A Novel Model Reference Adaptive Controller of Quad-Rotor on Autonomous Take-off and Landing with Ground Effect

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***Abstract* — The ground effect has a significant impact on the control performance of quad-rotors. In order to assess and reduce the influence, the flow field expressions of the quad-rotor in hover is derived first, which is based on unstructured grid and Spalart-Allmaras turbulence model. Second, the model of ground effect is formulated in terms of the velocity on rotor plane and altitude. Taking the influence of ground effect, a model reference adaptive control law is proposed for the altitude channel. The simulation results show that the ground effect should not be ignored when the altitude is less than 3 times the rotor radius, but the effect on velocity in rotor plane due to ground effect decreases as the altitude increases. The altitude control method based on model reference adaptive technique reduces the influence of ground effect during takeoff and landing of the quad-rotor.**

Keywords - quad-rotor; ground effect; vertical take-off and landing; numerical calculation; model reference adaptive control (MRAC)

I. INTRODUCTION

Compared with common Unmanned Aerial Vehicle (UAV), quad-rotor has lower cost and can make more complex flight such as hovering flight and vertical take-off and landing. Quad-rotor also use rotors to provide power. Therefore, it will also generate ground effect (GE) like helicopter during take-off and landing, and the ground effect will have a significant impact on the autonomous control.

In general, the control of quad-rotor will ignore ground effect, or the altitude is considered to be high enough and will not be influenced by ground effect. In the process of take-off and landing, many researches take ground effect as interference which is not modeled. Currently, the methods of anti-interference control of quad-rotor are mainly PID method [1], robust control [2], back-stepping method [3] and neural network [4].

Robust control has better interference suppression, so Sampaio proposed an optimal $H\infty$ robust controller to the stability problem of Micro Aerial Vehicles (MAVs) in a Software-in-the-Loop platform [5,6]. The synthesis of the robust controller is grounded by the γ-iteration algorithm, which results in a MIMO optimal controller bounded by an attenuation level. Cabecinhas proposed a controller while rejecting constant force disturbances and the constant force disturbance is estimated through the use of a sufficiently smooth projector operator [7]. The controller consists of a nonlinear adaptive state feedback controller that asymptotically stabilizes the closed-loop system in the presence of force disturbances. Alexis took atmospheric turbulence as additive disturbances and which is more complex. His control scheme is computed based on a piecewise affine (PWA) model of the quad-rotor's attitude dynamics [8,9]. The switchings among the PWA models are ruled by the rate of the rotation angles and for each PWA system a corresponding predictive controller is computed. The experimental results show that the quad-rotor is not influenced by wind while performing accurate attitude tracking.

The model of quad-rotor is nonlinear, so many uncertainties will influence its flight such as wind, the ignored items of model and the perturbation of parameters. Besides using controller based on linearized model, nonlinear controller is also available such as back-stepping controller. Back-stepping controller is designed for the quad-rotor altitude and attitude stabilization in the existence of external disturbances and measurement noise. Basri designed a controller consists of back-stepping controller which can automatically select its parameters on-line by a fuzzy supervisory mechanism [10,11], and the simulation results indicate that it can stabilize the quad-rotor helicopter with better performance than linear design techniques. Lee designed an adaptive back-stepping hovering control for a quad-rotor with model parameter uncertainties [12]. In the research, the back-stepping based technique is utilized to design a nonlinear adaptive controller with can compensate for the motor thrust factor and the drag coefficient of a quad-rotor.

The size of quad-rotor is usually small, so its model has more uncertainties than common UAV. Furthermore, the weight of quad-rotor is light and its power is low, so the ability of resisting wind is also weaker. Therefore, the hybrid control of variety methods is common in the research of anti-interference. Gao proposed a hybrid control method based on back-stepping and fuzzy adaptive PID [13]. In the circumstances with or without disturbance, the method selects the back-stepping-based control or the fuzzy adaptive PID. Rabhi proposed an algorithm based on fuzzy control to ensure the stability of the quad-rotor, and the nonlinear model is represented by a Takagi-Sugeno (T-S) fuzzy model [14].

Taking advantage of nonlinear approximation, neural network control is also studied. In reference [15,16], a fuzzy neural network is developed to approximate the uncertainty function, and a robust compensator was proposed to confront the approximate errors and external disturbances. The method based on neural network has good performance on un-modeled errors, and the stability can be proved using Lyapunov stability theory. However, it is not sure whether the method will reduce the flight performance.

