ERP Analysis Based on 2-D Time-Variant Logistic-CML Model

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***Abstract* -- Although electroencephalographic (commonly known as EEG) signals are accepted as a representative non-stationary time series, the event-related potential (ERP) is known to be a complicated signal produced by nonlinear dynamical processes. In this research, we built a 2-D time-invariant Logistic–Coupled Map Lattice (CML) for use in ERP analysis. The time-variant maximal Lyapunov exponent of ERP and the global correlation dimension were calculated using this model, and some quantized parameters that reflect the global characteristics of the system were obtained, enabling the discovery of information that traditional methods cannot identify.**

Keywords -- ERP, 2-D time-variant Logistic-CML model, time-variant maximal Lyapunov exponent

1. I. INTRODUCTION

The development of knowledge about the external world is advancing very quickly, leading to rapid technological enhancements. However, this is not the case for human beings, especially regarding the brain, which controls the human mental processes. The analysis of electroencephalogram (EEG) signals makes it possible to study brain functions and cognition processes, and provides new opportunities for the development of a correlative domain. However, the EEG activity and information processes are very complex. Hence, distilling useful information with modern signal processing technology is the key to studying brain activities and diagnosing certain diseases. Over recent years, many methods have been proposed, and there have been advances in clinical analysis and applications. However, because the inherent characteristics of real signals have been neglected, the models that have been built and the information distilled from the signals are too simple. Thus, the development of technology concerning biomedical signal processes is not sufficiently advanced.

The synchronization and chaos found in EEG activities indicate that the brain is a nonlinear dynamical system, and the event-related potential (ERP) is known to be a complicated signal produced by typical nonlinear dynamical processes. It is the nonlinear dynamics that place the observed quantities into the whole variable space of a system. Thus, we can discover useful information about these signals by studying the parameters of the system and determining its inherent dynamical characteristics. This method has been widely used in feature extraction, pattern recognition, dynamic detection, and secret communication [1,2].

The fact that many physical systems, including the nervous system, are nonlinear spatio-temporal systems makes the problem more complex. This nonlinearity in both time and space means that the signals have the characteristics of a space-time structure. In this case, we have to think of the spatial relevance of the system, or we will obtain incorrect results. The spatial effects have often been ignored in traditional nonlinear dynamical analysis. For example, when the correlated dimension and maximal Lyapunov exponent are calculated in EEG nonlinear analysis, the signals are obtained from different leads. Although the process of parameter estimation involves all leads, the direct study object is a time series from a single lead. Thus, the result is a topographical synopsis of a nonlinear EEG parameter, and this cannot reflect the spatial structure of the brain. If we use a coupled map lattice (CML) model to describe the nonlinear space-time dynamical system, the development of the real system can be qualitatively simulated, and some measurements can be defined to effectively find the nonlinear dynamical characteristics of the signals [3-6].

Furthermore, the usual nonlinear dynamical analysis of EEG signals only considers a given brain activity. In this case, the system can be viewed as invariable. However, because of many physio-chemical changes, the oxygen saturation of blood and conductibility of the brain will change momentarily, and these changes will influence the system. We can obtain some dynamical parameters to represent and measure a system if it is invariable, but if the system state and the system are time-variant, customary methods will be inaccurate. Hence, we must use a time-variant model.

Therefore, to determine the inherent characteristics of EEG signals, we propose an improved model based on a CML, which is a mature area of nonlinear dynamics. We build a 2-D time-invariant Logistic-CML model, and calculate the dynamic parameters of EEG signals under different stimulation patterns. We then compare these parameters, and attempt to explain the results.

2. II. THE CML MODEL

A CML model is a continuous-state dynamical system in discrete time and space. Consider the expression:

$$x_{n+1}(i,j)=f\left(\sum_{m=-K}^{K}\sum_{n=-K}^{K}\varepsilon(i+m,j+n)x_n(i+m,j+n)-\varepsilon(i,j)x_n(i,j)\right) \tag{1}$$

where f is a local nonlinear map, n represents discrete time steps, (i,j) are the lattice coordinates, and K indicates the number of lattice points that are related to the current lattice. ε is a spatial coupling coefficient that satisfies:

$$\sum_{m=-K}^{K}\sum_{n=-K}^{K}\varepsilon(i+m,j+n)-\varepsilon(i,j)=1 \tag{2}$$

