The Research on Congestion Rate of Single Item Picking System in Narrow Aisles under Equivalent Picking and Walking Speed

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***Abstract* — It constructs Markov model for the congestion rate of narrow aisles single item picking system under equal picking and walking speed. For describing the status of the transfer, there are innovative ideas. Get the analytical expression of picking area, picking density and congestion rate. We analyze various factors on the impact of congestion rate. The conclusion is that the larger picking area, the lower congestion rate, and within a certain range picking density, the congestion reach the maximum. Our research provides guidance to overcome picking congestion, improve the picking efficiency and customer satisfaction.**

Key words - Congestion, Order picking, Narrow aisles, Markov

I. INTRODUCTION

Reduce logistics costs, shorten customer response time, is the key to improve the operating efficiency of the storage system. Merchant Warehouse has characteristics of multi-customers, multi-species, multi-frequency, and small batch. Consumers want to get goods faster and faster, therefore, the storage picking efficiency has a direct impact on the response time of consumer demand. Warehouse picking system is one of the most highly labor-intensive and the most costly activities, the time spent on the warehouse picking system is about 60% of the total time in the library operations. Picking costs are about 9 times the cost of other stacking, loading and unloading, transportation, etc, accounting for 55%[1] of the total cost of warehousing. Research on the picking operation is very important to improve the efficiency of warehouse operation. However, a lot of storage centers due to the shelf layout, storage, routing strategy and other issues, there is a traffic jam due to the picker waiting for the other picker in the aisles. It is a waste of human cost, but also a waste of time, which greatly affects the whole storage. Because of the hazards caused by congestion cannot be ignored.

Congestion is the bottleneck affecting the efficiency of the picking system. Overcoming the picking congestion and improving the picking efficiency is the effective way to improve customer satisfaction, reduce logistics costs and improve service level of supply chain. Through the reading of domestic and foreign literature, scholars from different perspectives, using different methods studied the main factors affect the picking time and cost of goods location layout, storage strategy, picking Strategies and methods, but research on the problem of congestion in formed of picking process is still relatively less.

Various factors affecting the congestion cannot be simply considered as only one kind. It should take all factors into consideration. So researchers often analyze simultaneously on two or more than two kinds of factors, or analyze the distribution and storage strategy at the same time, or analyze order wave and storage strategy at the same time, while others analyze storage strategy and picking path at the same time, such as Felix (2011) [2] Dima (2011) [3], Jason (2012) [4,5], Nima (2013) [6].

In 2006, Gue and Meller[2] studied congestion of the annular shelf layout of the narrow aisles. Their study shows that in the narrow aisles, the picker will meet congestion within aisles and more jam with the increase in the number of pickers, lighten with increase of the picking zone, picker walking speed quicker cause more serious congestion. Skufca (2005) [3] has established a model of more pickers congestion in the narrow aisles system, which is based on the assumption of unlimited walking speed and single item picking system. In 2009, Pratik and Meller[4] studied on congestion between the two pickers of the annular shelf layout of the wide aisles. They believe that congestion in a wide aisles layout than the narrow aisles has eased. Congestion is an outstanding problem in the high turnover and multi pickers system. In 2013, Soondo Hong[5] further verified the results of Gue and Pratik, and has made the improvement in the congestion estimation algorithm.

On the basis of the research of the foreign scholars, the concept of state transfer is needed to be further clarified. On the basis of the research of Gue and Meller, we continue to study the narrow aisles of single picking system, which is different from Gue is the definition of state transfer is different, so different state transfer matrices are obtained.

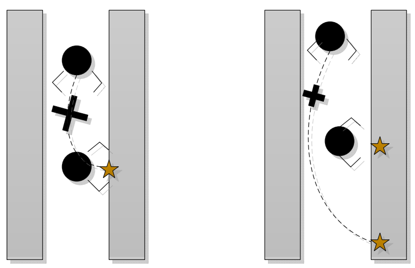
II. THE GENERATION OF PICKING CONGESTION

There contain a lot of links in the distribution center of the picking operation, including acceptance of orders, order sorting, goods sorting, collection and classification. These processes interact with each other, and the purpose is to improve the overall efficiency of the picking system.