The researches on ground effect during autonomous take-off and landing of quad-rotor are less. Referring to the ground effect of helicopter, it can be known that the velocity will reduce rapidly when helicopter flies close to ground. The phenomenon causes unstable up-and-down motion near the ground and which makes helicopter cannot finish normal landing. According to the actual flight data, the ground effect should not be ignored. Because quad-rotor has lighter weight and lower power, it is more easily affected by ground effect.

In view of the control problem brought by ground effect during autonomous take-off and landing, a model reference adaptive control method is proposed for altitude control of quad-rotor. First, the ground effect of quad-rotor is analyzed. Second, a model of ground effect is established based on fluid numerical calculation. Then a model reference adaption controller is designed for altitude control. The simulation results show that the method can reduce the influence of ground effect, and make sure the quad-rotor can finish autonomous take-off and landing successfully.

II. GROUND EFFECT OF QUAD-ROTOR IN AUTONOMOUS TAKE-OFF AND LANDING

Ground effect is a special characteristic when aircraft flies close to ground [17]. Because the ground blocks the air flow under the rotor, the speed of air flow will reduce, then the pressure on lower surface will increase and the lift will increase finally. Using experimental method to study ground effect is time-consuming and high-cost. With the development of fluid computing technology, the numerical simulation method has attracted more attention in recent years.

Fluent is often used for Computational Fluid Dynamics (CFD), and it uses the infinite volume method. The method divides the flow field into many discrete control volumes, and solves the continuity equation and momentum equation. Compared to other CFD method, it does not need coordinate transformation and can be easily applied to structured grid and unstructured grid. Thus, the method is suitable to solve complex problem of fluid flow.

A. Model of Quad-rotor in Gambit

The software Gambit is used to generate gird. Quad-rotor is divided into 5 separate volumes. Each rotor is a volume and the body is a volume as in Fig. 1. Rotor and body is connected by interface boundary conditions. The type of grid is unstructured. Compared to structured grid, it is suitable for complex shape.

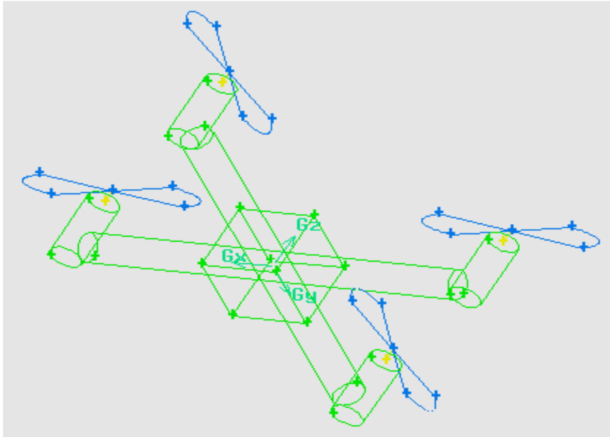

Figure 1. Model of quad-rotor in gambit

The quality of grid is measured by smoothness and skewness. In Fluent, it is acceptable that the maximum skew of gird is no more than 0.95, and the maximum skew of gird in this paper is 0.88. Since the rotor keeps rotating, the turbulence effect and viscous effect is significant. Therefore, it is better to make the grid of quad-rotor dense.

B. Turbulence Model

Fluent provides 6 turbulence models, and in general they are One-Equation Model, Two-Equation Model, Standard $k-\omega$, 4-Equation v2f, Reynolds Stress Model and $k-kl-\omega$ Transition Model. All of them are based on Reynolds-averaged Navier-Stokes equations (RANS), but they are suitable in different situations. The choice of turbulence model depends on the property of fluid, accuracy requirements, available computing resources and simulation time.

Spalart-Allmaras Model has better reliability, availability and efficiency than other models. It is a One-Equation Model for solving turbulent viscosity, and is suitable for the problems of fluid in tube and rotating machine in aerospace engineering. Compared with other turbulence models, it has better performance on the prediction of turbulent mixing layer, plane wake flow and vortex [18]. Vortex is an important part in the ground effect of quad-rotor, so Spalart-Allmaras Model is a better choice.

C. Boundary Condition

After grid is determined, boundary conditions should be set according to actual physical environment. Then the solution of control equations describes the flow field around quad-rotor. To study the ground effect of quad-rotor, the boundary conditions are considered as below.

