# Unequal-Interval Multivariate New-Information Optimization MGM (1, n) Model for Material Property Analysis

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***Abstract* — Unequal-interval multivariate MGM (1, n) model for material properties analysis has defects including low precision and adaptability. Utilizing new-information theory and grey system modeling method, this paper proposes an unequal-interval multivariate new-information optimization MGM (1, n) model for material properties analysis. The m-th component of original data sequence is regarded as the initial condition of grey differential equation, initial correction as the design variable, and the mean relative error as objective function. Background value of unequal new-information MGM (1, n) model is studied. Based on exponential, integral characteristic of grey model, and the new-information theory, inhomogeneous exponential function was utilized to fit one-accumulation generating sequence, thus proposing reconstruction method for background value of new-information MGM (1, n) mode. The construction equation of background value is also treated here. With high precision and adaptability, the model is suitable for both equal-interval and unequal-interval modeling. Examples have verified the practicality and reliability of the model.**

***Keywords - material property analysis; multivariate; background value; new-information; optimization; unequal-interval sequence***

## I. INTRODUCTION

Material property analysis involves a variety of factors, which makes accurate modeling difficult. Thus, it has important theoretical and practical value to use information technology to effectively excavate internal information of system and establish an accurate model of material analysis. Grey system theory is that system with all known system information is regarded as the white system, system with all unknown information as black system, and systems in between as grey system. Material property has a variety of factors, so it can be studied by the theory and method of grey system which have both known and unknown information. Grey model is an important part of grey system theory. It has diversified type mainly including GM (1, 1), GM (1, N), MGM (1, N) [1-3], GOM (1, 1) [4] and GRM (1, 1) [5]. In social, economic and engineering systems, there are often multiple variables with inherent links between each others. MGM (1, N) model is to extend GM (1, 1) model in the case of n variables, but it is not a simple combination of GM (1, 1) models. Compared with GM (1, n) model which only needs to establish a single *n*-element first-order differential equation, MGM (1, N) model establishes *n* *n*-element differential equations. By simultaneous solution, parameters in MGM model can reflect the mutual influence and restraint among multiple variables [6]. In Literature [2], the first component of sequence $\mathbf{X}^{(1)}$ was regarded as the initial condition of grey differential equation, establishing optimized MGM (1, N) model after correction. In Literature [7], based on new information priority principle in grey system theory, multivariate MGM (1, N) model was established regarding the *n*-th component of $\mathbf{X}^{(1)}$ as the initial condition of grey differential equation. In Literature [8], with initial condition of the *n*-th component, multivariable new-information MGM (1, N) model was established through optimization and correction of initial values and background value coefficient q. The background values were in the form of $z_i^{(1)} = qx_i^{(1)}(k+1) + (1-q)x_i^{(1)}(k)$ where $q \in [0,1]$. However, these MGM (1, N) models were all equal-interval model. In Literature [9], background values after homogeneous exponential function fitting were utilized to establish an unequal-interval multivariate MGM (1, N) model. The model had some inherent defects in modeling mechanism because homogeneous exponential function was more universal. In Literature [10], background values of the established multivariate unequal-interval MGM (1, N) model were obtained through average generation, so the precision of the model should be further improved. Multivariate unequal-interval GM (1, N) model in Literature [11] was established utilizing background value fitted with non-homogeneous exponential function, improving the precision of the model. Literature [12] analyzed the construction method of background value in multivariate grey model MGM (1, m). Utilizing vector continued fraction theory, it proposed background value reconstructed with trapezoidal equation of rational interpolation and numerical integration as well as extrapolation, effectively improving accuracy of simulation and predictive. Like most grey system models, the model was multivariate unequal-interval MGM (1, m) based on unequal-interval sequence. However, the original data obtained in practical work are often unequal-interval sequences, so it has certain practical and theoretical significance to establish multivariate MGM (1, m) model of unequal-interval sequences. This work adopted the modeling ideas of Literature [13], utilizing modeling method of new-information unequal-interval GM (1, 1) to study the background value construction of new-information unequal-interval MGM (1, n) model. Based on new-information principle, we proposed reconstruction method of background value in unequal-interval new-information MGM (1, n) model, as well as the equation of background value. The *m*-th component of original data sequence was regarded as an initial condition of grey differential equations; the initial correction as design variable; the mean relative error as objective function. Thus, multivariate unequal-interval grey MGM (1, n) model of new-information optimization was established for analysis of material properties. The optimization function *fmincon* of Matlab was written. If $n=1$, then unequal-interval new-information optimization MGM (1, n) model will degenerate into GM (1, 1) model; if $B=0$, then unequal-interval new-information optimization MGM (1, n) model is the combination of *n* unequal-interval new-information optimization GM (1, n) models. Unequal-interval new-information optimization MGM (1, n) model can be utilized for modeling, prediction, fitting and processing of data. According to the value of *n* in practical circumstances, we can obtain the necessary unequal-interval new-information models: MGM (1,2), MGM (1,3), MGM (1,4), etc. The model with high precision and adaptability is not only suitable for equidistant modeling, but also for non-equidistant modeling. Examples show that the model is practical and reliable, and it is worth widely applying in engineering and other related fields.

