivalent vice translation moving element set, equivalent rotation moving element set can be expressed on planar six-dimensional topological expressing line with interval.

(3) The DOF can be obtained through different operation with the analysis results displayed in planar six-dimensional topological expressing line with interval, union operation adopted for serial mechanism, intersection operation adopted between branches for parallel mechanism, union operation adopted between serial branch and parallel mechanism for hybrid mechanism, the number and constitute can be seen from the last analysis and operation results on six topological expressing line.

(4) This work presented a new method for the analysis of DOF, it can be applied to verify the DOF of special mechanism, also can be used to found new mechanism.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

ACKNOWLEDGEMENT

This work is supported by the Fundamental Research Funds for the Central Universities（No. 3142015023), research fund on science and technology of Hebei province (No. Z2015072), National Natural Science Foundation (No. 50975024).

REFERENCES

[1] F. Freudenstein, L. Dobrjanskyj, “On a theory of type synthesis of mechanisms”, in: *Proceedings of 11th International Congress of Applied Mechanics*, 1964, pp. 420-428.

[2] K. C. Lu, “ Graph theory and its application”, Beijing: *Tsinghua University press*, 1984, pp. 3-20.

[3] X. H. Wang , J. J. Zhang, and S. C. Fan, “Topological transformation of parallel mechanism based on topological features”, *mechanical science and technology*, vol. 24, no. 11, pp. 1310-1312, 2005.

[4] Y. J. Luo, Z. Kang, and Z. Y. Wu, “Considering the uncertainty of flexible mechanism topology optimization design”, *Chinese Journal of mechanical engineering*, vol. 47, no. 22, pp. 43-48, 2011.

[5] X. T. Niu, Z. K. Li, and X. M. Zhang, “Topology extraction of compliant mechanism”, *machine design and research* , vol. 29, no. 1, pp. 18-21, 2013.

[6] C. H. Suh, C. W. Radcliffe, “Kinematics and Mechanisms Design”, New York: John Wiley&Sons, Inc.1978.

[7] G. N. Sandor, A. G. Erdman, “Advance Mechanism: Analysis and Synthesis”, Prentice Hall, Englewood Cliffs, NJ, 1984.

[8] Z. Huang, Y. S. Zhao, and T. S. Zhao, “Higher spatial mechanism”, .Beijing: *Higher Education Press*, 2006, pp. 115-135.

[9] Z. Huang, J. F. Liu, and Y. W. Li, “The general formula for the DOF in the 150 years”, *Journal of Yanshan University*, vol. 35, no. 1, pp. 1-14, 2011.