(1) Velocity inlet. When quad-rotor hovers, the fluid flow above it at infinite distance is static and the velocity is 0 there.

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(2) Pressure outlet. Because of the existence of ground effect, back flow should not be ignored when quad-rotor hovers at low altitude. So pressure outlet is more suitable for the convergence.

(3) At present, two methods are widely used in the research on ground effect and they are method of images and surface singularity method [19]. The method of images is easy to use and adopted in this study. Because of symmetry, the normal component on ground of actual induced velocity and induced velocity of image offsets each other and the sum of them is 0. The case happens to meet the boundary conditions of tangential flow on solid boundary.

(4) Wall. This boundary condition is used to define the fluid and solid regions. In hovering state, rotor plane is velocity inlet, bottom surface is wall and other surfaces are pressure outlet.

D. Numerical Calculation Method

In the research of ground effect of quad-rotor, there are several moving flow fields and the problem involves relative rotation, which cause it is hard to get an accurate result by conventional CFD methods. Flunet provides a few models to handle it, such as Multi-Reference Frame (MRF), Mixing Plane, and Sliding Mesh. MRF and Mixing Plane is suitable for the calculation of steady flow and Sliding Mesh is for unsteady flow.

MRF is the most simple and economical model than the others. It is an approximation to the area with different rotation or speed. When the flow on the boundary almost mixes each other, MRF can be used. The model provides only an approximation of flow field, so it is taken as an initial flow field for Sliding Mesh here.

Sliding Mesh is used to handle unsteady problem in the study. There are more than two areas which have relative motion with each other and each area has one interface with others at least. To ensure the movement of flow perpendiculars to interface, the interface is designed to cylinder.

The study focuses on the transient flow field of quad-rotor, so the calculation uses Sliding Mesh. In view of that Sliding Mesh requires an initial flow field with high precision, the initial flow field is obtained by MRF.

III. SYSTEM DESCRIPTION

A. Analysis and Modeling of Ground Effect

The grid and the boundary conditions are shown in Fig. 2.

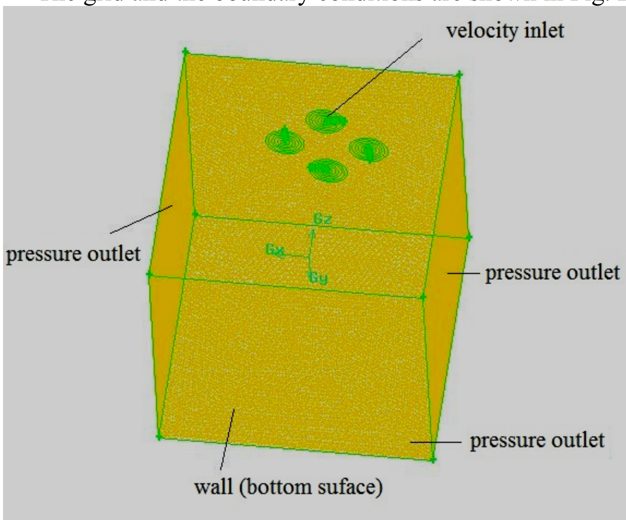
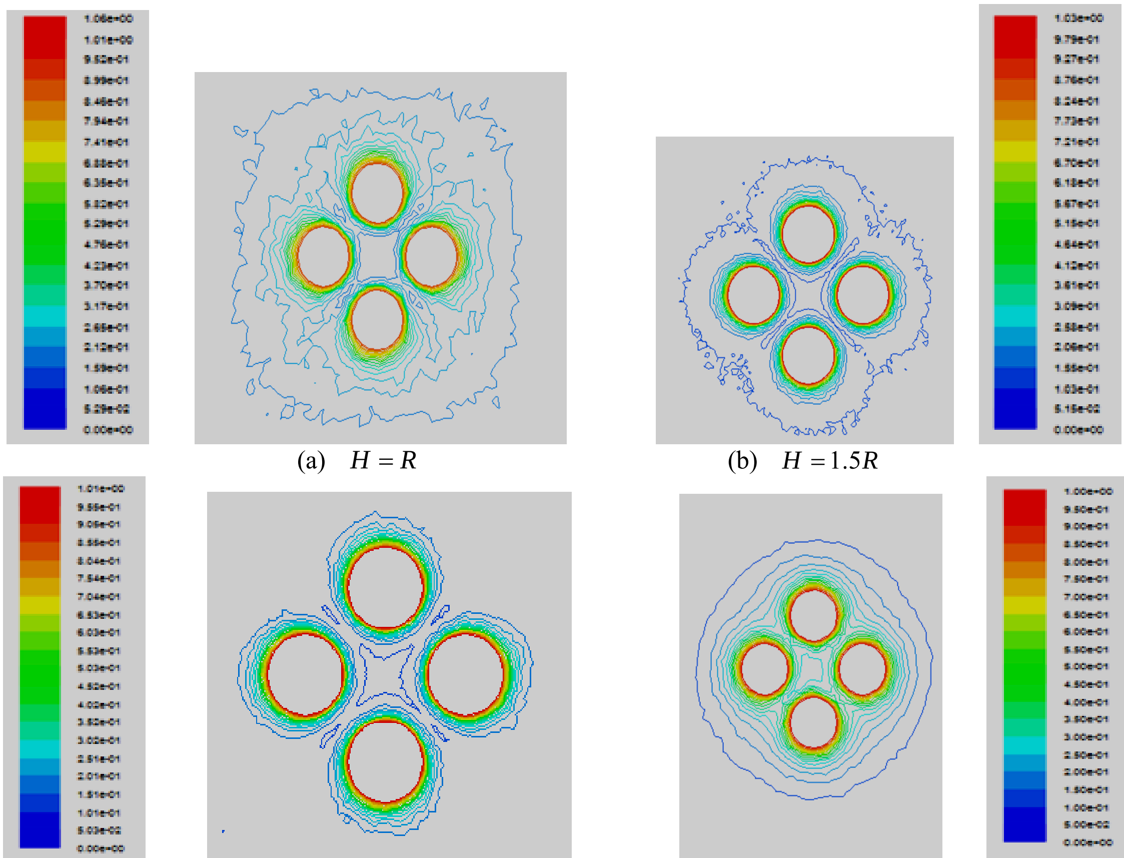