In fact, ε indicates the degree of interrelationship between the current lattice and an associated lattice. Thus, we can obtain this parameter by calculating the degree of correlation between the lattices, expressed as:

$$\hat{R}_{xy}(m)=\frac{1}{N}\sum_{n=0}^{N-|m|-1}x(n)y^*(n+m) \tag{3}$$

To ensure that ε satisfies (2), we must normalize the correlation coefficient:

$$\varepsilon(i+m,j+n)=\frac{R(i+m,j+n)}{\sum_{j=-K}^{K}\sum_{n=-K}^{K}R(i+m,j+n)},\quad m=-K,\ldots,K,\quad n=-K,\ldots,K \tag{4}$$

where $R(i,i+j)$ is the correlation coefficient between the signals at (i,j) and $(i+m,j+n)$.

We believe that the similarity between a real measurement sequence and the predicted sequence obtained from a model indicates that the model can represent the real system. To gain such a model, we must resolve the following optimization problem:

$$\min_{\langle\varepsilon,U_f\rangle}\|x(i,j)-\hat{x}(i,j)\| \tag{5}$$

where $x(i,j)=(x_1(i,j),\ldots,x_n(i,j))$ is a real signal in (i,j), and $\hat{x}(i,j)=(\hat{x}_1(i,j),\ldots,\hat{x}_n(i,j))$ is the predicted signal at that point. ε can be determined by (4), and U_f comes from f, an undetermined parameter set. The nonlinear map can take many forms. A logistic map is frequently used:

$$f(x)=\lambda x_n(1-x_n),\ \lambda\in(0,4),\ x_n\in[0,1] \tag{6}$$

We can obtain a Logistic-CML model as follows:

$$x_{n+1}(i,j)=\mu x_n^{'}(i,j)(1-x_n^{'}(i,j)) \tag{7}$$

where μ is the map coefficient.
The optimization problem becomes:

$$\min_{\mu}\|x(i,j)-\hat{x}(i,j)\| \tag{8}$$

Resolving these problems, we obtain:

$$\mu=\frac{sum(X(i,j))}{sum(x(i,j))} \tag{9}$$

where

$$X_n(i,j)=x_{n-1}^{'}(i,j)(1-x_{n-1}^{'}(i,j)),\quad n\geq 2 \tag{10}$$

Ordinarily, when building a time-variant model, we aim to find the map relationship between μ and the measurement signals. However, when there is not a one-to-one correspondence between μ and the measurement signals, we cannot find a one-value function to fit the curve of μ. It is known that radial basis functions (RBFs) can theoretically replace this arbitrary function, and have a universal approximation capability [7 – 10]. Thus, we built the time-variant model with a universal approximation capability using the Logistic function as the local map and an RBF to express the changes in map coefficient:

$$x_{n+1}(i,j)=\sum_{l=1}^{L}w(l)\exp\left(\frac{-\|\mu(n,i,j)-a(l)\|^2}{s}\right)x_n^{'}(i,j)(1-x_n^{'}(i,j)) \tag{11}$$

where $\mu(n,i,j)$ is the map coefficient at time n at coordinates (i,j).

We studied the degree of similarity between real measurement signals and predicted signals obtained from the model to measure the ability of the model to simulate a real system. If the model can accurately track the variation of the system, we consider the model to be able to generate the states and variations of a real system. To ensure the complexity of the test data, we adopted EEGs as the simulation signal.

We took a sequence of 1000 values. The first 800 were used to train the model parameters, and the final 200 values were used to test the predicted data obtained from the model. The scale of the grid was 5 × 5, and the terminal condition was assumed to be periodic. The correlation coefficient (Equation 3) was adopted to measure the degree of similarity between the measured and predicted data. A correlation coefficient close to 1 indicates better prediction accuracy.

