A New Approach to Analyze the Degrees of Freedom of Serial Mechanisms

Jianguo LUO[1,a*] , Zehao BU [2,b] , Jianyou HAN [3,c]

1. *Mechanical & Electrical Department*, North China Institute of Science and Technology, Beijing, 101601;
2. *Graduate School*, North China Institute of Science and Technology, Beijing, 101601;
3. *School of Mechanical Engineering*, Beijing University of Science & Technology, Beijing, 100083

[a]luojg_1598@126.com, [b]bzhao@163.com , [c]jyhan@bjust.edu.cn

Abstract — **In studying methods for calculating and analyzing the degree of freedom of existing mechanism, it is found that there are some shortcomings. Using basic physical performance and motion mathematical representation, we study the connection and the difference between the input and output motion of the mechanism. Through analyzing the mathematical representation of the input physical motion element and the output terminal trace element of the mechanism, we propose a new definition of freedom, and define input and output degrees of freedom. Using the theory of main auxiliary motion, three situations of pure rotation input, pure translation input, translation input and rotation input are analyzed to obtain the general formulae of input and output motion transformation and their degree of freedom. Through in-depth analysis, the conditions and rules of the output motion of first-order, second-order and third-order associated motion of the mechanism are derived. With the help of the research method and conclusion, we can find parallel 4 and 5 degrees of freedom mechanism with same branch chain, which is a new scientific theory and effective method for the analysis of degree of freedom and mechanism type synthesis.**

Keywords - Input degree of freedom, Output degree of freedom, Primary motion, Associated motion, trajectory, Mathematical representation

I. INTRODUCTION

The calculation of degree of freedom is the most basic problem of mechanism analysis, according to the IFToMM standard [1], degree of freedom is defined as number of independent parameters needed to determine the types of mechanism or the kinematic chain, that is the number of independent parameters in order to determine the position of all members of the mechanism or kinematic chain under a certain position. For a long time, the Grubler-Kutzbach formula (G-K formula) has been used to recognize almost all the degree of freedom of the planar mechanism and some spatial mechanisms, and it is only based on the most basic arithmetic operations. It has made outstanding contributions to the development of mechanical discipline and the civilization progress of human society, and all the people who pay attention to study the degree of freedom in the whole world also made great contribution to it. However, there are counter examples in history, in 1978, the United States professor Suh [2], in 1980, Shigley and Uicker [3], in 1984, Sandor and Erdman [4], in 1987, Mabie and Reinholtz [5], in 1998, Eckhardt [6], in 1999, professor Tsai of University of Maryland [7] , in 2000, professor Merlet of France [8], in 2004, international famous scholar Waldron and Kinzel [9] pointed out that the G-K formula can not adapt to the examples and cases in the corresponding books. It has been more than 150 years to find the universal freedom formula of mechanism, professor Huang zhen of Yanshan University in 2011 published a monograph, Research on the degree of freedom of mechanism, a general formula which have been finding for 150 years [10]. In this book, three conditions discussed, the general application (including open chain, single chain, parallel mechanism and other mentioned mechanisms), classical mechanism and parallel mechanism, mechanism specified in the bar calculating formula of degree of freedom, have strong adaptability and scientific nature. All the calculations about the degree of freedom are based on the number of components, the number of joints, joint degree of freedom, the number of constraints, parallel redundant constraints, number of independent constraints on the component. These parameters include the physical elements of the mechanism, the characteristics of the motion elements, the relationship between the whole motion elements and the relationship of the local motion elements. Because these factors are explicit or implicit, both are structural factors or motion elements. So for the space structure which is complex and special, parameters of the structural elements and some other dominant elements are easy to define, but for the whole and partial characteristics in motion elements which are implicit parameters are not easy to determine, there are some inconvenience in the use of the process.

Existing theory and method haven’t distinguish the degree of freedom of the input and output motion, and the internal relationship and difference between the input and output motion elements are not found, there is no research on the connection and difference between the input and output of the mechanism from the basic physical performance and basic mathematical representation of the motion. The theoretical and computational analysis methods of physical mathematical nature of the degree of freedom can not be obtained. Based on the above thinking, it led research team to redefine degree of freedom, redefine the degree of freedom of input and output of mechanism, starting from the mathematical representation and the relationship between the motion elements, through the introduction of theory of main-vice motion, general method of analysis and calculate the degree of freedom of mechanism found then. The analysis process and the conclusion of the degree of freedom of the serial chain are presented in three cases of pure rotation input, pure translation input, translation and rotation input. The conditions and rules of the existence of the first order and the second order and the third order associated motion on output terminal of mechanism given after in-depth analysis. With the aid of research methods and conclusions found what Merlet said there is no same branched parallel mechanism with 4 or 5 degree of freedom. In this way, new scientific theories and effective methods found for the analysis of the degree of freedom and the mechanism type synthesis.

II. MAIN MOTION AND ASSOCIATED MOTION

Motion is introduced into the mechanism through the input motion joint, and through the integrated action of multi joints transfered into the motion of the output joint or component. The basic motion elements of the input joint include linear motion and rotation motion.The number and composition of input motion elements can be determined, when the input motion elements have the characteristics as shown in Fig.1 and Fig.2, when there are two or more than two parallel axes rotation input joints, the output trajectory set is a circular ring plane or a circular plane, as shown in Fig.3. A number of different diameter circles contained in circular ring or circular plane, the output end with the same rotation motion of the input motion is defined as the output main motion.The plane contains numerous pairwise orthogonal linear trajectory, different linear motion can be achieved at the end of the output from the input motion is defined as the output associated motion.

$$1D(2R_i)->1D(1R_i)+\{1D(1P_j)+1D(1P_k)\} \quad (1)$$

$$1D(mR_i)->1D(1R_i)+\{1D(1P_j)+1D(1P_k)\} \quad (2)$$

the part in {} is the output motion characteristics, the natural number $m \geq 2$, i, j, k are the space three orthogonal axes (the same below).

