A Hybrid Asymptotic Waveform Evaluation (AWE) Technique to Compute Wideband Electromagnetic Characteristics of Conductor-Dielectric Mixed Objects

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***Abstract* — Deriving detailed models with highly efficient computations of electromagnetic scattering from complex objects have been an important problem in broadband electromagnetic computation fields. In this paper, the Asymptotic Waveform Evaluation, AWE, technique is used to study the wideband electromagnetic scattering characteristics of conductor and dielectric mixed objects. First, the surface current on the mixed objects is expanded in a Taylor series. Secondly, using Padé approximations, the coefficients are matched to rational functions. Using the functions, the electric/magnetic current distribution is calculated at any frequency within the given band. Finally, the wideband radar cross section of the mixed objects can be computed. Numerical results are shown to validate the accuracy and computational efficiency of the proposed technique.**

Keywords-mixing objects; asymptotic waveform evaluation; method of moments

I. INTRODUCTION

With the development of computational electromagnetic, more and more researchers pay attention to the study of broad-band electromagnetic scattering characteristics. In the area of radar stealth and anti-stealth technology research, radar target identification, complex antenna system design and modern electric system EMC analysis, we often need to simulate the electromagnetic characteristics of some complex objects which is composite of conductor and medium. With these backgrounds, it is very important to study the wideband electromagnetic scattering of conduct and dielectric mixing objects.

Based on electric filed integral equation(EFIE),the Method of Moments(MoM) is an important method to study the scattering characteristics of an arbitrary-shaped objects[1,2].The principle of the MoM is to transform the integral equations into the difference equations. The main work of the MoM is to use the computer to solve the algebraic equations. So, in the process of computing, the size of the matrix is related to the number of occupied memory and the speed of the calculations. Especially, when the Radar Cross Section(RCS) is changed dramatically with the frequency, in order to obtain the accurate broadband response, the frequency scanning interval must be very small, which will lead to a huge amount of computational effort.

To overcome this disadvantage, some efficient approximation methods have been considered. Among them, the Asymptotic Waveform Evaluation(AWE) is the hotspot of these researches. In the time analysis of digital system design, the AWE technique was first applied in 1990[3,4]. Then, it was presented to solve scattering proble. Using the Taylor series, the surface current distribution at any frequency can be obtained. Therefore, there is no essential to solve the matrices' equation repeatedly point by point at the given frequency[5,6]. Therefore the AWE technique can effectively reduce the computational complexity of the iterative solution of the equation. Because of these advantages, the AWE has been used in many domains, such as surface-wire junctions structures[7], antennas mounted on conducting platform[8], half-space structure[9], thin dielectric ellipsoidal shell[10]. However, the AWE technique has not been applied in the conduct and dielectric mixing objects yet.

In this paper, the integral equation of conduct and dielectric mixing objects is elaborated based on the surface equivalence principle and boundary conditions firstly. Then the AWE technique has been applied in the mixing objects. Comparing with the conventional MoM, AWE technique can improve characteristic of impedance matrix and accelerate iterative converging. The numerical examples show the accuracy and efficiency of the proposed method.

II. THEORY

A. *Integral Equation of Conduct and Dielectric Mxing Objects*

Supposing there is N perfectly conducting objects and M dielectric objects in the free space, which their cross section are arbitrary shape. As shown in figure1. S_{cn} denotes the surface of the N-th conducting objects, S_{dm} denotes the surface of the M-th dielectric objects. Supposing TM waves is the incident fields, the integral equations using equivalent principle are as follows:

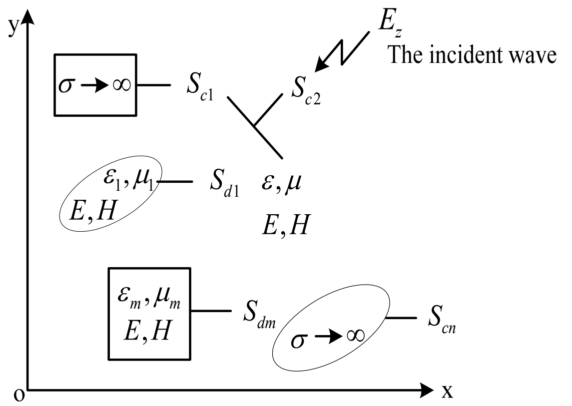


Figure 1. Scattering of Conduct and Dielectric Objects

$$\vec{E}=\vec{E}^i+\vec{E}^s(\vec{J}_c,\vec{J}_d,\vec{M}_d) \tag{1}$$

$$\vec{H}=\vec{H}^i+\vec{H}^s(\vec{J}_c,\vec{J}_d,\vec{M}_d) \tag{2}$$

Where $\vec{E}$ and $\vec{H}$ are the total field, $\vec{E}^i$ and $\vec{H}^i$ are the incident field, $\vec{E}^s$ and $\vec{H}^s$ are the scattering field. $\vec{J}_c$ is an equivalent current on the conducting surface, $\vec{J}_d$ is an equivalent current on the dielectric surface, $\vec{M}_d$ is an equivalent magnetic current on the dielectric surface.

Further, $\vec{E}^s$ and $\vec{H}^s$ are determined by

$$\vec{E}^s=\sum_{n=1}^{N}[-j\omega\vec{A}(\vec{J}_{cn})-\nabla V(\vec{J}_{cn})]+\sum_{m=1}^{M}-j\omega\vec{A}(\vec{J}_{dm})$$
$$-\nabla V(\vec{J}_{dm})-\frac{1}{\varepsilon}\nabla\times\vec{F}(\vec{M}_{dm})] \tag{3}$$

$$\vec{H}^s=\sum_{n=1}^{N}\frac{1}{\mu}\nabla\times\vec{A}(\vec{J}_{cn})+\sum_{m=1}^{M}[-j\omega\vec{F}(\vec{M}_{dm})-\nabla U(\vec{M}_{dm})$$
$$+\frac{1}{\mu}\nabla\times\vec{A}(\vec{J}_{dm})] \tag{4}$$

Where the subscript ***n*** denotes the ***N-th*** conducting objects and ***m*** denotes the ***M-th*** dielectric objects.

B. Method of Moments of Conduct and Dielectric Mixing Objects

Assume the conducting objects and the dielectric objects are in $\vec{z}$ axial direction, and the dielectric objects are characterized by a complex dielectric constant ε_r . Supposing *C* is the circumference of the conducting object, and *S* is the section of the dielectric object. Considering there is no field in the conducting object, while the rest is the superposition of the scattering field of the conducting objects and the dielectric objects. When the incident wave is TM waves, the equation is as follows:

$$E_z^{\ s}(\vec{\rho})=-\frac{\omega\mu_0}{4}\int_l J_z(\vec{\rho}^{\,'})H_0^{(2)}[k_0|\vec{\rho}-\vec{\rho}^{\,'}|]dl-$$
$$\frac{j}{4}\iint_s H_0^{(2)}[k_0|\vec{\rho}-\vec{\rho}^{\,'}|][k^2(\vec{\rho}^{\,'})-k_0^{\ 2}]E_2(\vec{\rho}^{\,'})ds \tag{5}$$

Where $k_0=\omega\sqrt{\mu_0\varepsilon_0}$,$k(\vec{\rho}^{\,'})=k_0\sqrt{\varepsilon_r}$.