## II. GREY MGM (1, N) MODEL OF UNEQUAL-INTERVAL MULTIVARIATE NEW-INFORMATION OPTIMIZATION

Definition 1: for sequence $\mathbf{X}_i^{(0)}=[x_i^{(0)}(t_1),x_i^{(0)}(t_2),\cdots,x_i^{(0)}(t_j),\cdots,x_i^{(0)}(t_m)]$, if $\Delta t_j=t_j-t_{j-1}\neq const$, where $i=1,2,\cdots,n$ and $j=2,\cdots,m$, then $\mathbf{X}_i^{(0)}$ is called as unequal-interval sequence, where $n$ is the number of variables, and $m$ the number of sequences for each variable.

Definition 2: for sequence $\mathbf{X}_i^{(1)}=\{x_i^{(1)}(t_1),x_i^{(1)}(t_2),\cdots,x_i^{(1)}(t_j),\cdots,x_{t_j}^{(1)}(t_m)\}$, if $x_i^{(1)}(t_1)=x_i^{(0)}(t_1)$, and $x_i^{(1)}(t_j)=x_i^{(1)}(t_{j-1})+x_i^{(0)}(t_j)\cdot\Delta t_j$, where $i=1,2,\cdots,n$, $j=2,\cdots,m$ and $\Delta t_j=t_j-t_{j-1}$, then $\mathbf{X}_i^{(1)}$ is called as one-accumulation generating sequence (1-AGO) of unequal-interval sequence $\mathbf{X}_i^{(0)}$.

Definition 3: Supposed the non-negative sequence $X_i^{(0)}=[x_i^{(0)}(t_1),\cdots,x_i^{(0)}(t_j),\cdots,x_i^{(0)}(t_m)]$, if $\Delta t_i=t_i-t_{i-1}\neq const$ where $i=2,\cdots,m$, $m$ is the sequence number of each variable, $X_i^{(r)}=[x_i^{(r)}(t_1)\ \ x_i^{(r)}(t_2)\ \cdots\ x_i^{(r)}(t_m)]$ is called as r-order accumulated generation of $X_i^{(0)}$.

According to the principle of matrix operations, it is obtained that $x_i^{(r)}=A_1x_i^{(r-1)}=A_1A_1x_i^{(r-2)}=\cdots=A_1^{\mathbf{r}}x_i^{(0)}$ where $A_1$ is first-order accumulated generation matrix and $A_1^r$ is called as the accumulated generation matrix of $r$.

The original multivariate data matrix is set as follows.

$$\mathbf{X}^{(0)}=\{\mathbf{X}_1^{(0)},\mathbf{X}_2^{(0)},\cdots,\mathbf{X}_n^{(0)}\}^{\mathbf{T}}=\begin{bmatrix}x_1^{(0)}(t_1) & x_1^{(0)}(t_2) & \cdots & x_1^{(0)}(t_m)\\ x_2^{(0)}(t_1) & x_2^{(0)}(t_2) & \cdots & x_2^{(0)}(t_m)\\ \cdots & \cdots & \cdots & \cdots\\ x_n^{(0)}(t_1) & x_n^{(0)}(t_2) & \cdots & x_n^{(0)}(t_m)\end{bmatrix}\tag{1}$$

The observed values of $\mathbf{X}^{(0)}(t_j)$ where $j=1,2,\cdots m$ at $t_j$ are $\mathbf{X}^{(0)}(t_j)=[x_1^{(0)}(t_j),x_2^{(0)}(t_j),\cdots,x_n^{(0)}(t_j)]$; $[x_i^{(0)}(t_1),x_i^{(0)}(t_2),\cdots,x_i^{(0)}(t_j),\cdots,x_i^{(0)}(t_m)]$ is an unequal-interval sequence because the interval $t_j-t_{j-1}$ is not a constant.