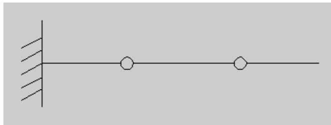

Figure 1. Two parallel axis rotation input branch

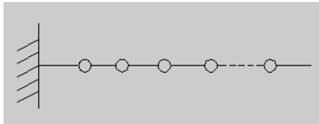

Figure 2. M parallel axis rotation input branch

Similarly, when the input motion elements have the characteristics which are shown in Fig.4, there are two orthogonal motion input joints, the output terminal trajectory is a rectangular plane, as shown in Fig.6. The rectangular plane contains numerous pairwise orthogonal linear trajectory, the output end with the same motion of the input motion is defined as the output main motion, the plane contains numerous different diameter circular trajectory, the end of the output which can achieved different rotation motion with the input motion is defined as the output associated motion.

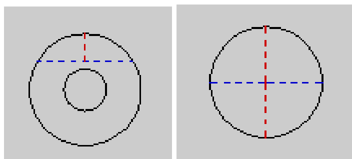

Figure 3. Output terminal trajectory set

According to the definition of the degree of freedom in the IFToMM terminology standards, we have to give two independent input parameters to determine the output terminal, the mechanism has two degree of freedom, which is inconsistent with the math representation of the output trajectory of two groups of orthogonal linear trajectory, one group of circle trajectory. So the straight line trajectory or circular trajectory in the same plane with the axis parallel is defined as one group(the same below). In this case, the orthogonal linear trajectory of two groups corresponding to the associated trajectory, the trajectory of one group circular corresponding to the main trajectory. The output end of the mechanism have two linear motion degree of freedom in the two dimensional plane, and one rotation degree of freedom in plane, the output end can achieve 3 degree of freedom.

The formula is expressed as follow:

According to the definition of the degree of freedom in the IFToMM terminology standards, we need to give one or two independent input parameters to make the output terminal realize some certain motion, this mechanism has 1 or 2 degree of freedom, which is inconsistent with the math representation of the output trajectory, two groups orthogonal linear trajectory and one group circle trajectory. In this case, the orthogonal linear trajectory of two groups are the main trajectory, one circular trajectory is the associated trajectory. The output end of the mechanism have two linear motion degree of freedom which consist in the two dimensional plane, and also have one rotation degree of freedom which consist in one plane, that means the output end can achieve three degree of freedom in space.

The formula is expressed as follow:

$$1D(1P_i)+1D(1P_j)->1D(1P_i)+1D(1P_j)+\{1D(1R_k)\} \quad (3)$$

Fig.5 show the case that three orthogonal translation input joints included, Fig.7 is the corresponding output end motion trajectory set. The output end of the mechanism have three linear motion degree of freedom. At the same time, there are three rotation degree of freedom, the output terminal have six degree of freedom.

The formula is expressed as follow:

$$1D(1P_i)+1D(1P_j)+1D(1P_k)->1D(1P_i)+1D(1P_j)+1D(1P_k)+\{1D(1R_i)+1D(1R_j)+1D(1R_k)\} \quad (4)$$

From the above analysis, we can see that Fig.1 and Fig.4 are the two basic structure forms of generating associated motion of mechanism, Fig.3 and Fig.6 are the corresponding basic mathematical representation form of mechanism output terminal motion, formula (1) and (3) are the corresponding basic calculation formula of the mechanism associated motion. Whether it is the main motion, or the associated motion, whether it is rotation motion elements, or translation elements, the output motion elements of mechanism have consistent character.

$1D(1P_i)+\{1D(1P_i)\}$ represent a class of basic mathematics element, the corresponding output terminal is one degree of freedom. $1D(1R_i)+\{1D(1R_i)\}$ represent another class of basic mathematics element, the corresponding output terminal of the mechanism is one degree of freedom. From above, it would be easy to get the input degree of freedom of mechanism (IN-DOF) and the output degree of freedom of mechanism (OUT-DOF) in the above mentioned three types of cases.

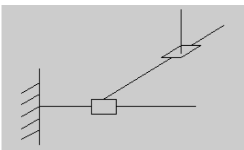

Figure 4. Two orthogonal translation input branch

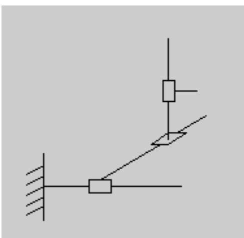

Figure 5. Three orthogonal translation input branch

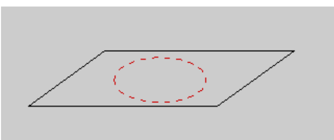

Figure 6 Output terminal trajectory set of chain in Fig.4

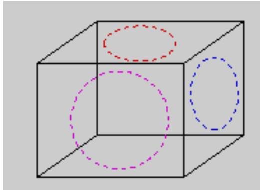

Figure 7 Output terminal trajectory set of chain in Fig. 5

Based on this result, we think that the input degree of freedom and output degree of freedom should be separated, and given new definitions.