Specially, for the conducting objects, the surface field equation is as follows:

$$\vec{E}=\vec{E}^i+\vec{E}^s=0 \tag{6}$$

For the dielectric objects, the surface field equation is as follows:

$$\vec{E}=\vec{E}^i+\vec{E}^s \tag{7}$$

The point matching method is used to calculate the integral equation, and the basis function and the test function are selected function δ , respectively. The conducting object boundary is divided into ***N*** segments, and the dielectric object boundary is divided into $\boldsymbol{N-n}$ segments, and then the equation (5) is changed to:

$$E^s(\vec{\rho})=\sum_{i=1}^{n}g(\vec{\rho},\vec{\rho}^{\,'})J(\vec{\rho}^{\,'})+\sum_{i=n+1}^{N}\alpha(\vec{\rho},\vec{\rho}^{\,'})E(\vec{\rho}^{\,'})$$
$$=\sum_{i=1}^{n}g_{ji}\cdot J_i+\sum_{i=n+1}^{N}\alpha_{ji}\cdot E_i \tag{8}$$

For the conducting objects, the equation (6) is changed to:

$$E^i + \sum_{i=1}^{n} g_{ji} \cdot J_i + \sum_{i=n+1}^{N} \alpha_{ji} \cdot E_i = 0 \qquad (9)$$

Its matrix form is as follows:

$$\overline{\overline{G}} \cdot \overline{J} + \overline{\overline{\alpha}} \cdot \overline{E} = -E^i$$

(10)

For the dielectric objects, the equation (7) is changed to:

$$E^i + \sum_{i=1}^{n} g_{ji} \cdot J_i + \sum_{i=n+1}^{N} \alpha_{ji} \cdot E_i = E$$

(11)

Its matrix form is as follows:

$$\overline{\overline{g}} \cdot \overline{J} + \overline{\overline{A}} \cdot \overline{E} = -E^i$$

(12)

And then the formula (10) and (12) are simultaneously, the matrix form is as follows:

$$\begin{vmatrix} G & \alpha \\ g & A \end{vmatrix} \begin{vmatrix} J \\ E \end{vmatrix} = \begin{vmatrix} -E^i \end{vmatrix}$$

(13)

Where

$$G_{ji} = \begin{cases} \dfrac{\eta}{4} k\Delta C_n H_0^{(2)} [k\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \\ \dfrac{\eta}{4} k\Delta C_n [1 - j\dfrac{2}{\pi} \ln(\dfrac{\gamma k\Delta C_n}{4e})] \end{cases} \qquad (14)$$

$$\alpha_{ij} = \frac{\eta}{4} k\Delta S_n H_0^{(2)} [k\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

$$i \in (1 \cdots n) \qquad j \in (n+1 \cdots N) \qquad (15)$$

$$g_{ij} = \frac{\eta}{4} k\Delta C_n H_0^{(2)} [k\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

$$i \in (n+1 \cdots N) \qquad j \in (1 \cdots n) \qquad (16)$$

$$A_{ji} = \begin{cases} \dfrac{\eta}{4} k\Delta S_n H_0^{(2)} [k\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} & i \neq j \\ \dfrac{\eta}{jk(\varepsilon_{rm} - 1)} & i = j \end{cases}$$

(17)

C. AWE Technique of Conductor and Dielectric Mixing Objects

In order to obtain the surface current distribution of the objects in the given frequency band, MoM must repeat the computations point by point. While using the AWE technique, $I(k)$ can be expanded in Taylor series:

$$I(k) = \sum_{n=0}^{\infty} m_n (k - k_0)^n \qquad (18)$$

$$m_n = Z^{-1}(k_0)[\frac{V^n(k_0)}{n!} - \sum_{i=1}^{n} \frac{Z^{(i)}(k_0) m_{n-i}}{i!}]$$

(19)

Where k_0 is the central wave number in the desired frequency band. $Z^{(i)}(k_0)$ is the ***i-th*** derivative with respect to k of $Z(k)$ at $k = k_0$. $V^n(k_0)$ is the ***n-th*** derivative with respect to k of $V(k)$ at $k = k_0$.

With the purpose of enlarging the convergence radius of the Taylor series, $I(k)$ can be expanded into rational functions by the Padé approximation.