To establish the model, one-accumulation is conducted on the original data, thereby generating a new matrix.

$$\mathbf{X}^{(1)}=\{\mathbf{X}_1^{(1)},\mathbf{X}_2^{(1)},\cdots,\mathbf{X}_n^{(1)}\}^{\mathbf{T}}=\begin{bmatrix}x_1^{(1)}(t_1) & x_1^{(1)}(t_2) & \cdots & x_1^{(1)}(t_m)\\ x_2^{(1)}(t_1) & x_2^{(1)}(t_2) & \cdots & x_2^{(1)}(t_m)\\ \cdots & \cdots & \cdots & \cdots\\ x_n^{(1)}(t_1) & x_n^{(1)}(t_2) & \cdots & x_n^{(1)}(t_m)\end{bmatrix}\tag{2}$$

where $x_i^{(1)}(t_j)$ meets Definition 2.

$$x_i^{(1)}(t_j)=\begin{cases}\sum_{j=1}^{k}x_i^{(0)}(t_j)(t_j-t_{j-1}) & (k=2,\cdots,m)\\ x_i^{(0)}(t_1) & (k=1)\end{cases}\tag{3}$$

Unequal-interval multivariate MGM (1, n) model is a first-order cubic differential equation set.

$$\begin{cases} \frac{dx_1^{(1)}}{dt} = a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \cdots + a_{1n}x_n^{(1)} + b_1 \\ \frac{dx_2^{(1)}}{dt} = a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + \cdots + a_{2n}x_n^{(1)} + b_2 \\ \cdots \\ \frac{dx_n^{(1)}}{dt} = a_{n1}x_1^{(1)} + a_{n2}x_2^{(1)} + \cdots + a_{nn}x_n^{(1)} + b_n \end{cases} \tag{4}$$

Denoting $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$, and $\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$, Eq. (4) can be expressed as:

$$\frac{dX^{(1)}(t)}{dt} = AX^{(1)}(t) + B. \tag{5}$$

According to new information priority theory of grey system, the first component of sequence $\mathbf{x}_i^{(1)}(t_j)$ was regarded as initial condition of grey differential equations, which would result in insufficient use of the new information. Therefore, we regarded the *m*-th component of $\mathbf{x}_i^{(1)}(t_j)$ as the initial condition, fully using the latest information. Continuous time response of Eq.(5) is

$$\mathbf{X}^{(1)}(t) = e^{At}\mathbf{X}^{(1)}(t_m) + \mathbf{A}^{-1}(e^{\mathbf{A}t} - \mathbf{I})\mathbf{B}. \tag{6}$$

where $e^{\mathbf{A}t} = \mathbf{I} + \sum_{k=1}^{\infty} \frac{\mathbf{A}^k}{k!} t^k$, and $\mathbf{I}$ is a unit matrix.

In order to identify $\mathbf{A}$ and $\mathbf{B}$, two-side integrals of Formula (4) are obtained at $[t_{j-1}, t_j]$.

$$x_i^{(0)}(t_j)\Delta t_j = \sum_{l=1}^{n} a_{il} \int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt + b_i \Delta t_j \tag{7}$$

where $i = 1,2,\cdots,n$ and $j = 2,3,\cdots,m$.