Degree of freedom (DOF) is defined as the representation number of moving dimensionality of an object in space.

Mechanism input degree of freedom(IN-DOF) is defined as the maximum number of independent input motion parameters when the mechanism or kinematic chain in exact position.

Mechanism output degree of freedom(OUT-DOF) is defined as the maximum representation number of the mechanism output terminal moving dimensionality in space representation number of moving dimensionality(NOMD) is defined as the number of circling motion around different rotating shaft and the number of of linear motion along different direction in space relative datum coordinate system.

The input degree of freedom(IN-DOF) is specified as the measure index of driving capability of mechanism, the output degree of freedom(OUT-DOF) is specified as the measure index of performance of mechanism.

III. THE ORDER AND COMPOSITION OF ASSOCIATED MOTION

First of all, if two or more rotation or translation inputs work together, the associated translation or rotation output is defined as first order associated motion, and the first-order associated motion is analyzed in two cases.

Case 1. One rotation input active elements, adding one parallel rotation input, it will get two first order translation output associated motion elements, the analysis process as follows:

$1D(2R_i)->1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}$,

IN-DOF=2, OUT-DOF=3.

Case 2. One translation input active elements, adding one orthogonal translation input, it will get one first order rotation output associated motion element, the analysis process as follow:

$1D(1P_i)+1D(1P_j)->1D(1P_i)+1D(1P_j)+\{1D(1R_k)\}$,

IN-DOF=2, OUT-DOF=3.

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From the above analysis, case 1 and case 2 both achieve more than one increment by adding one input degree of freedom to existed branch with one input degree of freedom, this kind of increase of input motion is defined as effective increase of class Y1. As for other cases, one or less than one increment degree of freedom by adding one input degree of freedom to existed branch with one input degree of freedom, this kind of increase of input motion is defined as invalid increase class of N1.

Graphs are used to represent the relationship between input and output motion elements, Fig.8 is the symbol graph that parallel rotating input motion elements generating the first order associated orthogonal translation output, solid circle represents the input rotation motion element, the dotted rectangle represents the first order associated output translation motion element, arrow indicates the process and direction, VP indicates the first order associated translation output. Fig.9 shows a orthogonal translation input gets first order associated orthogonal rotation output symbol, the solid rectangle represents the motion input translation element, the dotted circle represents first order associated output rotation motion element, arrow indicates the process and direction, VR indicated first order associated rotation output.

From the above analysis, we can see the existence of first order associated motion must under the comprehensive effect of at least two orthogonal translation input motion elements, or at least more than two parallel rotation input motion elements. This is the necessary and sufficient conditions for the existence of first order associated motion, this conclusion can be obtained through reduction to absurdity.

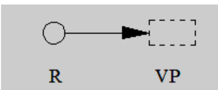

Figure 8. First order associated translation output symbol.

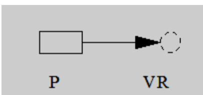

Figure 9. First order associated rotation output symbol

Secondly, if one translation or rotation input and one or more first order associated translation or rotation output work together again, the generated associated rotation or translation output is defined as second order associated motion, due to the first order associated motion class Y1 have two cases of efficient increase, the second order associated motion will conduct in-depth analysis based on these two situations.

Case 1. Adding one translation input active element $1D(1P_i)$, and interact with the first order associated output motion elements $\{1D(1P_j)+1D(1P_k)\}$, then it will get two second order associated motion elements $\{\{1D(1R_k)+1D(1R_j)\}\}$,analysis procedure as follows:

$$1D(2R_i) -> 1D(1R_i)+\{1D(1P_j)+1D(1P_k)\} \,|+ 1D(1P_i)$$
$$-> 1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_j)+1D(1R_k)\}\}$$

In this formula, $|+$ indicates first order associated increased input motion(the same below), through this process, the original IN-DOF=2 and OUT-DOF=3 extended to IN-DOF=3 and OUT-DOF=6, that is first order associated increase one IN-DOF to achieve the result of three increase of OUT-DOF, this increase of input motion is defined as efficient increase of class Y2, the equation can be simplified as the equivalent:

$$1D(2R_i)+1D(1P_i) -> 1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}$$
$$+\{\{1D(1R_j)+1D(1R_k)\}\},$$

IN-DOF=3，OUT-DOF=6.

Case 2. Adding one rotation input active elements $1D(1R_k)$, interact with the first order associated output motion element $\{1D(1R_k)\}$,it will get two second order associated motion elements$\{\{1D(1P_j)+1D(1P_k)\}\}$, the original IN-DOF=2 and OUT-DOF=3 extended to IN-DOF=3 and OUT-DOF=3, that is adding one input motion, no increase of OUT-DOF of mechanism. Increasing the input motion only change the space of output motion, output terminal representation of moving dimensionality in space unchanged, this kind of increase of input motion is defined as invalid increase of class N2, calculation process as follow:

$$1D(1P_i)+1D(1P_j) -> 1D(1P_i)+1D(1P_j)+\{1D(1R_k)\} \,|+ 1D(1R_k)$$
$$-> 1D(1P_i)+1D(1P_j)+\{1D(1R_k)\}$$
$$+\{\{1D(1P_i)+1D(1P_j)\}\}$$

the equation can be simplified as the equivalent:

$$1D(1P_i)+1D(1P_j)+1D(1R_k) -> 1D(1P_i)+1D(1P_j)+$$
$$\{1D(1R_k)\}+\{\{1D(1P_i)+1D(1P_j)\}\}$$

IN-DOF=3， OUT-DOF=3.