Figure 2. Grid and boundary conditions

Using Fluent to solve the flow field has several steps: (1) Import the grid model of Gambit. (2) Check the grid. (3) Set the computational environment. (4) Select the property of fluid. (5) Chose algorithm of CFD. (6) Set boundary conditions and number of iteration. (7) Initialize the flow field and start calculation.

After calculation, the velocity of ground effect at different altitudes is shown in Fig. 3. Here, R is the radius of rotor and H is altitude.



(a) $H = R$ (b) $H = 1.5R$

(c) $H = 2R$ (d) $H = 3R$

Figure 3. Velocity of ground effect at different altitudes.

Import the data from Flunet into Matlab for post-processing. Basing on the velocity of ground effect on rotor plane and different altitudes, the curve is shown in Fig. 4. After the curve fitting of least square method, the relationship between the velocity of ground effect on rotor plane and altitude is Equation (1).

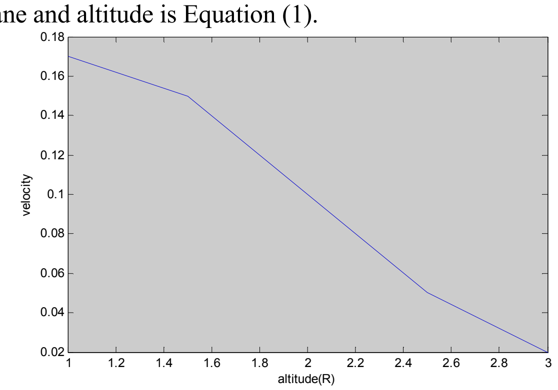


Figure 4. Velocity of ground effect on rotor plane at different altitudes

$$V_{GE} = 0.0333H^3 - 0.2057H^2 + 0.3145H + 0.028 \quad (1)$$

Here V_{GE} is the velocity of ground effect on rotor plane and H is altitude.

B. *Mathematical Model of Quad-rotor*

Assumption [20]:

(1) The structure is supposed to be rigid and strictly symmetrical.

(2) The center of mass and the body fixed frame origin are assumed to coincide.

(3) The moment is proportional to the DC motor voltage.

(4) The change of attitude angle range is limited into $(-5^{\circ}, 5^{\circ})$.

(5) The air resistance can be ignored at low speed.

The quad-rotor is motivated by four motors and can lead to three attitudes, i.e. yaw, pith, roll. The front and rear rotors rotate in a clockwise direction while the left and right rotors rotate in a counter-clockwise direction to balance the torque created by the spinning rotors.