We denote $z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt$. Traditionally, trapezoidal area $z_i^{(1)}(t_j)\Delta t_j$ was utilized for calculation of background value, so when time interval is small, constructed background value is appropriate due to the gentle change of sequence data. When the sequence changes drastically, constructed background value will cause large errors to the model. If background value of new-information model is constructed according to traditional methods, the obtained model parameter will be the same with that of the non new-information model, which is unreasonable. Regarding $z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt$ as the background value of $x_i^{(1)}(t_j)$ ($[t_{j-1}, t_j]$), the estimated parameter matrix $\hat{\mathbf{A}}$ and parameter $\hat{\mathbf{B}}$ are more suitable for Whitening Eq.(4). According to quasi-exponential principles of grey prediction model and unequal-interval GM(1,1) modeling method [17], we set $x_i^{(1)}(t) = A_i e^{B_i t} + C_i$, where $A_i, B_i, C_i$ are undetermined coefficients. Denote $x_i^{(1)}(t) = G_i e^{a_i(t-t_m)} + C_i$, where $a_i$, $G_i$ and $C_i$ are undetermined coefficients, and $x_i^{(1)}(t_j) = G_i e^{a_i(t-t_m)} + C_i$. When the data are known, these coefficients can be determined the grey modeling method.

$x_i^{(1)}(t_j)$ is accumulatively reduced as:

$$\begin{aligned} x_i^{(0)}(t_j) &= \frac{x_i^{(1)}(t_j) - x_i^{(1)}(t_{j-1})}{\Delta t_j} \\ &= \frac{G_i(1-e^{a_i \Delta t_j})}{\Delta t_j} e^{-a_i(t_j - t_m)} \\ &= g_i e^{-a_i(t_j - t_m)} \end{aligned} \tag{8}$$

where $g_i = \frac{G_i(1-e^{a\Delta t_j})}{\Delta t_j} = \frac{G_i(1-(1+(a_i\Delta t_j)+\frac{(a_i\Delta t_j)^2}{2!}+\cdots)}{\Delta t_j}$.

When $a_i$ and $\Delta t_j$ are small, take the first two expansion terms of $e^{a_i \Delta t_j}$.

$$g_i = \frac{G_i(1-e^{a_i\Delta t_j})}{\Delta t_j} = \frac{G_i(-a_i\Delta t_j)}{\Delta t_j} = -G_i a_i ,$$

$$\frac{x_i^{(0)}(t_j)}{x_i^{(0)}(t_{j-1})} = \frac{e^{-a_i(t_j - t_m)}}{e^{-a_i(t_{j-1} - t_m)}} = e^{-a_i \Delta t_j}.$$

Then

$$a_i = \frac{\ln x_i^{(0)}(t_j) - \ln x_i^{(0)}(t_{j-1})}{\Delta t_j} (j = 2,3,\cdots,m). \tag{9}$$

where $j = 2,3,\cdots,m$.

Substitute Eq. (9) into Eq. (8), and

$$\begin{cases} g_i = \dfrac{x_i^{(0)}(t_j)}{e^{-a_i(t_j-t_m)}} = \dfrac{x_i^{(0)}(t_j)}{[x_i^{(0)}(t_j)/x_i^{(0)(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}} \\ G_i = \dfrac{x_i^{(0)}(t_j)\Delta t_j[x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}}{1-\dfrac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_j)}} \end{cases} \quad (10)$$

Solving through initial value condition $x_i^{(1)}(t_m)=G_ie^{a_i(t_m-t_m)}+C_i=G_i+C_i$, we can obtain

$$C_i = x_i^{(0)}(t_m)-G_i$$

$$= x_i^{(0)}(t_m)-\frac{x_i^{(0)}(t_j)\Delta t_j[x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}}{1-\dfrac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_{j-1})}} \quad (11)$$

Eq.(9) and Eq.(11) are substituted into the calculation equation of background value $\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt$, and

$$\begin{aligned} z_i^{(1)}(t_j) &= \int_{t_{j-1}}^{t_j} x_i^{(1)}dt \\ &= -\frac{\Delta t_j x_i^{(0)}(t_j)}{a_i}+C_i\Delta t_j \\ &= \frac{(\Delta t_j)^2 x_i^{(0)}(t_j)}{\ln x_i^{(0)}(t_j)-\ln x_i^{(0)}(t_j)}+x_i^{(0)}(t_m)\Delta t_j \\ &\quad -\frac{x_i^{(0)}(t_m)(\Delta t_j)^2[x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}}{1-\dfrac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_j)}} \end{aligned} . \quad (12)$$

Denoting $\mathbf{a}_i = (a_{i1}, a_{i2}, \cdots, a_{in}, b_i)^{\mathbf{T}}$, the value of $a_i$ is obtained as $\hat{\mathbf{a}}_i$ by the least square method.