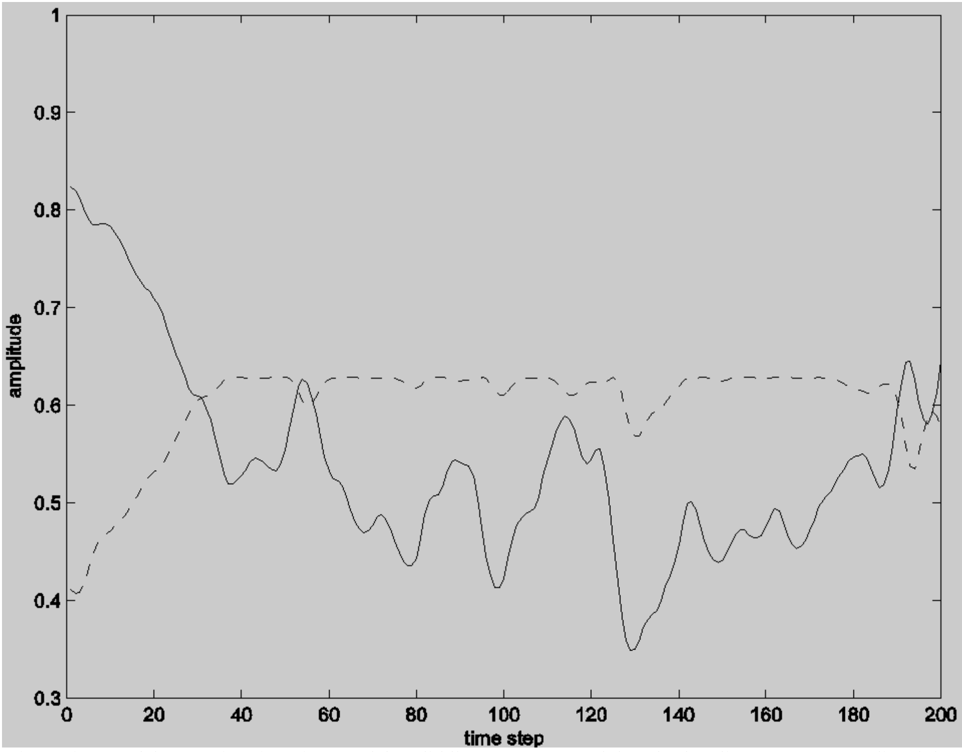

Fig.1. Fitness of the 2-D time-invariant model. Solid lines represent real data, broken lines represent predicted data

TABLE 1 CORRELATION COEFFICIENTS BETWEEN REAL AND PREDICTED DATA

Lattice No.	1	2	3	4	5
correlation coefficients	0.47391	-0.41707	-0.84756	-0.81963	-0.76874
Lattice No.	6	7	8	9	10
correlation coefficients	-0.27154	-0.78942	-0.87355	-0.75239	-0.85445
Lattice No.	11	12	13	14	15
correlation coefficients	-0.73569	-0.67754	-0.84271	-0.7127	-0.83114
Lattice No.	16	17	18	19	20
correlation coefficients	-0.32372	-0.52124	-0.68166	-0.73239	-0.76061
Lattice No.	21	22	23	24	25
correlation coefficients	-0.63208	-0.089111	-0.50181	-0.68453	-0.74493

Fig.1 and table 1 show that the 2-D time-invariant Logistic-CML cannot provide a good fit to the EEG signals. The results of fitting the 2-D time-variant Logistic-CML model to the real system is shown in Fig. 2 and table 2.