From the above, we can know that only the case 1 can increase one IN-DOF of mechanism, the increment of OUT-DOF is greater than 1. Graphs are used to represent the relationship between input and output motion elements, Fig.10 shows the parallel rotation input motion elements generating first order associated orthogonal translation output, this output combined with first order orthogonal translation input creating the second-order associated rotation motion, the two layer dotted circle represents the second order(vice) output rotation motion element, the dashed arrow indicates the participation process and direction of the associated motion elements, VVR indicates

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second order associated rotation output, the meaning of the remaining symbols are the same to Fig.8 and Fig.9.

From the above analysis, the existence of second order associated motion must under the comprehensive effect of more than two parallel rotation input motion elements and one translation input element which is parallel to the axis of rotation input. This is the necessary and sufficient conditions for emerging of second order associated motion, this conclusion can be obtained through reduction to absurdity.

Finally, if one translation or rotation input and one or more second order associated translation or rotation output interact again, the generated associated rotation or translation output is defined as third order associated motion, due to the fact that the second order associated motion of class Y2 class has only one case of efficient increase, the third order associated motion will conduct in-depth analysis based on this situation.

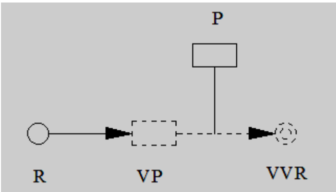

Figure 10. Second order associated rotation output symbol

Case 1. Adding one rotation input active elements $1D(1R_k)$, and interact with the second order associated motion element $\{\{1D(1R_k)+1D(1R_j)\}\}$, it will get two third order associated motion elements $\{\{\{1D(1P_i)+1D(1P_j)\}\}\}$. The original IN-DOF=2 and OUT-DOF=3 extended to IN-DOF=3 and OUT-DOF=6, then further extended to IN-DOF=4 and OUT-DOF=6, that is adding one IN-DOF to second order associated motion, the output of mechanism will increase 0 OUT-DOF. Increasing the input motion only change the space of output motion, output terminal representation of moving dimensionality in space unchanged, this kind of increase in input motion is defined as invalid increase of class N3, calculation process as follows:

$$\begin{aligned}1D(2R_i)->1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}|+1D(1P_i)\\ ->1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_j)+1D(1R_k)\}\}\|+1D(1R_k)\\ ->1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_j)+1D(1R_k)\}\}+\\ \{\{\{1D(1P_j)+1D(1P_k)\}\}\}\end{aligned}$$

the equation can be simplified as the equivalent

$$\begin{aligned}1D(2R_i)+1D(1P_i)+1D(1R_k)->1D(1R_i)+\{1D(1P_j)\\ +1D(1P_k)\}+\{\{1D(1R_j)\\ +1D(1R_k)\}\}+\{\{\{1D(1P_j)+1D(1P_k)\}\}\}\end{aligned}$$

IN-DOF=4，OUT-DOF=6.

In this formula, $\|+$ indicates second order associated increased input motion, the mechanism already has 6 degree of freedom before increasing input motion. It does not need to add any input motion elements, thus there is no third order associated motion. If increasing input motion element, it will only have effect on the dimension interval of the object, and will not affect the number of output terminal representation of moving dimensionality in space. It can be known that the third order associated motion does not exist.

IV.ANALYSIS ON THE DEGREE OF FREEDOM OF SERIAL MECHANISM

Joint and branch chain are the basic components of the serial mechanism, parallel mechanism and hybrid mechanism, the degree of freedom of 1 to 6 joint serial chain is analyzed, which lays the foundation for the analysis of the mechanism. In order to get the main motion and associated motion of serial mechanism general motion chain input motion elements, rapid judging on IN-DOF and OUT-DOF of serial mechanism, we will analyze three cases of the pure rotation input (as shown in Table 1), pure translation input (as shown in Table 2), the translation and rotation input (such as table 3 to11) and give the analysis process and results.

In the above analysis and results, the natural number $l,m,n\geq 2$, i,j,k is three orthogonal axis direction of space (the same below).

From the above, we can know that the OUT-DOF of serial mechanism can be 1, 2, 3, 4, 5, 6.

From the above we can know that the OUT-DOF of pure translation input joint serial mechanism can be 1, 3, 6. There is no OUT-DOF of 2, 4, 5 of the pure translation input joints serial mechanism.

From the above we can know that the OUT-DOF of the serial mechanism composed by 2 to 6 translation and rotation inputs can be 2, 3, 4, 5, 6.

Through the analysis of above three cases, serial chain of 6 translation or(and) rotation joint, the IN-DOF and OUT-DOF can be obtained from table 1 to 11, check the corresponding form of composition, then we can know the specific OUT-DOF and the specific motion elements, how the motion elements generated. This work opens up a new scientific and effective way for the analysis and calculation of the degree of freedom.

According to the definition of mechanism input degree of freedom, we can find a rule from table 1 to 11 of the input-output motion conversion process.

When the branch consists of rotation input motion element, whether rotation motion axis parallel or orthogonal, regardless of the number of rotation input motion elements, they are independent input motion parameters which were needed when the mechanism or motion chain is in a certain type, namely the rotation input motion elements are inevitable to mechanism IN-DOF. The translation input

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motion elements, either their axis are parallel or orthogonal, regardless of the number of translation input motion elements, every one which combined with another one or more translation input motion elements can be used as input motion parameters, namely translation elements input motion is probable on mechanism IN-DOF.