$$\hat{\mathbf{a}}_i = [\hat{a}_{i1}, \hat{a}_{i2}, \cdots, \hat{a}_{in}, \hat{b}_i]^{\mathbf{T}} = (\mathbf{L}^T\mathbf{L})^{-1}\mathbf{L}^T\mathbf{Y}_i \quad (13)$$

where

$$\mathbf{L} = \begin{bmatrix} z_1^{(1)}(t_2) & z_2^{(1)}(t_2) & \cdots & z_n^{(1)}(t_2) & \Delta t_2 \\ z_1^{(1)}(t_3) & z_2^{(1)}(t_3) & \cdots & z_n^{(1)}(t_3) & \Delta t_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ z_1^{(1)}(t_m) & z_2^{(1)}(t_m) & \cdots & z_n^{(1)}(t_m) & \Delta t_m \end{bmatrix} \quad (14)$$

and

$$\mathbf{Y}_i = [x_i^{(0)}(t_2)\Delta t_2, x_i^{(0)}(t_3)\Delta t_3, \cdots, x_i^{(0)}(t_m)\Delta t_m]^{\mathbf{T}} \quad (15)$$

Then discrimination values of $\mathbf{A}$ and $\mathbf{B}$ can be obtained.

$$\hat{\mathbf{A}} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \cdots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \cdots & \hat{a}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \hat{a}_{n1} & \hat{a}_{n2} & \cdots & \hat{a}_{nn} \end{bmatrix}, \quad \hat{\mathbf{B}} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \cdots \\ \hat{b}_n \end{bmatrix} \quad (16)$$

The calculation value of new-information MGM(1,n) model is:

$$\hat{\mathbf{X}}_j^{(1)}(t_i) = e^{\hat{\mathbf{A}}(t_j-t_m)}\mathbf{X}^{(1)}(t_m) + \hat{\mathbf{A}}^{-1}(e^{\hat{\mathbf{A}}(t_j-t_m)}-\mathbf{I})\hat{\mathbf{B}} \quad (17)$$

The *m*-th component of original data sequence is utilized as the initial condition of grey differential equation for correction. $X_i^{(0)}(t_m)$ is replaced by $X_i^{(0)}(t_m)+\beta_i$, where $\beta$ is the vector with the same dimension number of variables, namely $\beta = [\beta_1, \beta_2, \cdots, \beta_n]^T$. The fitting value of reduced original data is:

$$\hat{\mathbf{X}}_i^{(0)}(t_j) = \begin{cases} \lim\limits_{\Delta t \to 0} \dfrac{\mathbf{X}_i^{(1)}(t_1) - \mathbf{X}_i^{(1)}(t-\Delta t)}{\Delta t}, & j=1 \\ (\hat{\mathbf{X}}_i^{(1)}(t_j) - \hat{\mathbf{X}}_i^{(1)}(t_{j-1}))/(t_j - t_{j-1}), & j = 2,3,\cdots,m \end{cases} \quad (18)$$

Absolute error of $ith$ variable is definite as $\hat{x}_i^{(0)}(t_j) - x_i^{(0)}(t_j)$; relative error (%) of $ith$ variable as $e_i(t_j) = \frac{\hat{x}_i^{(0)}(t_j) - x_i^{(0)}(t_j)}{x_i^{(0)}(t_j)} * 100$; the average error of $ith$ variable as $\frac{1}{m}\sum_{j=1}^{m}\left|e_i(t_j)\right|$.

The average error of all the data is:

$$f = \frac{1}{nm}\sum_{i=1}^{n}(\sum_{j=1}^{m}\left|e_i(t_j)\right|) \quad (19)$$

Average error $f$ is regarded as the objective function; $\beta$ as the design variable, solving through the optimization function *fmincon* of Matlab2014a or other optimization methods.

If $n = 1$, then unequal-interval new-information optimization MGM (1, n) model will degenerate into GM (1, 1) model; if $B = 0$, then unequal-interval new-information optimization MGM (1, n) model is the combination of *n* unequal-interval new-information optimization GM (1, n) models. Unequal-interval new-information optimization MGM (1, n) model can be utilized for modeling, prediction, fitting and processing of data. According to the value of *n* in practical circumstances, we can obtain the necessary unequal-interval new-information models: MGM (1,2), MGM (1,3), MGM (1,4), etc.