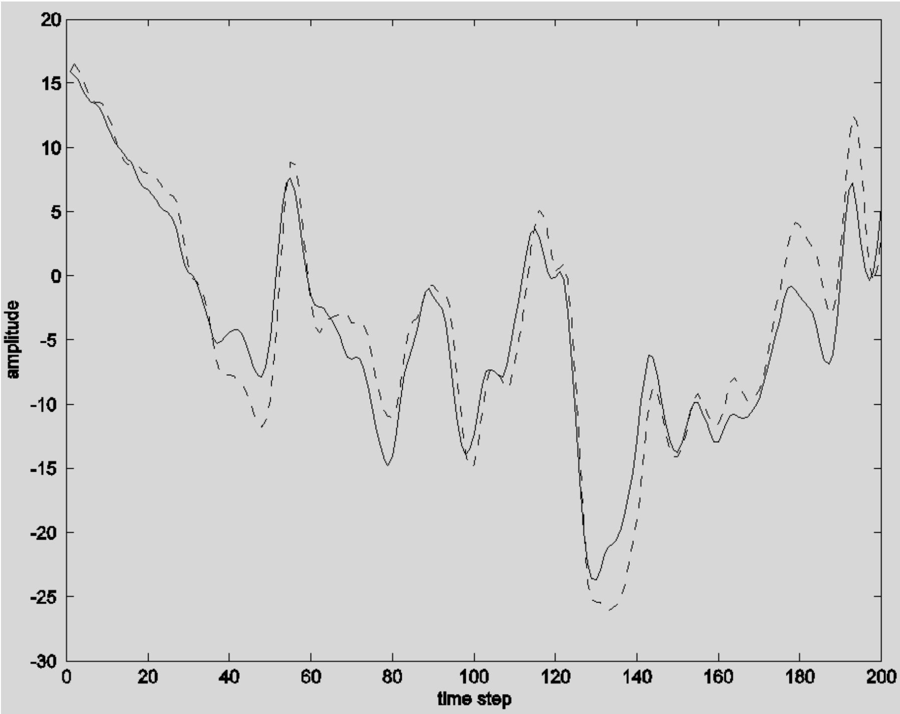

Fig.2. Fit of the 2-D time-variant Logistic-CML model in one lattice. Solid lines represent real data, broken lines represent predicted data

TABLE 2 CORRELATION COEFFICIENTS BETWEEN REAL AND PREDICTED DATA

Lattice No.	1	2	3	4	5
correlation coefficients	0.97066	0.98723	0.979	0.96955	0.97384
Lattice No.	6	7	8	9	10
correlation coefficients	0.9888	0.98857	0.95721	0.9696	0.9786
Lattice No.	11	12	13	14	15
correlation coefficients	0.93034	0.97882	0.98358	0.96796	0.92895
Lattice No.	16	17	18	19	20
correlation coefficients	0.92017	0.96564	0.96531	0.96082	0.92165
Lattice No.	21	22	23	24	25
correlation coefficients	0.95068	0.95529	0.95976	0.95598	0.95269

From the results, it is clear that the 2-D time-variant Logistic-CML model can provide a good fit to the real system in every lattice. Because this model uses the logistic function as a local map, we not only obtain many interesting dynamical behaviors, but also a time-variant model that is related at every time step. We use this model to deal with the ERP.

III. ERP DATA AND ANALYSIS

A. Materials and EEG Recording

Participants were asked to recognize the target stimulus within the cue scope. The stimuli were presented on a color display in the order ‘background scope cue target stimulus.’ The background was white, and the scope cues were black loops of different scales centered in the center of the screen. There were 11 stimuli, one of which was the target stimulus. The target was a transverse semicircle with uphill or downhill convexity. Other semicircles were abstracted stimuli with left-forward convexities. These stimuli were located at random points on the background such that they would not overlap. The abstracted stimuli always appeared on the background, whereas the target stimulus always appeared in the cue scope.

As shown in Fig.3, the background was first presented for 300 ms. A black loop was then randomly presented as the cue for a further 300 ms. The three kinds of loop appeared with the same probability. When the loop disappeared, one target stimulus and 10 abstracted stimuli appeared. The probability of the target stimulus having uphill or downhill convexity was the same. The stimuli were shown for 1000 ms, giving the participants enough time to recognize the target stimulus. The screen was black between the scope cue and the target stimulus. The inter-stimulus interval (ISI) were divided into short ISI of 400–600 ms and long ISI of 600–800 ms.