Define $v=(x,y,z,\psi,\theta,\phi)\in R^6$, let $\xi=(x,y,z)\in R^3$ presents the position, $\eta=(\psi,\theta,\phi)\in R^3$ respectively denote yaw, pitch and roll angle. Then we can get that the kinetic energy of motion $T_{trans}=1/2m\xi^T\xi$ and the kinetic energy of rotation $T_{rot}=1/2m\eta^T\eta$, where m represents the mass of airframe. Ignoring the line movement and using the Lagrange method, the dynamic model of the four-rotor helicopter can be presented as Equation (2) [21].

$$\begin{cases}\ddot{\psi}=\dfrac{K_{tc}}{J_y}l(V_f+V_b)+\dfrac{K_{tn}}{J_y}l(V_r+V_l)\\ \ddot{\theta}=\dfrac{K_f}{J_p}l(V_r-V_l)\\ \ddot{\phi}=\dfrac{K_f}{J_r}l(V_f-V_b)\end{cases}\qquad(2)$$

where K_{tc}, K_{tn} are respectively counter rotation propeller torque-thrust constant and normal rotation propeller torque-thrust constant. J_y is equivalent moment of inertia about the yaw axis, J_p is equivalent moment of inertia about the pitch axis, J_r is equivalent moment of inertia about the roll axis. K_f is the propeller force-thrust constant which is found by experiment. V_f, V_b, V_r and V_l respectively represent the front, back, right and left motor voltage of the system.

IV. ALTITUDE CONTROLLER BASED ON MODEL REFERENCE ADAPTIVE CONTROL

Control strategy focuses on the altitude channel. A model reference adaptive controller is designed based on the original closed-loop system and its structure is shown in Fig. 5.

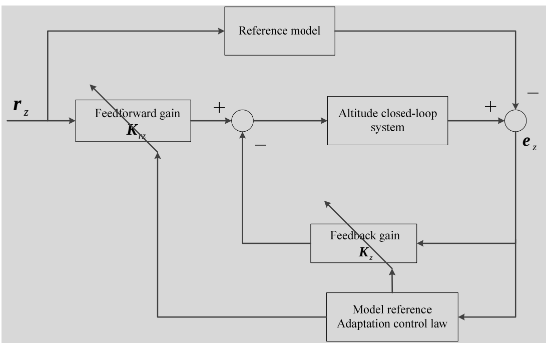


Figure 5. Model reference adaptive control on altitude channel

After linearization, the state equation of altitude channel is:

$$\begin{cases}\dot{\boldsymbol{x}}_z(t)=\boldsymbol{A}_z\boldsymbol{x}_z(t)+\boldsymbol{B}_z\boldsymbol{r}_z(t)\\ \boldsymbol{y}_z(t)=\boldsymbol{C}_z\boldsymbol{x}_z(t)\end{cases}\qquad(3)$$

Though the ground effect is expressed in Equation (1), it is not accurate. Because the actual situation is complex, the model reference adaptive controller is designed to deal with both ground effect and uncertain interference.

When there is no ground effect, $\boldsymbol{A}_z$, $\boldsymbol{B}_z$ and $\boldsymbol{C}_z$ nearly do not change. So their value is respectively constant $\bar{\boldsymbol{A}}_z$, $\bar{\boldsymbol{B}}_z$ and $\bar{\boldsymbol{C}}_z$. Taking ground effect into consideration, the original state equation changes and the performance of original controller goes bad. At this time, model reference adaptive control is needed. According to Equation (3), the reference model is chosen as:

$$\begin{cases}\dot{\boldsymbol{x}}_{new}=\boldsymbol{A}_{new}\boldsymbol{x}_{new}+\boldsymbol{B}_{new}\boldsymbol{r}_z\\ \boldsymbol{y}_z=\boldsymbol{C}_{new}\boldsymbol{x}_{new}\end{cases}\qquad(4)$$

