Research on the Scale Test Method to Calculate the Attenuation Characteristics of Plane Shock Wave Pressure

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***Abstract* — Based on the similarity theory, this paper analyzed the influence parameters on plane shock wave pressure (PSWP), and proposed a scale test method to calculate the attenuation characteristics of PSWP of the original model. The scale model test and the original model test were simulated by using LS-DYNA code. Fitting the results of the scale model test, it obtained the dimensionless empirical model of attenuation characteristics of PSWP in oxygen-free copper within a certain scope of scale distance. Comparing the simulated values of PSWP of the original model with the values computed by the dimensionless empirical model at typical points, and the maximum relative errors of shock wave's peak pressure, pulse width and impulse are respectively less than 0.5%, 2% and 0.1%. The results are in good agreement with each other, which verifies the feasibility of this scale test method. This method provides a new idea to research on the attenuation characteristics of plane shock wave pressure in solid materials.**

***Keywords* - impact dynamics; scale test method; plane shock wave pressure; attenuation characteristics**

I. INTRODUCTION

PSWP has been widely used in explosive percussion [1], synthesis of materials of high energy density [2] and the research of dynamic characteristic [3]. For example, it is needed to know the attenuation characteristics of PSWP in solid materials to design the sample rooms to research the synthesis efficiency [4] of the high energy density materials, and the shock initiation criterion [5] of condensed explosive is established by the energy arriving in the surface between shell and the explosive. So, it is of great significance to research the attenuation characteristic of shock wave's peak pressure, pulse width and impulse in solid materials.

The research studied by Erkman et al [6] had shown that, before sparse wave, causing the attenuation of PSWP, didn't catch up with the shock wave, the main performance of PSWP decaying in the target board was the pulse width's attenuation. After the sparse wave overtook the shock wave, the peak vales of shock wave would decrease rapidly. So far, the relevant scholars have carried out extensive research studies on the attenuation characteristics of shock wave's peak pressure in different materials. Tang et al [7] studied the attenuation rule of peak pressure of shock wave in LY12-M aluminum. Cheng et al [8] used the light gas gun to drive flyers hitting target board at different speeds and obtained the attenuation models of peak pressure in aluminum foam material. Wang et al [9] used the flyer, which was 10mm in thickness, impacting LY12 aluminum target board at the speed of 670 m/s and studied the attenuation characteristic of shock wave's peak pressure in C30 concrete.

When the peak pressure of the PSWP was 10GPa, the sensors could be used to measure the pressure were only manganin gauge [10] and PVDF piezoelectric sensor[11]. However, the life of the two kinds of sensors under the pressure of dozens of GPa was short (about 2us), so these sensors couldn't fully measure the plane shock wave pressure-time history curve when the pulse width was a microsecond. As a result, it was impossible to get the attenuation characteristic of PSWP in solid materials, but the technology of scaled simulation [12] could realize the problem.

In this paper, based on the similarity theory, we analyzed the similar conditions and present a scale test method of attenuation characteristics of PSWP, and developed the numerical simulation tests of the scale model and the original model to verify the feasibility of this scale test method.

II. RESEARCH ON THE DIMENSIONLESS RELATIONSHIPS OF THE ATTENUATION CHARACTERISTIC OF PSWP

A. The Influenced Parameters on the Attenuation Characteristic of PSWP

When the flyer impacts the target board in parallel, the typical waveform of shock wave pressure-time history curve in the center of the impact surface is shown in figure 1. The three characteristic indexes of the plane shock wave intensity are: the peak value, the pulse width (the holding time of the peak pressure of the platform) and the impulse.