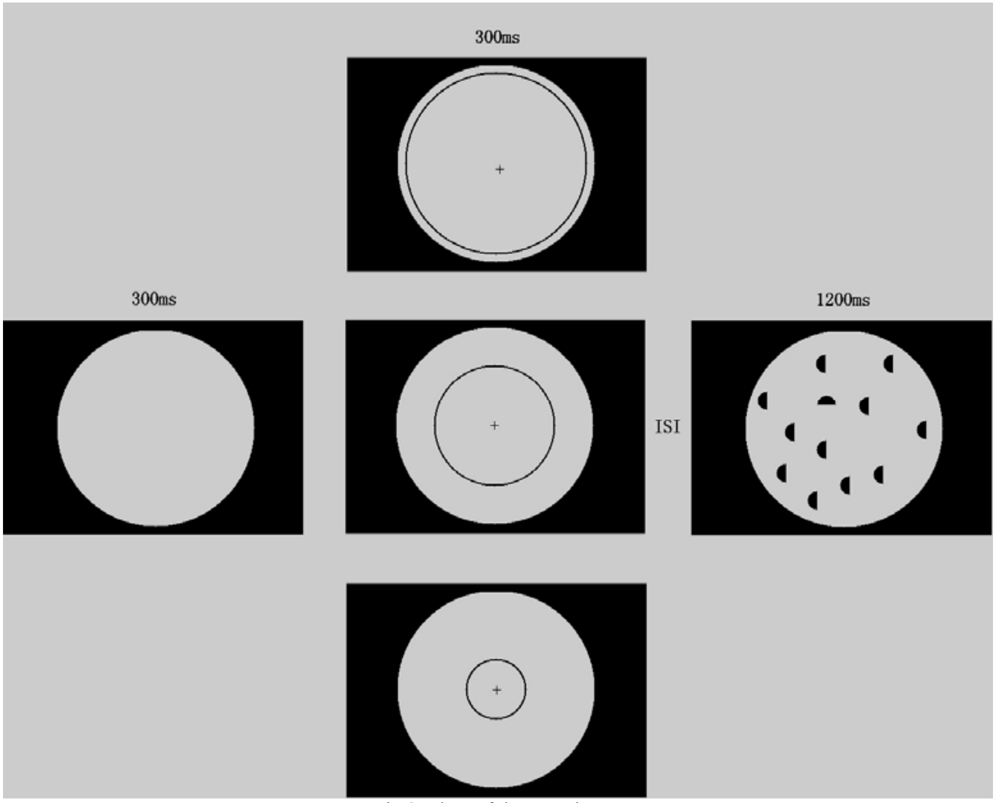

Fig.3. Flow of the experiment

Ten male students (aged 22–25, dextral) participated in the experiment. We adopted 10–20 systems to record EEGs shown as Fig.4, with electrodes on earlobes (A1, A2) selected as the reference electrodes.

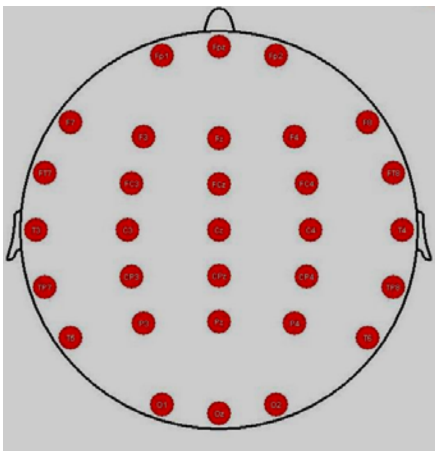

Fig. 4. Sketch map about electrode-mounting

The sample frequency was 1000 Hz. We recorded EEGs from 400 ms before the stimuli were presented to 1000 ms after they had disappeared. The participants were asked to press the right or left mouse button according to whether the convexity of the target stimulus was uphill or downhill. We recorded both the accuracy and speed of reaction.

B. ERP Analysis

To comprehensively describe the characteristics of the data, we used the 2-D time-variant Logistic-CML model to calculate several kinds of EEG for different stimulus modes.

Traditionally, a serial signal is extracted from a single channel, from which the phase space is reconstructed, and then the maximal Lyapunov exponent is calculated. However, this is not sufficiently accurate to deal with a time-variant system such as the brain. We can obtain a time-invariant Logistic-CML model using the time-variant Logistic-CML model at every time step. In a given

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time step, the dynamic characteristics of the real system are determined. We have proved that the time-invariant Logistic-CML model can simulate a system with fixed dynamical characteristics. Thus, the 2-D time-variant Logistic-CML model can describe the real system. We can obtain time series of arbitrary length in every step using this model, then calculate the maximal Lyapunov exponent at every time step.

Consider a 1-D model in a certain time step. We construct a new state $\{\bar{x}_m(i)\}$ by adding an infinitesimal disturbance $\{dx_0(i)\}$ to state $\{x_m(i)\}$ of the model at time m, that is:

$$\bar{x}_m(i) = x_m(i) + dx_0(i),\ i = 1,\ldots,L \tag{12}$$

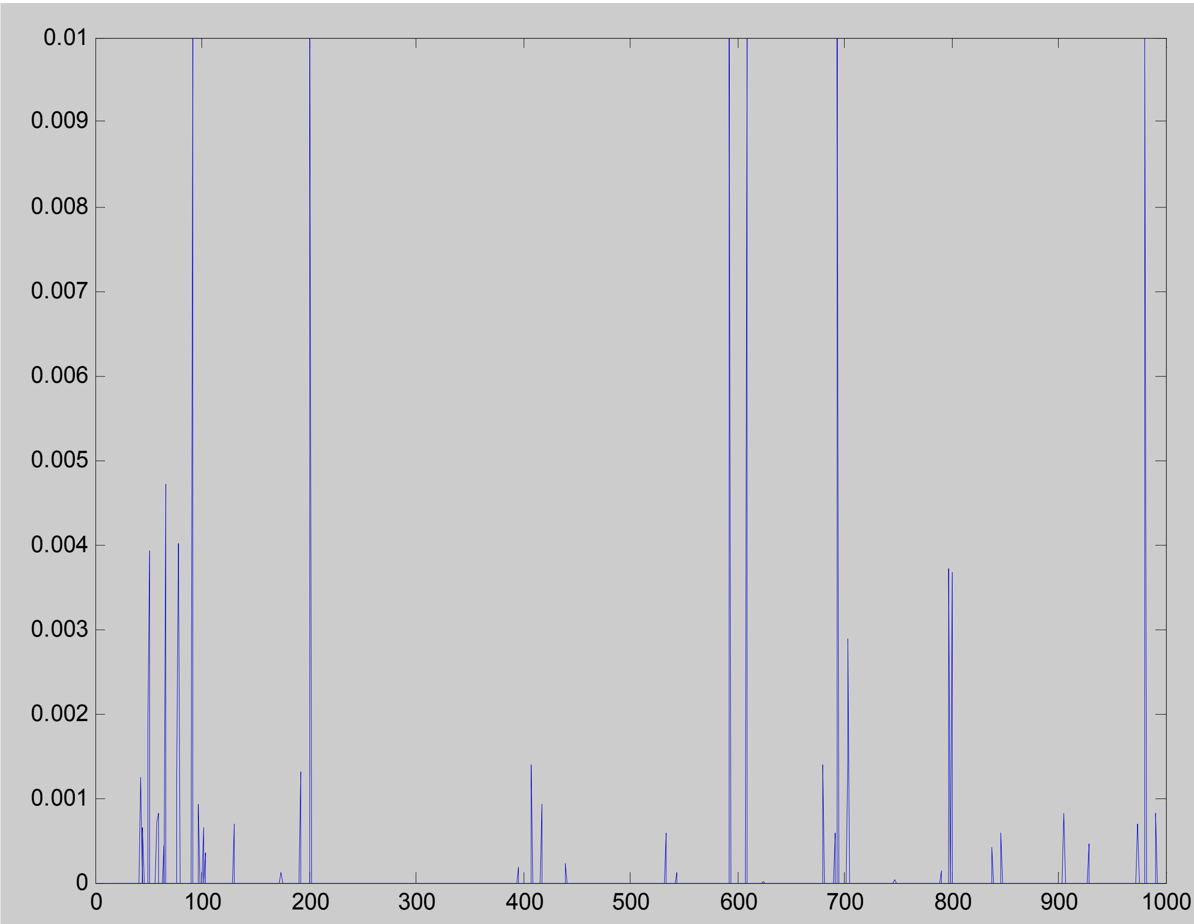
Where L is the maximal coordinate in the lattice. Iterating the CML system, we obtain $\{x_{m+n}(i)\}$, $\{\bar{x}_{m+n}(i)\}$, and the difference $\{dx_n(i)\}$ between them. We can prove that the maximal Lyapunov exponent can be described as:

$$\lambda = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \log \frac{\|dx_n\|}{\|dx_{n-1}\|} \tag{13}$$

where

$$\|dx_n\| = \sqrt{\sum_{i=1}^{L} [dx_n(i)]^2} \tag{14}$$

Extending (13) and (14), we obtain the calculation method for the 2-D model. We constructed a 5 × 5 grid using adjacent leads shown as table 3, and calculated the maximal Lyapunov exponent from 200 ms before the stimulus until 800 ms after the stimulus.