The controller is designed as:

$$\boldsymbol{h}_z=-\boldsymbol{K}_z\boldsymbol{x}_z+\boldsymbol{K}_{rz}\boldsymbol{r}_z\qquad(5)$$

where $\boldsymbol{r}_z$ is the desired input. $\boldsymbol{K}_{rz}$ and $\boldsymbol{K}_z$ is feedforward gain matrix and feedback gain matrix respectively. Suppose we have $\tilde{\boldsymbol{K}}_{rz}$ and $\tilde{\boldsymbol{K}}_z$ which makes the closed-loop system is same to the reference model, then set $\Delta\boldsymbol{K}_{rz}=\boldsymbol{K}_{rz}-\tilde{\boldsymbol{K}}_{rz}$ and $\Delta\boldsymbol{K}_z=\boldsymbol{K}_z-\tilde{\boldsymbol{K}}_z$. Define error vector $\boldsymbol{e}_z=\boldsymbol{x}_{new}-\boldsymbol{x}_z$, so

$$\dot{\boldsymbol{e}}_z=\boldsymbol{A}_{new}\boldsymbol{e}_z+\boldsymbol{B}_{new}\tilde{\boldsymbol{K}}_{rz}^{-1}(\Delta\boldsymbol{K}_z\boldsymbol{x}_z-\Delta\boldsymbol{K}_{rz}\boldsymbol{r}_z)\qquad(6)$$

Lyapnove function is chosen as:

$$L=\boldsymbol{e}_z^T\boldsymbol{P}_{new}\boldsymbol{e}_z+\boldsymbol{tr}(\Delta\boldsymbol{K}_z^T\Gamma\Delta\boldsymbol{K}_z+\Delta\boldsymbol{K}_{rz}^T\Gamma\Delta\boldsymbol{K}_{rz})\qquad(7)$$

where Γ is positive definite matrix. Because Equation (4) comes from the closed-loop system without ground effect, the system is stable. So there is a matrix $\boldsymbol{P}_{new}$ and it satisfies $\boldsymbol{A}_{new}^T\boldsymbol{P}_{new}+\boldsymbol{P}_{new}\boldsymbol{A}_{new}=-\boldsymbol{Q}_z$ where $\boldsymbol{Q}_z$ is positive definite matrix. In addition, $\tilde{\boldsymbol{K}}_z$ and $\tilde{\boldsymbol{K}}_{rz}$ change slowly compared to $\boldsymbol{K}_z$ and $\boldsymbol{K}_{rz}$ respectively, so they can be taken as constants.

$$\begin{aligned}\dot{L}=&\boldsymbol{e}_z^T(\boldsymbol{A}_{new}^T\boldsymbol{P}_{new}+\boldsymbol{P}_{new}\boldsymbol{A}_{new})\boldsymbol{e}_z\\ &+2\boldsymbol{e}_z^T\boldsymbol{P}_{new}\boldsymbol{B}_{new}\tilde{\boldsymbol{K}}_{rz}^{-1}(\Delta\boldsymbol{K}_z\boldsymbol{x}_z-\Delta\tilde{\boldsymbol{K}}_{rz}\boldsymbol{r}_z)\\ &+2\boldsymbol{tr}(\Delta\boldsymbol{K}_z^T\Gamma\Delta\dot{\boldsymbol{K}}_z+\Delta\boldsymbol{K}_{rz}^T\Gamma\Delta\dot{\boldsymbol{K}}_{rz})\end{aligned}\qquad(8)$$

According to the property of matrix trace, select:

$$\begin{cases}\Delta\dot{\boldsymbol{K}}_z=\dot{\boldsymbol{K}}_z=-\boldsymbol{B}_{new}^T\boldsymbol{P}_{new}\boldsymbol{e}_z\boldsymbol{x}_z^T\operatorname{sgn}(l)\\ \Delta\dot{\boldsymbol{K}}_{rz}=\dot{\boldsymbol{K}}_{rz}=\boldsymbol{B}_{new}^T\boldsymbol{P}_{new}\boldsymbol{e}_z\boldsymbol{x}_z^T\operatorname{sgn}(l)\end{cases}\qquad(9)$$

where $\operatorname{sgn}(l)$ is determined by $\tilde{\boldsymbol{K}}_{rz}$ whether it is positive or negative definite matrix. From Equation (8) and (9), we obtain:

$$\dot{L}=\boldsymbol{e}_z^T(\boldsymbol{A}_{new}^T\boldsymbol{P}_{new}+\boldsymbol{P}_{new}\boldsymbol{A}_{new})\boldsymbol{e}_z=\boldsymbol{e}_z^T(-\boldsymbol{Q}_z)\boldsymbol{e}_z<0\qquad(10)$$