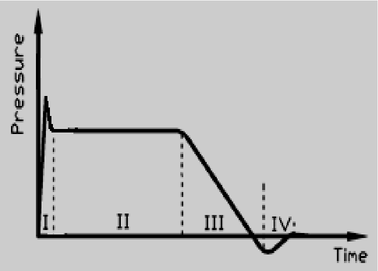


Fig .1The Typical Waveform of Pressure-time History Curve in the Impact Surface

Based on the establishment of PSWP, the parameters affecting the shock wave's peak pressure P_x, pulse width τ_x and impulse I_x in the propagation distance x were as follows:

1) Material parameters:

(a) Material parameters of flyer: density ρ_f, *Grüneisen* coefficient γ_f, the constants C_f and S_f in Hugoniot, elastic modulus E_f, Poisson ratio v_f, yield limit Y_f;

(b) Material parameters of target board: density ρ_t, *Grüneisen* coefficient γ_t, the constants C_t and S_t in Hugoniot, elastic modulus E_t, poisson ratio v_t, yield limit Y_t;

2) Structure parameters: flyer's thickness D_f, flyer's diameter H_f, velocity of flyer V_f, target board's thickness D_t and target board's diameter H_t.

B. Scale Test Method

Select length dimension *M*, quality dimension *L*, and time dimension *T* as the basic dimensions, the dimensions of these six parameters: γ_f、S_f、v_f、γ_t、S_t and v_t are $M^0L^0T^0$, and the other parameters' physical dimensions as shown in table I.

TABLE I. PHYSICAL DIMENSIONS OF THE PARAMETERS

Parameters	D_f	ρ_f	Y_t	H_f	V_f	C_f	E_f	Y_f	H_t	D_t	ρ_t	C_t	E_t	x	P_x	τ_x	I_x
M	0	1	1	0	0	0	1	1	0	0	1	0	1	0	1	0	1
L	1	-3	-1	1	1	1	-1	-1	1	1	-3	1	-1	1	-1	0	-1
T	0	0	-2	0	-1	-1	-2	-2	0	0	0	-1	-2	0	-2	1	-1

Based on the second similar law [12] , the functional relationships between the PSWP and all the affecting parameters can be expressed as follows:

$$\begin{cases} P_x = f\left(D_f,\rho_f,Y_t,H_f,V_f,C_f,E_f,Y_f,H_t,D_t,\rho_t,C_t,E_t,x,\gamma_f,\gamma_t,S_f,S_t,v_f,v_t\right) \\ \tau_x = g\left(D_f,\rho_f,Y_t,H_f,V_f,C_f,E_f,Y_f,H_t,D_t,\rho_t,C_t,E_t,x,\gamma_f,\gamma_t,S_f,S_t,v_f,v_t\right) \\ I_x = h\left(D_f,\rho_f,Y_t,H_f,V_f,C_f,E_f,Y_f,H_t,D_t,\rho_t,C_t,E_t,x,\gamma_f,\gamma_t,S_f,S_t,v_f,v_t\right) \end{cases} \tag{1}$$

In the physical process, D_f, ρ_f and Y_t are selected as the basic physical quantities, Equation (1) can be written into the dimensionless relationships as follows:

$$\begin{cases} \dfrac{P_x}{\rho_t V_f^2} = f\left(\dfrac{x}{D_f},\dfrac{H_f}{D_f},\dfrac{H_f}{H_t},\dfrac{H_t}{D_t},\dfrac{V_f^2\rho_f}{Y_t},\dfrac{V_f}{C_f},\dfrac{E_f}{Y_t},\dfrac{Y_f}{Y_t},\dfrac{\rho_f}{\rho_t},\dfrac{E_t}{Y_t},\dfrac{V_f}{C_t},\gamma_f,\gamma_t,S_f,S_t,v_f,v_t\right) \\ \dfrac{\tau_x V_f}{D_f} = g\left(\dfrac{x}{D_f},\dfrac{H_f}{D_f},\dfrac{H_f}{H_t},\dfrac{H_t}{D_t},\dfrac{V_f^2\rho_f}{Y_t},\dfrac{V_f}{C_f},\dfrac{E_f}{Y_t},\dfrac{Y_f}{Y_t},\dfrac{\rho_f}{\rho_t},\dfrac{E_t}{Y_t},\dfrac{V_f}{C_t},\gamma_f,\gamma_t,S_f,S_t,v_f,v_t\right) \\ \dfrac{I_x}{\rho_t D_f V_f} = h\left(\dfrac{x}{D_f},\dfrac{H_f}{D_f},\dfrac{H_f}{H_t},\dfrac{H_t}{D_t},\dfrac{V_f^2\rho_f}{Y_t},\dfrac{V_f}{C_f},\dfrac{E_f}{Y_t},\dfrac{Y_f}{Y_t},\dfrac{\rho_f}{\rho_t},\dfrac{E_t}{Y_t},\dfrac{V_f}{C_t},\gamma_f,\gamma_t,S_f,S_t,v_f,v_t\right) \end{cases} \tag{2}$$