TABLE 1. KINEMATIC ANALYSIS OF PURE ROTATION INPUT

Inout type	Input-output conversion formula	IN-DOF	OUT-DOF
Rotation input	$1D(1R_i) \longrightarrow 1D(1R_i)$	1	1
	$1D(mR_i) \longrightarrow 1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}$	*m*	3
	$1D(mR_i)+1D(1R_j) \longrightarrow 1D(1R_i)+1D(1R_j)+\{1D(1P_j)+1D(1P_k)\}$	*m*+1	4
	$1D(1R_i)+1D(1R_j) \longrightarrow 1D(1R_i)+1D(1R_j)$	2	2
	$1D(lR_i)+1D(mR_j) \longrightarrow 1D(1R_i)+1D(1R_j)+\{1D(1P_j)+1D(1P_k)\}+\{1D(1P_i)+1D(1P_k)\}$	*l*+*m*	5
	$1D(1R_i)+1D(1R_j)+1D(1R_k) \longrightarrow 1D(1R_i)+1D(1R_j)+1D(1R_k)$	3	3
	$1D(mR_i)+1D(1R_j)+1D(1R_k) \longrightarrow 1D(1R_i)+1D(1R_j)+1D(1R_k)+\{1D(1P_j)+1D(1P_k)\}$	*m*+2	5
	$1D(lR_i)+1D(mR_j)+1D(nR_k) \longrightarrow 1D(1R_i)+1D(1R_j)+1D(1R_k)+\{1D(1P_j)+1D(1P_k)\}+\{1D(1P_i)+1D(1P_k)\}+\{1D(1P_i)+1D(1P_j)\}$	*l*+*m*+*n*	6

Table 2. Kinematic analysis of pure translation input

Input type	Input-output conversion formula	IN-DOF	OUT-DOF
Translation input	$1D(1P_i) \longrightarrow 1D(1P_i)$	1	1
	$1D(mP_i) \longrightarrow 1D(1P_i)$	*m*	1
	$1D(1P_i)+1D(1P_j) \longrightarrow 1D(1P_i)+1D(1P_j)+\{1D(1R_k)\}$	2	3
	$1D(1P_i)+1D(1P_j)+1D(1P_k) \longrightarrow 1D(1P_j)+1D(1P_j)+1D(1P_k)+\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}$	3	6
	$1D(lP_i)+1D(mP_j)+1D(nP_k) \longrightarrow 1D(1P_i)+1D(1P_j)+1D(1P_k)+\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}$	*l*+*m*+*n*	6

TABLE 3. KINEMATIC ANALYSIS OF 1 TRANSLATION AND 1 ROTATION INPUT

Input type	Input-output conversion formula	IN-DOF	OUT-DOF
1 translation 1 rotation	$1D(1R_i)+1D(1P_j) \longrightarrow 1D(1R_i)+1D(1P_j)$	2	2
	$1D(1R_i)+1D(1P_i) \longrightarrow 1D(1R_i)+1D(1P_i)$	2	2

TABLE 4. KINEMATIC ANALYSIS OF 1 TRANSLATION AND 2 ROTATION INPUT

Input type	Input-output conversion formula	IN-DOF	OUT-DOF
1 translation 2 rotation	$1D(1P_i)+1D(2R_j) \longrightarrow 1D(1P_i)+1D(1R_j)+\{1D(1P_i)+1D(1P_k)\}+\{\{1D(1R_j)\}\}$	3	3
	$1D(1P_i)+1D(2R_i) \longrightarrow 1D(1P_i)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_j)+1D(1R_k)\}\}$	3	6
	$1D(1P_i)+1D(1R_j)+1D(1R_k) \longrightarrow 1D(1P_i)+1D(1R_j)+1D(1R_k)$	3	3

TABLE 5. KINEMATIC ANALYSIS OF 1 TRANSLATION AND 3 ROTATION INPUT

Input type	Input-output conversion formula	IN-DOF	OUT-DOF
1 translation 3 rotation	$1D(1P_i)+1D(3R_j) \longrightarrow 1D(1P_i)+1D(1R_j)+\{1D(1P_i)+1D(1P_k)\}+\{\{1D(1R_j)\}\}$	3	4
	$1D(1P_i)+1D(3R_i) \longrightarrow 1D(1P_i)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_j)+1D(1R_k)\}\}$	4	6
	$1D(1P_i)+1D(1R_i)+1D(1R_j)+1D(1R_k) \longrightarrow 1D(1P_i)+1D(1R_i)+1D(1R_j)+1D(1R_k)$	4	4
	$1D(1P_i)+1D(2R_j)+1D(1R_k) \longrightarrow 1D(1P_i)+1D(1R_j)+\{1D(1P_i)+1D(1P_k)\}+\{\{1D(1R_j)\}\}$	4	3
	$1D(1P_i)+1D(2R_i)+1D(1R_j) \longrightarrow 1D(1P_i)+1D(1R_i)+1D(1R_j)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_j)+1D(1R_k)\}\}$	4	6

TABLE 6. KINEMATIC ANALYSIS OF 2 TRANSLATION AND 1 ROTATION INPUT

Input type	Input-output conversion formula	IN-DOF	OUT-DOF
2 translation 1 rotation	$1D(2P_j)+1D(1R_i) \longrightarrow 1D(1P_j)+1D(1R_i)$	3	2
	$1D(1P_i)+1D(1P_j)+1D(1R_k) \longrightarrow 1D(1P_i)+1D(1P_j)+1D(1R_k)+\{1D(1R_k)\}$	3	3