## III. APPLICATION EXAMPLES OF MODEL

Example 1: Based on the influence data of absorption on the mechanical properties of pure PA66 in Reference [10], mechanical property tests were conducted on PA66 samples with different water absorption rates. Along with changing water absorption rates, changes of PA66 bending strength, flexural modulus and tensile strength experimental data were obtained. $X_1^{(0)}$ is the bending strength (MPa); $X_2^{(0)}$ the bending elastic modulus (GPa); $X_3^{(0)}$ the tensile strength (MPa). The data are shown in Table 1 and Table 2.

TABLE 1 THE AFFECTING DATA OF WATER ABSORPTION TO MECHANICAL PROPERTIES OF PA66 SAMPLES FROM NO.1 TO NO.5

| No. | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Water Absorption $t_j$ / % | 0 | 0.0607 | 0.1071 | 0.1662 | 0.2069 |
| $X_1^{(0)}$ | 83.4 | 84.9 | 84.5 | 84.2 | 84.4 |
| $X_2^{(0)}$ | 2.63 | 2.64 | 2.61 | 2.65 | 2.66 |
| $X_3^{(0)}$ | 84.2 | 84.4 | 86.3 | 84.3 | 81.3 |

TABLE2 THE AFFECTING DATA OF WATER ABSORPTION TO MECHANICAL PROPERTIES OF PA66 SAMPLES FROM NO.6 TO NO.9

| No. | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| Water Absorption $t_j$ / % | 0.4344 | 0.5243 | 0.8524 | 0.9756 |
| $X_1^{(0)}$ | 78.4 | 75.4 | 59.5 | 54.1 |
| $X_2^{(0)}$ | 2.52 | 2.32 | 1.90 | 1.72 |
| $X_3^{(0)}$ | 74.9 | 75.7 | 73.2 | 66.9 |

An unequal-interval new-information optimization MGM (1, 3) was established according to the proposed method, with model parameters as follows:

$$\mathbf{A} = \begin{vmatrix} 0.0468 & -15.6730 & -0.0620 \\ 0.0069 & -0.6888 & -0.0011 \\ 0.0954 & -0.7070 & -0.2612 \end{vmatrix}, \quad \mathbf{B} = \begin{vmatrix} 80.5759 \\ 2.5492 \\ 81.7480 \end{vmatrix},$$

$$\beta = \begin{vmatrix} -1.9699 \\ -0.0098714 \\ -5.648 \end{vmatrix}.$$

Fitting value of $X_3^{(0)}$ is

$\hat{X}_3^{(0)}$ = [84.2, 83.7215, 82.8874, 82.0685, 81.2964, 79.2375, 76.824, 73.7069, 70.4031]

Absolute error of $X_3^{(0)}$ is

$q$ = [0, -0.67847, -3.4126, -2.2315, -0.0036492, 4.3375, 1.124, 0.50693, 3.5031]

Relative error of $X_3^{(0)}$ (%) is

$e$ = [0, -0.80387, -3.9543, -2.6471, -0.0044885, 5.791, 1.4848, 0.69252, 5.2363]

The mean relative error of $X_3^{(0)}$ is: 2.2905%; mean relative error of the model is 3.2854%, which proves high accuracy of the model. For the information model without optimization, the mean relative error of $X_3^{(0)}$ is 2.8387%; the mean relative error of model is 3.5258%. The maximum relative error of $X_3^{(0)}$ is 5.2363%, which is smaller than that of Literature [10] (-6.1048%).

Example 2: 600N-loaded **TiN** film was tested under relative sliding velocity of 0.314m / s, 0.417m / s, 0.628m / s, 0.942m / s, 1.046m / s, respectively. Table 3 shows the data of film friction test.