For the scale model,when the flyer's material, the target board's material and the speed of flyer are the same as the original model, there are fifteen parameters are the same as the original model:

$$\left(V_f,\rho_f,C_f,E_f,Y_f,\gamma_f,S_f,v_f,\rho_t,C_t,E_t,Y_t,\gamma_t,S_t,v_t\right) = const \tag{3}$$

When $(H_f/D_f)_m = (H_f/D_f)_p$, $(H_f/H_t)_m = (H_f/H_t)_p$, $(H_t/D_t)_m = (H_t/D_t)_p$, the dimensionless relationships of attenuation characteristics of PSWP can be expressed as follows:

$$\begin{cases} \dfrac{P_{xm}}{\rho_{tm}V_{fm}^2} = f(\dfrac{x_m}{D_{fm}}) \\ \dfrac{\tau_{xm}V_{fm}}{D_{fm}} = g(\dfrac{x_m}{D_{fm}}) \\ \dfrac{I_{xm}}{\rho_{tm}D_{fm}V_{fm}} = h(\dfrac{x_m}{D_{fm}}) \end{cases} \tag{4}$$

The subscript *p* says original model, *m* says scale model.

Due to the flyer's material, target's board material, and the velocity of flyer of the scale model are the same as the original model, so the influences of ρ_{tm}, V_{fm} in Equation 4 on P_{xm}, τ_{xm} and I_{xm} remain the same. When the units of $x_m, D_{fm}, P_{xm}, \tau_{xm}$ and I_{xm} in Equation 4 are regulated, we can turn the Equation 3 into the magnitude relationships of these physical quantities as shown in Equation (5):

$$\begin{cases} P_{xm} = f(x_m/D_{fm}) \\ \tau_{xm}/D_{fm} = g(x_m/D_{fm}) \\ I_{xm}/D_{fm} = h(x_m/D_{fm}) \end{cases} \tag{5}$$

Finally, fitting the results of the scale model test can get the quantitative relationships as follows:

$$\begin{cases} P_{xm} = a\cdot(x_m/D_{fm})^{\alpha} & k_1 \le x_m/D_{fm} \le k_2 \\ \tau_{xm}/D_{fm} = b\cdot(x_m/D_{fm})^{\beta} & k_3 \le x_m/D_{fm} \le k_4 \\ I_{xm}/D_{fm} = c\cdot(x_m/D_{fm})^{\gamma} & k_5 \le x_m/D_{fm} \le k_6 \end{cases} \tag{6}$$

In Equation (6), *a*, *b*, *c*, α, β, γ, k_1, k_2, k_3, k_4, k_5 and k_6 are constants.

Thus, the dimensionless relationships of attenuation characteristic of PSWP of the original model can be expressed as follows:

$$\begin{cases} P_{xp} = a\cdot(x_p/D_{fp})^{\alpha} & k_1 \le x_p/D_{fp} \le k_2 \\ \tau_{xp}/D_{fp} = b\cdot(x_p/D_{fp})^{\beta} & k_3 \le x_p/D_{fp} \le k_4 \\ I_{xp}/D_{fp} = c\cdot(x_p/D_{fp})^{\gamma} & k_5 \le x_p/D_{fp} \le k_6 \end{cases} \tag{7}$$

To sum up, we should use the similar method to predict the attenuation characteristics of PSWP by using the small model of full geometry similarity, and the following four similar conditions must be satisfied between the scale model and the original model:

1) The materials of the flyer and target board between scale model and original model are the same;

2) Scale model and original model are geometrically similar in structure;

3) The speed of flyers between scale model and original model are the same;

4) The similar parameters related to the size of the flyer and target board are the same: $(H_f/D_f)_m = (H_f/D_f)_p$, $(H_f/H_t)_m = (H_f/H_t)_p$ and $(H_t/D_t)_m = (H_t/D_t)_p$.