$i \neq j$

$i = j$

$$I(k) = \frac{\sum_{i=0}^{L} a_i (k - k_0)^i}{1 + \sum_{j=0}^{M} b_j (k - k_0)^j}$$

(20)

Generally, $b_0 = 1$, $L = M$ or $L = M + 1$

The unknown coefficient of a_i and b_j in the formula (20) are solved as follows:

$$\begin{bmatrix} m_L & m_{L-1} & m_{L-2} & \cdots & m_{L-M+1} \\ m_{L+1} & m_L & m_{L-1} & \cdots & m_{L-M+2} \\ m_{L+2} & m_{L+1} & m_L & \cdots & m_{L-M+3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{L+M-1} & m_{L+M-2} & m_{L+M-3} & \cdots & m_L \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_M \end{bmatrix} = - \begin{bmatrix} m_{L+1} \\ m_{L+2} \\ m_{L+3} \\ \vdots \\ m_{L+M} \end{bmatrix}$$

(21)

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_L \end{bmatrix} = \begin{bmatrix} m_0 & 0 & 0 & \cdots & 0 \\ m_1 & m_0 & 0 & \cdots & 0 \\ m_2 & m_1 & m_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_L & m_{L-1} & m_{L-2} & \cdots & m_{L-M} \end{bmatrix} \begin{bmatrix} 1 \\ b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix}$$

(22)

III. NUMERICAL COMPUTATION

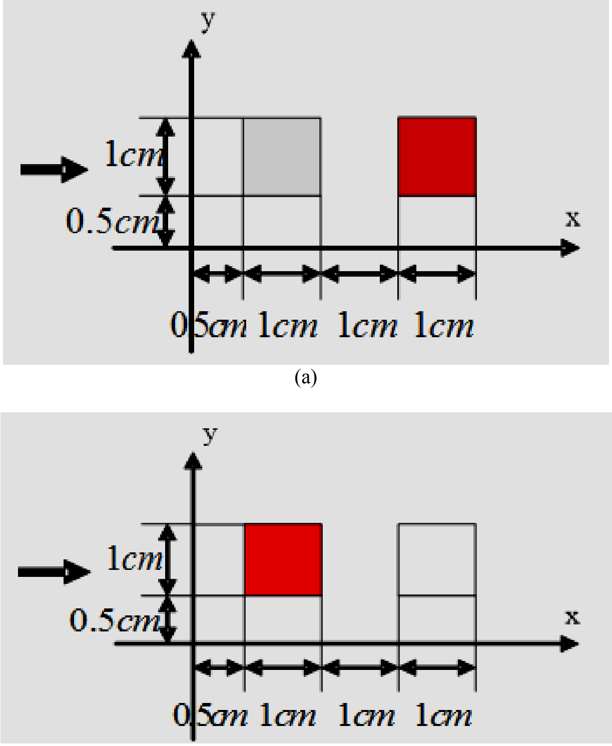
Two examples are introduced to illustrate the accuracy and efficiency of the AWE technique. The central frequency for AWE and MOM is 2.5GHz respectively in the examples. All the simulations are fulfilled on a personal computer with the Intel ® Core™2 Duo CPU with 2.1GHz and 2.0 GB RAM.

A. Single Conductor and Dielectric Mixing Objects

There are single conduct and dielectric mixing objects illuminated by the TM wave incident, which expression is $\vec{E}_z^{\,i} = e^{-jkx}$.As shown in figure 2. The size of each object is $0.01 \times 0.01 m^2$. The relative dielectric constant of dielectric object is 4. The distance between two bodies is $0.01m$. Now the electromagnetic scattering characteristics in the wide frequency band from 1GHz to 4GHz are analyzed. Here the AWE technique is in conjunction with the Padé approximation. To evaluate the effectiveness of the AWE technique in the wideband scattering characteristics of the mixing objects, the results are compared with the MoM point by point. The comparison results are shown in figure 3. The figures show that the calculation results of AWE method basically coincide with the conventional MoM. If the TM wave incident to the conduct objects at first, as shown in figure 3(a), we can see that the curves are basically consistent from 1GHZ to 3GHZ，while the TM wave incident to the dielectric objects at first, as shown in figure 3(b), we can see that the curves are basically coincident from 2GHZ to 4GHZ. The reasons of this phenomenon is coupling relationship between the conductor and dielectric objects. The comparison of the CPU time required by AWE and the conventional MoM are also given in Table 1.