TABLE 3 DATA OF FILM FRICTION TEST

| NO ( $j$ ) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Sliding velocity (m/s) | 0.314 | 0.471 | 0.628 | 0.942 | 1.046 |
| Friction Coefficient ( $\mu$ ) | 0.251 | 0.258 | 0.265 | 0.273 | 0.288 |
| Wear Rate ( $\omega$, $\times 10^{-5} mg/m$) | 7.5 | 8 | 8.5 | 9.5 | 11 |

Sliding velocity was denoted by $t_j$ ; friction coefficient by $X_1^{(0)}$ ; wear rate by $X_2^{(0)}$ . Then an unequal-interval MGM (1, 2) model was established according to the proposed method, and the parameters were as follows.

$$\mathbf{A} = \begin{vmatrix} -1.1618 & 0.0407 \\ -101.2603 & 3.6243 \end{vmatrix}, \quad \mathbf{B} = \begin{vmatrix} 0.1817 \\ 0.8559 \end{vmatrix}, \quad \beta = \begin{vmatrix} -0.049212 \\ -1.3718 \end{vmatrix}$$

Fitting value of $X_1^{(0)}$ was:

$\hat{X}_1^{(0)}$ = [0.25676, 0.258, 0.26169, 0.27276, 0.2897]

Absolute error of $X_1^{(0)}$ was:

$q$ = [0.0057598, 1.802e-006, -0.0033124, -0.00023628, 0.0016993]

Relative error of $X_1^{(0)}$ was:

$e$ = [2.2948, 0.00069845, -1.25, -0.08655, 0.59004]

The mean relative error was 0.8444%, and the mean relative error of the model was 1.3384%. But for the information model without optimization, the mean relative error was 0.98783%, and mean relative error of the model was 1.4981%, which proved that the model had high precision. Literature [14] utilized equal-interval MGM (1, 3) model, with the mean relative error of 1.6225%.

Example 3: On CA6140 common lathe with YT14 carbide cutting tools outside circle, guaranteeing the tool geometric parameters and cutting speed is constant, only changing the cutting depth, the cutting force measured is shown in Table 4 [15].

TABLE 4 CUTTING EXPERIMENT DATA WHEN F=0.02MM/R

| No. | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $a_p$ / mm | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |
| $F_{1z}$ / N | 838.98 | 1060.45 | 1261.79 | 1483.25 | 1704.72 |
| $F_{1y}$ / N | 255.10 | 290.16 | 355.22 | 420.28 | 469.08 |

Cutting depth $a_p$ was denoted by $t_j$, the main cutting force $F_{1z}$ by $X_1^{(0)}$ and the main cutting force $F_{1y}$ by $X_2^{(0)}$. Then an unequal-interval MGM (1, 2) model was established according to the proposed method, and the parameters were as follows.

According to the present paper method to establish unequal-interval MGM (1, 2) model, where $t_j$ is the cutting depth $a_p$, $X_1^{(0)}$ is the main cutting force $F_{1z}$ and $X_2^{(0)}$ is the main cutting force $F_{1y}$. The parameters of the model are as follows

$$A=\begin{vmatrix} -1.0015 & 5.4693 \\ -0.2286 & 1.3132 \end{vmatrix},\ B=\begin{vmatrix} 878.3494 \\ 274.7248 \end{vmatrix},\ \beta=\begin{vmatrix} 17.791699 \\ -8050.3258 \end{vmatrix}$$

Fitting value of $X_1^{(0)}$ was:

$\hat{F}_{1z}$ = [824.13648, 990.92847, 1282.3852, 1491.5916, 1704.6253]

Absolute error of $X_1^{(0)}$ was:

$q$ = [-14.8435,-69.5215, 20.5952, 8.34157, -0.0947043]

Relative error of $X_1^{(0)}$ was:

$e$ = [-1.7692,-6.5559, 1.6322, 0.56238,-0.0055554]

The average relative error of $F_{1z}$ is 2.105% and the average relative error of the model is 2.1744%, which proved that the model had high precision.

## IV. CONCLUSION

(1) There is higher bias on fitting approximation non-homogenous series for building the unequal-interval multivariate MGM (1, n) model for material properties analysis. In fact, there is a lot of approximation non-homogenous series.

(2) For multivariable unequal-interval sequence with mutual influence among variables, we proposed the method of constructing background value of new-information unequal-interval MGM (1, n) model based on the new-information principle. According to exponential and integral characteristics of grey model and the new information theory, homogeneous exponential function was utilized to fit one-accumulation generating sequence. Besides, the construction equation of background value was also given. Applying new information theory and grey system modeling method, the *m*-th component of original data sequence was regarded as initial condition of grey differential equation; initial correction as the design variable; the mean relative error as objective function.