TABLE 7. KINEMATIC ANALYSIS OF 2 TRANSLATION AND 2 ROTATION INPUT

Input type	Input-output conversion formula	IN-DOF	OUT-DOF
2 translation 2 rotation	$1D(2P_j)+1D(2R_i) \longrightarrow 1D(1P_j)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_i)\}\}$	4	3
	$1D(2P_i)+1D(2R_i) \longrightarrow 1D(1P_i)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_k)+1D(1R_j)\}\}$	4	6
	$1D(2P_j)+1D(1R_i)+1D(1R_k) \longrightarrow 1D(1P_j)+1D(1R_i)+1D(1R_k)$	4	3
	$1D(2P_j)+1D(1R_j)+1D(1R_k) \longrightarrow 1D(1P_j)+1D(1R_j)+1D(1R_k)$	4	3
	$1D(1P_i)+1D(1P_j)+1D(2R_k) \longrightarrow 1D(1P_i)+1D(1P_j)+1D(1R_k)+\{1D(1P_j)+1D(1P_i)\}+\{1D(1R_k)\}+\{\{1D(1R_k)\}\}$	4	3
	$1D(1P_i)+1D(1P_j)+1D(2R_i) \longrightarrow 1D(1P_i)+1D(1P_j)+1D(1R_i)+\{1D(1R_k)\}+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}\}$	4	6
	$1D(1P_i)+1D(1P_j)+1D(1R_j)+1D(1R_k) \longrightarrow 1D(1P_i)+1D(1P_j)+1D(1R_j)+1D(1R_k)+\{1D(1R_k)\}$	4	4

TABLE 8. KINEMATIC ANALYSIS OF 2 TRANSLATION AND 3 ROTATION INPUT

Input type	Input-output conversion formula	IN-DOF	OUT-DOF
2 translation 3 rotation	$1D(2P_j)+1D(3R_i)\longrightarrow 1D(1P_j)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_i)\}\}$	5	3
	$1D(2P_i)+1D(3R_i)\longrightarrow 1D(1P_i)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_k)+1D(1R_j)\}\}$	5	6
	$1D(2P_j)+1D(1R_i)+1D(1R_j)+1D(1R_k)\longrightarrow 1D(1P_j)+1D(1R_i)+1D(1R_j)+1D(1R_k)$	5	4
	$1D(2P_j)+1D(2R_i)+1D(1R_k)\longrightarrow 1D(1P_j)+1D(1R_i)+1D(1R_k)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_i)\}\}$	5	5
	$1D(2P_i)+1D(2R_i)+1D(1R_k)\longrightarrow 1D(1P_i)+1D(1R_i)+1D(1R_k)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_k)+1D(1R_j)\}\}$	5	6
	$1D(1P_i)+1D(1P_j)+1D(3R_i)\longrightarrow 1D(1P_i)+1D(1P_j)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{1D(1R_k)\}+\{\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}\}$	5	6
	$1D(1P_i)+1D(1P_j)+1D(3R_k)\longrightarrow 1D(1P_i)+1D(1P_j)+1D(1R_k)+\{1D(1P_j)+1D(1P_i)\}+\{1D(1R_k)\}+\{\{1D(1R_k)\}\}$	5	3
	$1D(1P_i)+1D(1P_j)+1D(1R_i)+1D(1R_j)+1D(1R_k)\longrightarrow 1D(1P_i)+1D(1P_j)+1D(1R_i)+1D(1R_j)+1D(1R_k)+\{1D(1R_k)\}$	5	5
	$1D(1P_i)+1D(1P_j)+1D(2R_k)+1D(1R_i)\longrightarrow 1D(1P_i)+1D(1P_j)+1D(1R_k)+1D(1R_i)+\{1D(1P_j)+1D(1P_i)\}+\{1D(1R_k)\}+\{\{1D(1R_k)\}\}$	5	4
	$1D(1P_i)+1D(1P_j)+1D(2R_i)+1D(1R_k)\longrightarrow 1D(1P_i)+1D(1P_j)+1D(1R_i)+1D(1R_k)+\{1D(1P_j)+1D(1P_k)\}+\{1D(1R_k)\}+\{\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}\}$	5	6

TABLE 9. KINEMATIC ANALYSIS OF 3 TRANSLATION AND 1 ROTATION INPUT

Input type	Input-output conversion formula	IN-DOF	OUT-DOF
3 translation 1 rotation	$1D(3P_i)+1D(1R_j)\longrightarrow 1D(1P_i)+1D_j(1R_j)$	4	2
	$1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(1R_j)\longrightarrow 1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(1R_j)+\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}$	4	6
	$1D(2P_i)+1D(1P_k)+1D(1R_j)\longrightarrow 1D(1P_i)+1D(1P_k)+1D(1R_j)+\{1D(1R_j)\}$	4	3
	$1D(2P_i)+1D(1P_k)+1D(1R_i)\longrightarrow 1D(1P_i)+1D(1P_k)+1D(1R_i)+\{1D(1R_j)\}$	4	4
	$1D(2P_i)+1D(1P_k)+1D(1R_k)\longrightarrow 1D(1P_i)+1D(1P_k)+1D(1R_k)+\{1D(1R_j)\}$	4	4