III. SIMULATION EXAMPLES

In order to identify the feasibility of this scale test method, the numerical simulation method is applied.

A. *Computing Models and Algorithm*

In order to research the attenuation characteristics of PSWP, which's peak pressure is 10 GPa in the oxygen-free copper. The flyer's material and the target board's material are oxygen-free copper, and the flyer's velocity calculated by the theoretical model in reference [14] is 0.0518cm/us.

Based on ANSYS/LS-DYNA software, the original model can be established, and the relative parameters: the flyer's thickness is 3cm, the flyer's diameter is 90cm, the target board's thickness is 45 cm, the diameter of target board is 90 cm, and the speed of flyer is 0.0518 cm/us. When $H_{fp}/H_{fm} = D_{fp}/D_{fm} = H_{tp}/H_{tm} = 3$, the scale model can be established, and the relative parameters: the flyer's thickness is 1 cm, the diameter of flyer is 30 cm, the speed of flyer is 0.0518 cm/us, the diameter of target board is 30 cm, and the thickness of target board is 15 cm.

In order to reflect the dynamic unloading behavior in oxygen free copper [15], the flyer and target board adopt Johnson-Cook model and *Grüneisen* equation of state. The related parameters of the oxygen free copper are shown in table II and table III. We use solid 164 unit to divide mesh, and take Lagrange algorithms. The contact between flyer and target board uses the eroding-surface-to-surface algorithm. Symmetry constraints are imposed on the symmetry surface between the flyer and target board, while fixed constraints are imposed around the target board. Due to the symmetry of the structure and load of the flyer and target board, it only simulates 1/4 of the model to reduce the computing time. The original model is shown in figure 2.

TABLE II. PARAMETERS OF THE JOHSSON-COOK EQUATION OF QXYGEN FREE COPPER [8]

A(×10^{11}Pa)	B(×10^{11}Pa)	C	n	m	$\dot{\varepsilon}_0$ (us)
0.0009	0.00292	0.31	0.025	1.09	1e-6

TABLE III. PARAMETERS OF *Grüneisen* EQUATION OF STATE OF QXYGEN FREE COPPER

C_0(cm/us)	S_1	S_2	S_3	γ_0	A	E_0	V_0
0.394	1.49	0.0	0.0	2.02	0.47	0.0	1.0

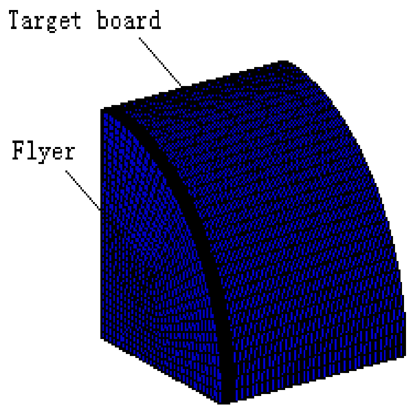


Fig .2 The Original Model

B. The Reliability Analysis of Simulation Models and Results

The pressure-time history curves in the center unit of each test's impact surface are shown in figure 3. The peak pressures of the platforms are 10GPa, and the comparison results of pulse width between the simulation values and the theoretical values calculated by the theoretical model in the reference [10] are shown in table IV.