Therefore, the model reference adaptive control law of the closed-loop system on altitude channel is:

$$\begin{cases}\boldsymbol{K}_z=\displaystyle\int_0^t-\boldsymbol{B}_{new}^T\boldsymbol{P}_{new}\boldsymbol{e}_z\boldsymbol{x}_z^T\operatorname{sgn}(l)d\tau+\boldsymbol{K}_{z0}\\ \boldsymbol{K}_{rz}=\displaystyle\int_0^t\boldsymbol{B}_{new}^T\boldsymbol{P}_{new}\boldsymbol{e}_z\boldsymbol{r}_z^T\operatorname{sgn}(l)d\tau+\boldsymbol{K}_{rz0}\end{cases}\qquad(11)$$

where $\boldsymbol{P}_{new}$ is the solution of $\boldsymbol{A}_{new}^T\boldsymbol{P}_{new}+\boldsymbol{P}_{new}\boldsymbol{A}_{new}=-\boldsymbol{Q}_z$. $\boldsymbol{Q}_z$ is unit matrix, and $\boldsymbol{r}_z$ is desired input. According to Equation (11) and the stability theory of Lyapnov, the general

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error of Equation (6) is global asymptotically stable, and the control performance will approach the situation of reference model in Equation (4).

V. SIMULATION VERIFICATION

In the simulation, the proposed method is tested in two circumstances. One has ground effect, and another does not. The system matrices of Equation (2) are $\boldsymbol{A}_z = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -300 & -100 & -16 \end{bmatrix}$, $\boldsymbol{B}_z = \begin{bmatrix} 0 \\ 0 \\ 300 \end{bmatrix}$ and $\boldsymbol{C}_z = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.

The PID method is compared with the method of this paper in Fig. 6. The initial input is 5R, and R is the radius of quad-rotor. At time of 5s, the input adds to 8R.The dashed line represents the performance of PID method when there is no ground effect. The solid line represents the performance of PID method when ground effect works. The line with point represents the MRAC method proposed in this paper when ground effect works.

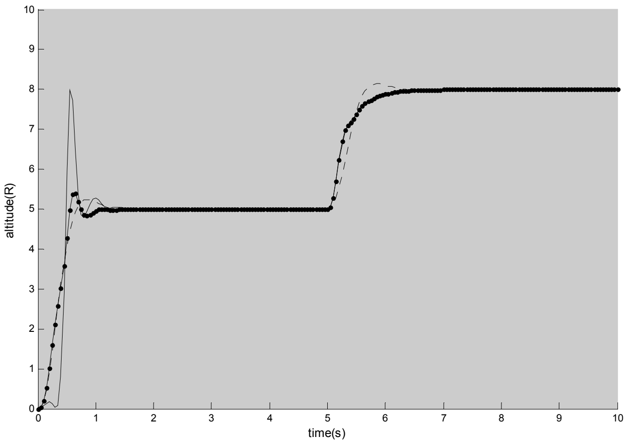

Figure 6. Influence of ground effect on altitude control

In Fig. 6, PID method can realize the altitude control when there is no ground effect (dashed line). It can be seen that because ground effect is ignored, the performance of PID method at time of 0s and 5s is no different. However, the hypothesis is not consistent with the actual situation.

When taking ground effect into account, the overshoot of PID method (solid line) increases and there are several oscillations in Fig. 6. The input is 5R before 5s, so ground effect works, and the overshoot and oscillations is severe. After 5s, the overshoot and oscillations return to the normal level. It is because ground effect can be ignored when altitude is higher than 5R.

In Fig. 6, the line with point is the method proposed in this paper. It also has some overshoots, but it is much less than PID. In addition, it has only one oscillation and converges quickly. At 5s, the altitude reaches 5R where ground effect can be ignored. After 5s, the performances of our method and PID show no different when input is same.

When ground effect works, the output of original controller deviates that of MRAC, so adaptive control law adjusts the parameters automatically. Fig. 6 shows that the method is able to ensure the control performance and reduce the influence brought by ground effect.

VI. CONCLUSIONS

In the process of take-off and landing, ground effect has a significant impact on the control of quad-rotor. In order to ensure the control performance, a method based on model reference adaptive control is proposed. Firstly, the model of ground effect is established by using numerical calculation. Secondly, the relationship between the velocity on rotor plane and altitude is obtained. Then, an adaptive control law is designed based on the original closed-loop system. The simulation results show that when taking ground effect into account, the method can adjust the parameters automatically. It ensures the control performance of altitude and realizes the stable take-off and landing of quad-rotor.

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