(3) In this paper, if $n=1$, the unequal-interval new-information optimization MGM (1, n) model will degenerate into GM (1, 1) model; if $B=0$, then unequal-interval new-information optimization MGM (1, n) model is the combination of *n* unequal-interval new-information optimization GM (1, n) models. Unequal-interval new-information optimization MGM (1, n) model can be utilized for modeling, prediction, fitting and processing of data. According to the value of *n* in practical circumstances, we can obtain the necessary unequal-interval new-information models: MGM (1,2), MGM (1,3), MGM (1,4), etc.

(4) The model with high precision and adaptability is not only suitable for equidistant modeling, but also for non-equidistant modeling. Examples show that the model is practical and reliable, and it is worth widely applying in engineering and other related fields.

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## REFERENCES

[1] S.F. Liu, Y.G. Dang, Z.G. Fang, et al., "Grey Systems and Application (Edition 3)", China Science Press, China, 2014.

[2] Y.X. Luo and J.Y. Li, "Application of Multi-variable Optimizing Grey Model MGM (1,n,q,r) to the Load-strain Relation", The 2009 IEEE International Conference on Mechatronics and Automation (ICMA 2009). Changchun, China, pp.4023-4027, 2009.

[3] Y.X. Luo, X. Wu, M. Li et al., "Grey dynamic model GM(1,N) for the Relationship of Cost and Variability", Kybernetes, vol. 38, pp. 435-440, 2009.

[4] Z.M. Song and J.L. Deng, "Accumulated Generating Operation in Opposite Direction and Its Use in Grey Model GOM(1,1)", Systems Engineering, vol. 19, pp. 66-69, 2001.

[5] B.H. Yang and Z.Q. Zhang, "The Grey model has been Accumulated Generating Operation in Reciprocal Number and Its Application". Mathematics in Practice and Theory, vol. 33, pp. 21-25, 2003.

[6] J. Zhai and J.M. Sheng, "MGM(1,n) Grey Model and its Application", Systems Engineering---Theory & Practice, vol. 17, pp. 109-113, 1997.

[7] Z.M. He and Y.X. Luo, "Application of New Information Multi-variable Grey Model NMGM(1,n) to the Load-strain Relation", 2009 International Conference on Intelligent Computation Technology and Automation (ICICTA 2009) , October10-11, Zhangjiajie, China, 2009.

[8] Y.X. Luo and W.Y. Xiao, "New Information Grey Multi-variable Optimizating Model NMGM(1,n,q,r) for the Relationship of Cost and Variabilit", 2009 International Conference on Intelligent Computation Technology and Automation (ICICTA 2009), October10-11, Zhangjiajie, China, 2009.

[9] F.X. Wang, "Multivariable Unequal-interval GM(1,m) Model and Its Application", Systems Engineering and Electronics, vol. 9, pp. 388-390, 2007.

[10] P.P. Xiong, Y.G. Dang and H. Zhu, "Research of Modeling of Multi-variable Unequal-interval MGM(1,m) Model", Control and Decision, vol. 26, pp. 49-53, 2011.

[11] P.P. Xiong, Y.G. Dang and Y. Yang, "Optimization of Background Value in Multi-Variable Non-Unequal-interval Model", 19th Chinese Conference on Grey Systems, pp. 277-281, 2010.

[12] L.Z. Cui, S.F. Liu and Z.P. Wu. "MGM(1,m) based on Vector Continued Fractions Theory", Systems Engineering, vol. 26, pp. 47-51, 2008.

[13] Y.X. Luo, "Unequal-interval New-information GM(1,1) Model and Its Application", Journal of Shenyang University of Technology, vol. 32, pp. 550-554, 2010.

[14] Y.X. Luo and X.Y. Che, "Grey Multi-variable Optimizing Model and Its Application to Analysis of Tribological Behaviors of the Film", Lubrication Engineering, vol. 33, pp. 58-61, 2008.

[15] Z.H. Han and H.X. Dong, "The Method and Grey MGM(1,n) Optimizing Model and It's Application to Metal Cutting Research", Natural Science Journal of Xiangtan University, vol. 30, pp. 117-120, 2008.