TABLE 10. KINEMATIC ANALYSIS OF 3 TRANSLATION AND 2 ROTATION INPUT

Input type	Input-output conversion formula	IN-DOF	OUT-DOF
3 translation 2 rotation	$1D(3P_i)+1D(2R_j)\longrightarrow 1D(1P_i)+1D(1R_j)+\{1D(1P_i)+1D(1P_k)\}+\{\{1D(1R_j)\}\}$	5	3
	$1D(3P_i)+1D(2R_i)\longrightarrow 1D(1P_i)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_k)+1D(1R_j)\}\}$	5	6
	$1D(3P_j)+1D(1R_i)+1D(1R_k)\longrightarrow 1D(1P_j)+1D(1R_i)+1D(1R_k)$	5	3
	$1D(3P_i)+1D(1R_i)+1D(1R_k)\longrightarrow 1D(1P_i)+1D(1R_i)+1D(1R_k)$	5	3
	$1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(2R_j)\longrightarrow 1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(1R_j)+\{1D(1P_i)+1D(1P_k)\}+\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}+\{\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}\}$	5	6
	$1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(1R_j)+1D(1R_k)\longrightarrow 1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(1R_j)+1D(1R_k)+\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}$	5	6
	$1D(2P_i)+1D(1P_k)+1D(2R_j)\longrightarrow 1D(1P_i)+1D(1P_k)+1D(1R_j)+\{1D(1P_i)+1D(1P_k)\}+\{1D(1R_j)\}+\{\{1D(1R_j)\}\}$	5	3
	$1D(2P_i)+1D(1P_k)+1D(2R_i)\longrightarrow 1D(1P_i)+1D(1P_k)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{1D(1R_j)\}+\{\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}\}$	5	6
	$1D(2P_i)+1D(1P_k)+1D(1R_j)+1D(1R_k)\longrightarrow 1D(1P_i)+1D(1P_k)+1D(1R_j)+1D(1R_k)+\{1D(1R_j)\}$	5	4
	$1D(2P_i)+1D(1P_k)+1D(1R_i)+1D(1R_k)\longrightarrow 1D(1P_i)+1D(1P_k)+1D(1R_i)+1D(1R_k)+\{1D(1R_j)\}$	5	5

TABLE 11. KINEMATIC ANALYSIS OF 3 TRANSLATION AND 3 ROTATION INPUT

Input type	Input-output conversion formula	IN-DOF	OUT-DOF
3 translation 3 rotation	$1D(3P_i)+1D(3R_j) \rightarrow 1D(1P_i)+1D(1R_j)+\{1D(1P_i)+1D(1P_k)\}+\{\{1D(1R_j)\}\}$	6	3
	$1D(3P_i)+1D(3R_i) \rightarrow 1D(1P_i)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_k)+1D(1R_j)\}\}$	6	6
	$1D(3P_j)+1D(1R_i)+1D(1R_j)+1D(1R_k) \rightarrow 1D(1P_j)+1D(1R_i)+1D(1R_j)+1D(1R_k)$	6	4
	$1D(3P_j)+1D(2R_i)+1D(1R_k) \rightarrow 1D(1P_j)+1D(1R_i)+1D(1R_k)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_i)\}\}$	6	4
	$1D(3P_i)+1D(2R_i)+1D(1R_k) \rightarrow 1D(1P_i)+1D(1R_i)+1D(1R_k)+\{1D(1P_j)+1D(1P_k)\}+\{\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}\}$	6	6
	$1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(3R_j) \rightarrow 1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(1R_j)+\{1D(1P_i)+1D(1P_k)\}+\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}+\{\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}\}$	6	6
	$1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(1R_i)+1D(1R_j)+1D(1R_k) \rightarrow 1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(1R_i)+1D(1R_j)+1D(1R_k)+\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}$	6	6
	$1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(2R_j)+1D(1R_k) \rightarrow 1D(1P_i)+1D(1P_j)+1D(1P_k)+1D(1R_j)+1D(1R_k)+\{1D(1P_i)+1D(1P_k)\}+\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}+\{\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}\}$	6	6
	$1D(2P_i)+1D(1P_k)+1D(3R_j) \rightarrow 1D(1P_i)+1D(1P_k)+1D(1R_j)+\{1D(1P_i)+1D(1P_k)\}+\{1D(1R_j)\}+\{\{1D(1R_j)\}\}$	6	3
	$1D(2P_i)+1D(1P_k)+1D(3R_i) \rightarrow 1D(1P_i)+1D(1P_k)+1D(1R_i)+\{1D(1P_j)+1D(1P_k)\}+\{1D(1R_j)\}+\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}\}$	6	6
	$1D(2P_i)+1D(1P_k)+1D(1R_i)+1D_j(1R_j)+1D_k(1R_k) \rightarrow 1D(1P_i)+1D(1P_k)+1D(1R_i)+1D(1R_j)+1D(1R_k)+\{1D(1R_j)\}$	6	5
	$1D(2P_i)+1D(1P_k)+1D(2R_j)+1D(1R_k) \rightarrow 1D(1P_i)+1D(1P_k)+1D(1R_j)+1D(1R_k)+\{1D(1P_i)+1D(1P_k)\}+\{1D(1R_j)\}+\{\{1D(1R_j)\}\}$	6	3
	$1D(2P_i)+1D(1P_k)+1D(2R_i)+1D(1R_k) \rightarrow 1D(1P_i)+1D(1P_k)+1D(1R_i)+1D(1R_k)+\{1D(1P_j)+1D(1P_k)\}+\{1D(1R_j)\}+\{\{1D(1R_i)+1D(1R_j)+1D(1R_k)\}\}$	6	6