(a) time-variant maximal Lyapunov exponents of spontaneous EEG

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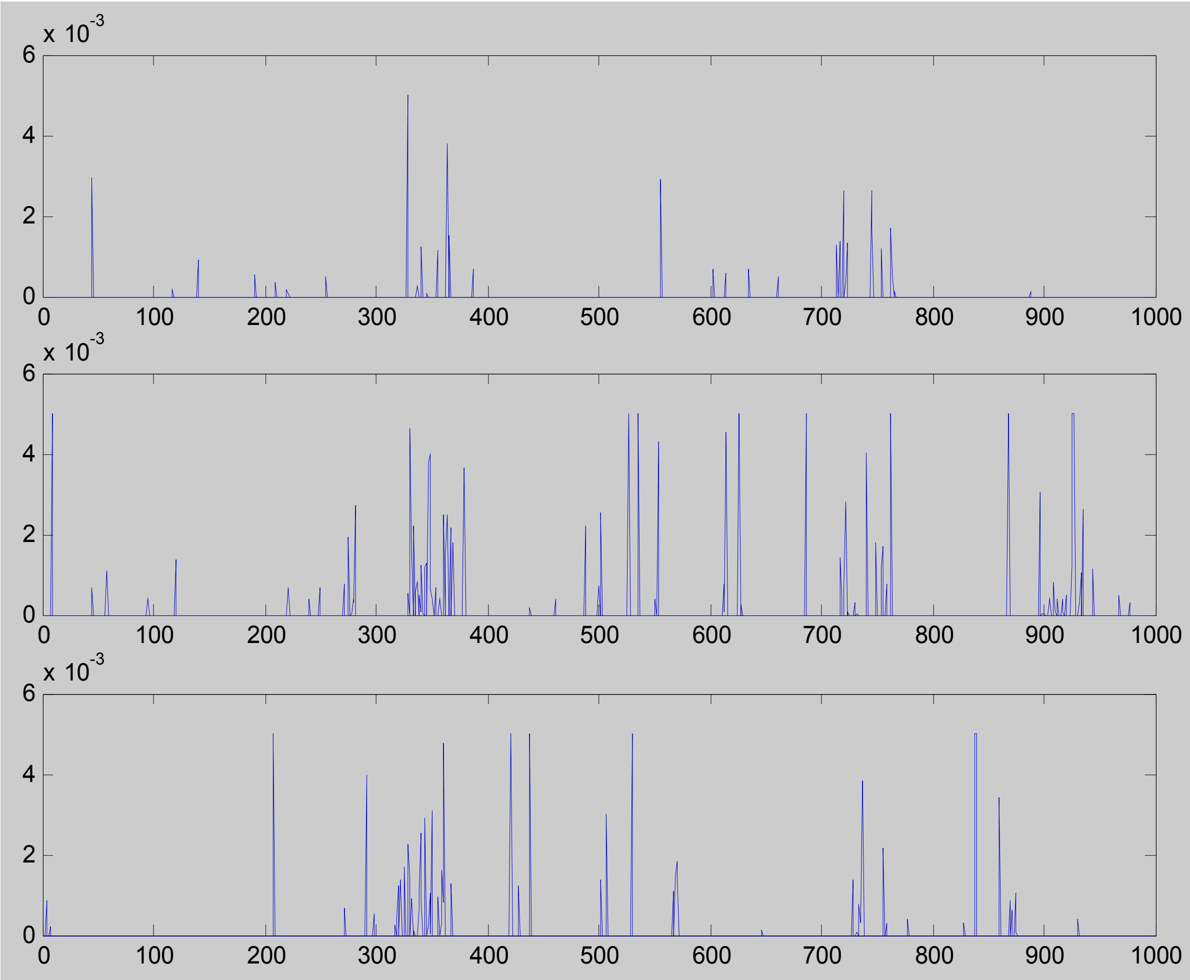

(b) time-variant maximal Lyapunov exponents of ERP from three cues: (top) small-scale cue, (middle) mid-scale cue, (bottom) large-scale cue
Fig.5. Time-variant maximal Lyapunov exponents of some signals, where the horizontal axis shows time and the vertical axis shows maximal Lyapunov exponent.

TABLE 3 DISTRIBUTION OF LEADS IN THE TEST

	1	2	3	4	5
1	F7	F3	Fz	F4	F8
2	F T7	F C3	F Cz	F C4	F T8
3	T3	C3	Cz	C4	T4
4	T P7	C P3	C Pz	C P4	T P8
5	T5	P3	Pz	P4	T6

The terminal condition was assumed to be periodic, that is, we took a random initial condition, and regarded the first 2000 steps as a transition. We used the subsequent 3000 steps to calculate the maximal Lyapunov exponent of spontaneous EEG and ERP signals in every step. The time-variant maximal Lyapunov exponent spectrum is shown in Fig.4.