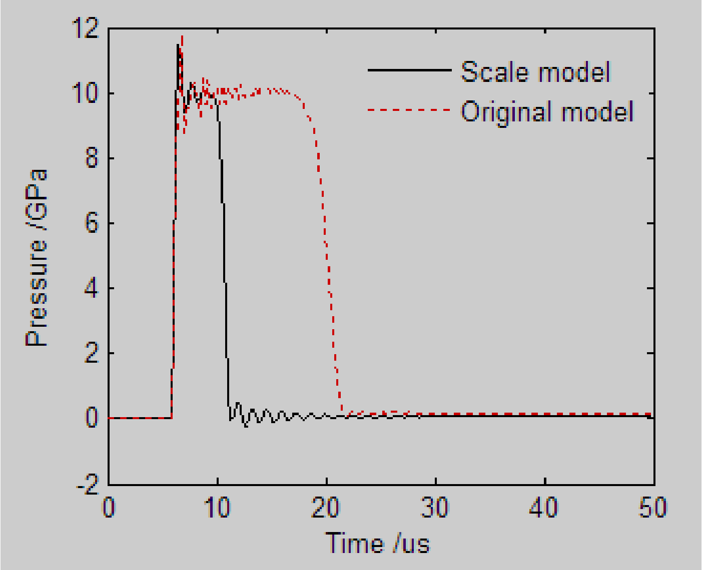


Fig .3. The Pressure-Time History Curves at the Center Unit of Each Test's Impact Surface

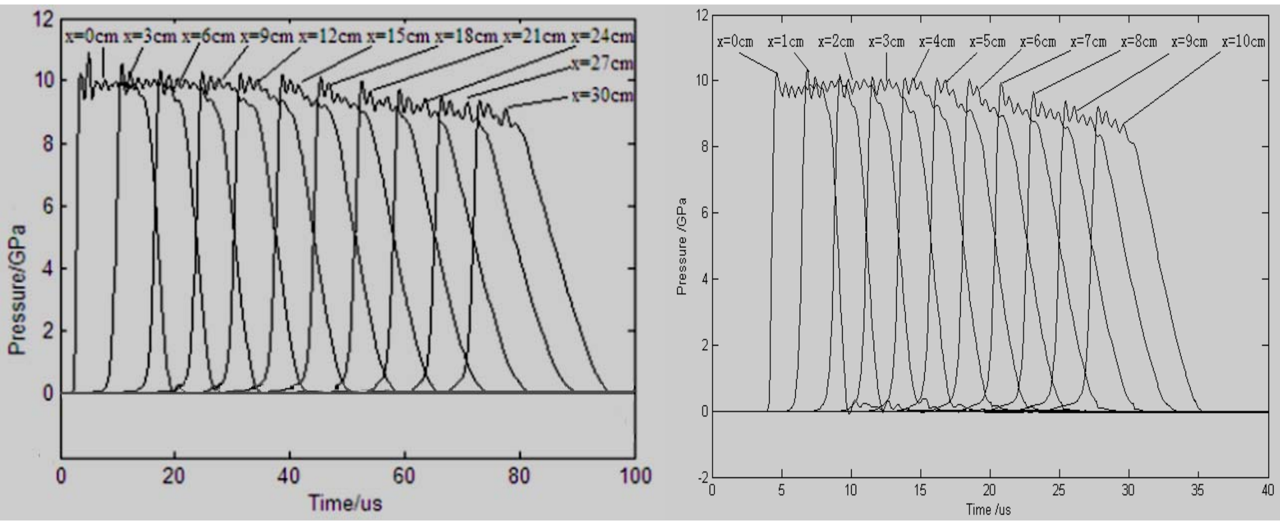
TABLE IV. THE COMPARISON RESULES OF PULSE WIDTH

	Theoretical values τ_0 (us)	**Simulation values τ_0' (us)**	$\left\|\frac{\tau_0-\tau_0'}{\tau_0}\right\|$ **(%)**
Scale model	3.62	3.54	2.21
Original model	10.87	10.52	3.22

From the figure 3 and table IV, the simulation results of the pulse width are in good agreement with the theoretical calculation. The relative errors of the pulse width are less than 4%, which indicates that the numerical simulation and simulation results are reliable. Therefore, the results obtained by numerical simulation can be used to further study the inherent law of plane shock wave propagation characteristics.

C. *Feasibility Analysis of the Scale Model Test*

LS-PREPOST software is used to extract the simulation results of the original model test and scale model test. The pressure-time history curves of the particles at the typical position of the axial direction of the center of the target plate are shown in figure 4.