TABLE 1. TOTAL CPU TIME OF THE AWE TECHNIQUE AND THE CONVENTIONAL MOM

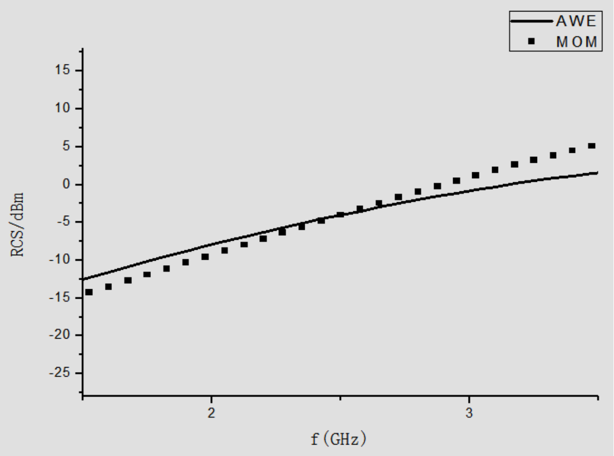
Problems	AWE technique	Conventional MoM	Time Multiplier
Example 1	9.063s	35.812s	3.95
Example 2	33.14s	132.891s	4.01



(a)

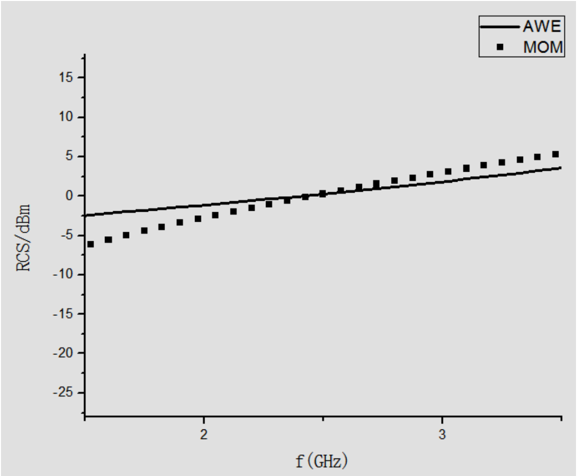
(b)

Figure.2 (a) Single Conduct and Single Dielectric Array 1
Figure.2 (b) Single Conduct and Single Dielectric Array 2
(non-filled area is conduct, filled area is dielectric).



(a)

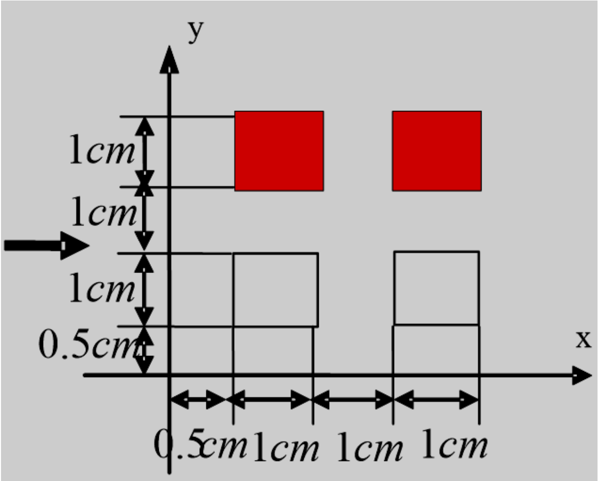
Figure.3(a) Broadband scattering characteristics corresponding to figure.2(a)



(b)
Figure.3(b) Broadband scattering characteristics corresponding to figure.2(b)