C. Results

We calculated several time-variant maximal Lyapunov exponent spectra for different EEG signals using the validated CML model, and determined which parameters reflect the characteristics of the system.

The maximal Lyapunov exponent is the most direct parameter by which a system can be determined to be chaotic—if the maximal Lyapunov exponent is non-negative, the system can be considered chaotic, and vice-versa. Generally, the maximal Lyapunov exponent can be calculated by reconstructing the phase space. The selection of the embedding dimension and time delay is very important when reconstructing the phase space, as inappropriate parameters will lead to incorrect results. The reconstruction theorem requires the time series to be sufficiently smooth, but EEGs rarely satisfy this requirement because of the presence of noise. Because of these limitations, the maximal Lyapunov exponent obtained by this method may not be precise, meaning that the results derived from it are inconsistent [11,12]. For example, one person may regard the brain as a low-dimensional chaotic system, whereas another person will not. The method in this paper does not require the reconstruction of the phase space, thus avoiding the problems associated with reconstruction. In addition, the results using the proposed method will be more precise, as we can obtain data series of arbitrary length. Most importantly, we can examine how the system varies with time to obtain a precise description of the system, rather than a blurry average value of the characteristics over a long period.

From Fig.5, we can see that the ERP under the visual stimulation pattern and spontaneous EEGs do not always come from a chaotic system. This may be the source of disagreement over whether the brain is a low-dimensional chaotic system. EEG signals are neither always chaotic nor always non-chaotic. Thus, if the method is sensitive to chaos, chaos will be regarded as the result; if not, non-chaos will be regarded as the result. Furthermore, the number of time steps over which the system presents chaotic characteristics differs according to the different signals. If this number is large enough, the chaotic characteristics will be obvious, and the system will be viewed as chaotic. In contrast, the system may be viewed as non-chaotic. These results reflect the characteristics of the system to a certain degree, but they are obviously imprecise. However, the time-variant maximal Lyapunov exponent spectra can precisely describe the dynamical characteristics of the system, providing a useful tool for EEG analysis and applications.

In the time-variant maximal Lyapunov exponent spectra of spontaneous EEGs, the appearance of time steps with chaotic characteristics is irregular. This is because the so-called spontaneous EEGs are not really spontaneous. During the measurement process, the participants perform some mental activities, and these may influence the EEG. At the same time, some external factors beyond our control may also influence the EEG. We cannot control or predict these influences, so the time-variant maximal Lyapunov exponent spectra have an irregular shape.

In the time-variant maximal Lyapunov exponent spectra of the ERPs, we find that there are more time steps when the system presents chaotic characteristics 100 ms after the stimulus has been presented, particularly in the 100–200 ms period, for the three kinds of cues. The appearance of these time steps becomes regular. Obviously, the task strengthens the chaotic characteristics of the system, with the main influence appearing 100 ms after the stimulus has appeared.

Comparing the three time-variant maximal Lyapunov exponent spectra of ERP under different scope cues, we can see that the mid-scale scope cue exerted the greatest influence on the degree of chaos in the system. This indicates that there is not a simple linear relation between the cue scopes and the influence on the system. That is, the influence does not increase or decrease with the size of the scope. This result may indicate that there is an optimal scope size for influencing the brain.

Although the time-variant maximal Lyapunov exponent spectrum of spontaneous EEGs has fewer time steps that exhibit chaotic characteristics and is irregular, its values are higher than those of the time-variant maximal Lyapunov exponent spectra of ERP. This may have been caused by the participants' mental expectations and degree of concentration, as not all mental activities and external stimuli can be controlled while the participants are observing the stimuli in the ERP experiment.

IV. CONCLUSION

This paper has proposed a method of nonlinear dynamical analysis for analyzing the complexity of EEG signals, and presented a model that exhibits a good fit to such complicated systems. Using this model, we analyzed ERP signals under fixed location cues. The time-variant maximal Lyapunov exponent spectra of spontaneous EEG and ERP signals were calculated, and the relationships between these parameters and brain information processes were determined. The study results indicate that brain signals are not always chaotic, but instead vary. The brain's chaotic characteristics strengthen and become regular as it processes information. ERP begins to be influenced by scope cues during the P2 period (120–260 ms). We found that the influence mainly occurs in this period. There is no simple linear relation between the scopes of cues and the influence on the system.

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