V. CASE ANALYSIS

Mechanism is a subject that studies the structure and motion of the mechanism in the machine, and it is the main branch of the mechanical principle. Studying the structure, motion and force of the mechanism of various mechanical and other common problems. The research content is divided into two aspects, the first is to study the existing mechanism, namely mechanism analysis (structure analysis, motion analysis and dynamic analysis). The second is design of new mechanism according to the requirements, namely the mechanism synthesis (structural synthesis, motion synthesis and dynamic synthesis). The mechanism analysis includes two parts, structure analysis and motion analysis. The former studies the composition of the mechanism and determines the possibility and certainty of its motion. In 1983, Professor Hunt pointed out in his paper that 5 degree of freedom of parallel mechanism which can work normally is impossible. In 2000, Merlet also mentioned in his monograph, it is impossible to construct a symmetrical parallel mechanism with the degree of freedom as 4 and 5. The parallel robot can be defined as the moving platform and the static platform which are connected by at least two independent kinematic chains, and the mechanism has 2 or more than 2 degree of freedom, and is driven by a parallel way.

Next, we will find the 4 and 5 degree of freedom parallel mechanism with the same branch chain, which makes up for the blank that parallel mechanism of 4 and 5 degree of freedom without the same branch chain, what is believed by Melert.

First of all, analysis process and conclusion of 5 degree of freedom mechanism given, and the mechanism diagram as shown in Fig.11.

$$1D(1P_y)+1D(1P_z)+1D(2R_z)- > 1D(1P_y)+1D(1P_z)+1D(1R_z)+ \{1D(1P_x)+1D(1P_y)\}+\{1D(1R_x)\},$$

Thus, we get the general formula of other similar mechanism of 5 degree of freedom.

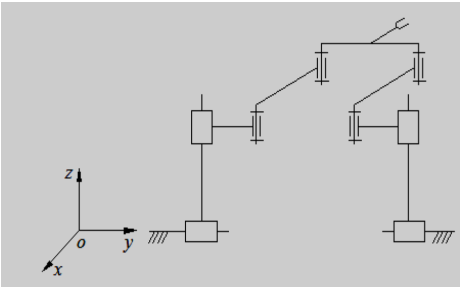
$$1D(1P_j)+1D(1P_k)+1D(2R_k)- > 1D(1P_j)+1D(1P_k)+1D(1R_k)+ \{1D(1P_i)+1D(1P_j)\}+\{1D(1R_i)\}$$

Secondly, the analysis process and conclusion of mechanism of 4 degree of freedom given, and the mechanism diagram as shown in Fig.12.

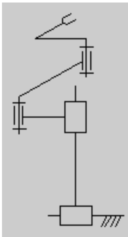
$$1D(1P_y)+1D(1P_z)+1D(1R_y)- > 1D(1P_y)+1D(1P_z) +1D(1R_y)+\{1D(1R_x)\}$$

Thus, we get the general formula of other similar mechanism of 4 degree of freedom.

$$1D(1P_j)+1D(1P_k)+1D(1R_j)- > 1D(1P_j)+1D(1P_k) +1D(1R_j)+\{1D(1R_i)\}.$$

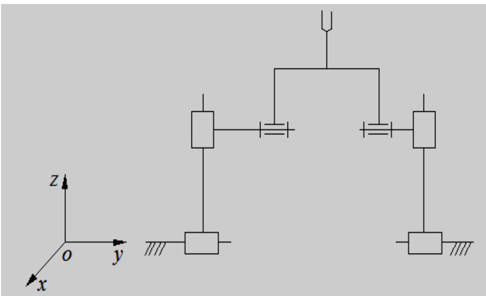


(a) Symmetric parallel mechanism of 5 degree of freedom.

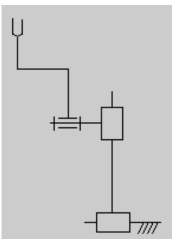


(b) Serial branch

Figure 11. Symmetric parallel mechanism of 5 degree of freedom and serial branch



(a) Symmetric parallel mechanism of 4 degree of freedom



(b) Serial branch

Figure 12. symmetric parallel mechanism of 5 degree of freedom and serial branch

During the past, people did not trust the universal formula to calculate the degree of freedom when developed new mechanisms, they had to use different methods for specific mechanisms or other complex computing methods. This cost too much time, and even after the entity model is manufactured, they found there is not the required degree of freedom. In order to get the necessary mechanism, it is impossible to succeed when there is no certain theoretical guidance. With the method proposed in this paper, we can synthesize the desired mechanism quickly.

VI. CONCLUSIONS

Based on the existing degree of freedom of mechanism calculation and analysis methods, its shortcomings found , from the perspectives of basic mathematical representation and basic physical motion to study the relationship and difference between input and output motion, conclusion can be drawn as following.

(1) The input degree of freedom and output degree of freedom are redefined respectively, the input degree of freedom(IN-DOF) is regard as the measuring method of the mechanism driving capability characteristics, the output degree of freedom(OUT-DOF) as the measuring method of the mechanism performance characteristics.

(2) Theory of main-vice motion introduced to find the general analysis and calculation method of the degree of freedom, and give three cases of serial branch, which include pure rotation input, pure translation input, translation and rotation input.

(3) From the perspective of the mathematical representation and relationship of mechanism motion elements, and by in-depth analysis, the existence and rule of the input first order, second order and third order associated motion are obtained.

(5) With the help of the research method and conclusion, we can find the parallel mechanism of 4 and 5 degree of freedom with the same branch chain, which is a new scientific theory and effective method for the analysis of the degree of freedom of mechanism and mechanism type synthesis.

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