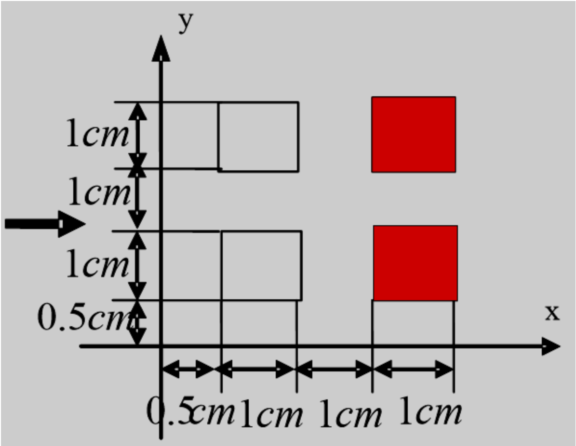
B. *Two Conductor and Dielectric Mixing Objects*

There are two conductors and two dielectric mixing objects. The conditions are the same as first example. Models are shown in figure 4. The comparison results between the AWE technique and the conventional MoM are shown in figure 5. In the central frequency, the figures show that the curves are fully agreement。But, because of coupling relations of the conductor and the dielectric objects, the results calculated by the two methods still have some deviations. The CPU time of AWE and the conventional MoM is listed in Table 1.

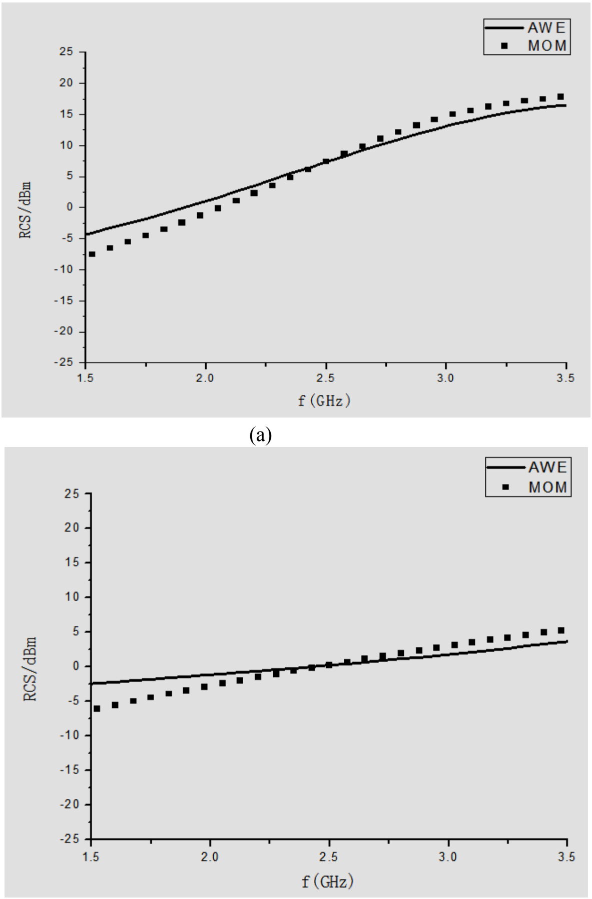


(a)

Figure.4 (a) Two Conduct and Two Dielectric Array 1



(b)
Figure.4 (b) Two Conduct and Two Dielectric Array 2 (non-filled area is conductor, filled area is dielectric).



(a)

(b)

Figure.5(a) Broadband scattering characteristics corresponding to figure.4(a)
Figure.5(b)Broadband scattering characteristics corresponding to figure.4(b).

IV. CONCLUSIONS

An efficient AWE technique is proposed to solve the wideband electromagnetic scattering characteristics of conduct and dielectric mixing objects. Firstly, the EFIE and the MFIE at a given frequency by the TM wave are obtained in terms of equivalent principle. Secondly, these equations are solved by using MoM to obtain the surface electric/magnetic current on the mixing objects. Finally, using the Padé approximation, the above currents can be calculated, and the wideband RCS can be computed. Numerical examples show the efficiency of the AWE technique is much higher than the MoM, and the accuracy of the AWE technique can be well approximated by the MoM, especially at the center frequency point.

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