(a) The Original Model Test

(b) The Scale Model Test

Fig.4 The Pressure-Time History Curves at the Typical Position of the Axial Direction of the Center of the Target Board

The pressure-time history curves of the typical points in figure 4 are analyzed and processed, set the pressure's unit as GPa, set the time's unit as us, set the distance's unit as cm, and set the impulse's unit as GPa.us. The shock wave's peak pressure P_{xm}, the scale pulse width τ_{xm}/D_{fm} and the scale impulse I_{xm}/D_{fm} of the scale model within a certain scale distance x_m/D_{fm} can be obtained as shown in table V. The shock wave's peak pressure P_{xp}, pulse width τ_{xp} and impulse I_{xp} of typical points of the original model can be obtained as shown in table VI.

TABLE V. SHOCK WAVE'S PEAK PRESSURE, SCALE PULSE WIDTH AND SCALE IMPULSE AT DIFFERENT TEST POINTS OF THE SCALE MODEL

	P_{xm} (GPa)				τ_{xm}/D_{fm} （us.cm^{-1})				I_{xm}/D_{fm} (GPa.us.cm^{-1})			
x_m/D_{fm}	7	8	9	10	1	2	3	4	1	2	3	4
	9.80	9.61	9.43	9.30	2.49	2.05	1.38	1.02	46.49	46.30	46.22	46.16

TABLE VI. SHOCL WAVE'S PEAK PRESSURE, PULSE WIDTH AND IMPULSE AT DIFFERENT TEST POINTS OF THE ORIGINAL MODEL

x_p/cm	22	25	29	4	7	10	4	7	10
	P_{xp} (GPa)			τ_{xp} (us)			I_{xp} (GPa.us)		
$D_{fp}=3cm$	9.74	9.58	9.38	6.80	4.82	3.71	139.33	138.86	138.51

Fitting the results of the scale model test can get the constants in Equation (6) as follows:

$$\begin{cases} a=13.0776, \alpha=-0.1483, & 7 \le x_m \le 10 \\ b=2.7104, \beta=-0.6345, & 1 \le x_m \le 4 \\ c=46.4836, \gamma=-0.0051, & 1 \le x_m \le 4 \end{cases} \tag{7}$$

So, based on the similar relationships between the scale model and the original model, the dimensionless relationships of attenuation characteristics of PSWP of the original model can be established as follows:

$$\begin{cases} P_{xp}=13.0776 \cdot (x_p/3)^{-0.1483}, & 7 \le x_p/3 \le 10 \\ \tau_{xp}/3=2.7104 \cdot (x_p/3)^{-0.6345}, & 1 \le x_p/3 \le 4 \\ I_{xp}/3=46.4836 \cdot (x_p/3)^{-0.0051}, & 1 \le x_p/3 \le 4 \end{cases} \tag{8}$$

The theoretical values of PSWP computed by the dimensionless relationships of the typical positions of the original model are shown in table VII.

TABLE VII THE THEORETICAL VALUES OF PSWP

x_p/cm	22	25	29	4	7	10	4	7	10
	P'_{xp} (GPa)			τ'_{xp} (us)			I'_{xp} (GPa.us)		
	9.73	9.55	9.34	6.78	4.74	3.78	139.24	138.85	138.59

The results obtained by comparing the simulated PSWP values of the typical positions with the values computed by the dimensionless relationships of the original model are shown in table VIII.

TABLE VIII THE COMPARISON RESULTS OF PSWP

x_p/cm	22	25	29	4	7	10	4	7	10
		$\frac{P_{xp}-P'_{xp}}{P_{xp}}$	(%)		$\frac{\tau_{xp}-\tau'_{xp}}{\tau_{xp}}$	(%)		$\frac{I_{xp}-I'_{xp}}{I_{xp}}$	(%)
	0.10	0.31	0.43	0.29	1.66	1.89	0.06	0.01	0.06

It can be seen from the table VIII, the relative errors of shock wave's peak pressure, pulse width and impulse are less than 0.5%, 2% and 0.1% respectively, which verifies the feasibility of this scale test method.

IV. CONCLUSION

In this paper, the influence parameters on the attenuation characteristics of PSWP in solid materials are analyzed by the second similar law, and a scale experimental method is put forward to predict the attenuation characteristic of PSWP was proposed.

The simulation examples indicate that the attenuation characteristics of PSWP of the scale model can predict the attenuation characteristics of PSWP of the original model in the oxygen-free copper, which verifies the feasibility of this scale experimental method.

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