Previous studies of picking usually are just for picking system with only one picker, but the enterprise in order to improve the throughput and response speed to clients, usually have multiple pickers working at the same time. If two or more than two pickers need to access a stock point, it is inevitable that the congestion will occur, and the congestion rate will increase when the picking density is high. Congestion phenomenon will lead to one part of pickers who suffer from congestion phenomenon is leisure, and reduce the operation efficiency of the selection system in the actual operation of the system. Picking operation as the key link in the market demand of enterprise response, and congestion will affect the whole supply chain efficiency. Picking congestion is a key factor in the planning and batching of district selection strategies of the picking system. When the storage system is relatively high and the length is relatively short, the chosen region is small, and there will be more likely congested. When the storage area needs to be used in batches, the picker is likely to walk in a certain way, which can also produce congestion. Picking congestion will reduce the productivity of the pickers. In general, the distribution center will choose to increase the number of the pickers to meet the required throughput, but this also increases the cost of labor. In order to improve the picking efficiency, it is necessary to study the picking congestion, congestion impact factors and as far as possible to reduce congestion occurrence which can effectively improve the picking efficiency.

In narrow aisles of the picking system, picking congestion phenomenon has two forms such as shown in Fig. 1. (a) Picking face congestion, that is when the two pickers choose goods on the same picking face to produce congestion. (b) Aisles congestion, which is when a picker is picking, another picker who is walking, but cannot pass through the congestion.

In narrow aisles layout, when the picking state is the upper picker is being picking, the downstream picker is waiting for picking, and the distance between them is one unit, congestion occurs. In this paper, we study the congestion problem of the picking system using the traditional rectangle layout and S-type path strategy. Because of the change of the distance between the pickers with no aftereffect characteristics, it is suitable for establishing the Markov model for distance variation among the pickers, and the validity of the model is verified by simulation and analysis of the factors affecting the congestion.



(a) Picking face congestion（b）Aisles congestion

Fig.1 Two types of narrow aisles congestion

III.MODEL HYPOTHESIS AND SYMBOL DESCRIPTION

A. *Description of Narrow Aisles Picking System*

Because of the variety of the picking system, and with realistic complexity, in order to describe the theoretical model simple, in this paper, we study the traditional shelf layout of narrow aisles containing n picking faces (Fig. 2). More than one picker is picking under the random storage and S-type path strategy. In one unit time period, the probability of the picking of the picker is p, walking probability is q. In the Single picking system that one can only pick one item at a picking face. If one is picking at this time period, then the probability that he will walk to the next picking face at the next period is 1.

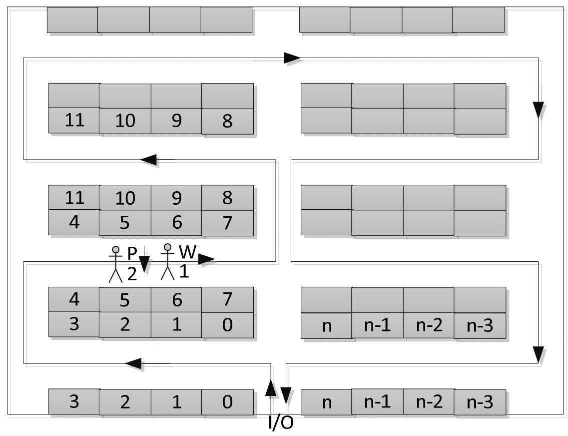


Fig.2 A Schematic Diagram of the Random Storage S-type Path Picking Under Traditional Layout

In this paper, we construct a congestion model under the equal picking and walking speed, that is to say, the time of walking is equal to the time of picking when pass a picking face. Expressed as the time ratio of picking and walking is $P:W=1:1$.

B. *Symbol Description*

In the traditional shelf layout system (see Fig. 2) to construct congestion model, make the following explanation of symbols.

(1)The order picking region has n picking faces. In the standard case, a picking face is a column of the tray shelf or a group of fluent shelves. In fact, each picking face may contain several picking location, for example, a column tray shelf may contain 3 or 4 layers of the tray rack. In this paper, the picking position of the picker is picking face.

(2)In this paper, narrow aisle picking system is studied, assuming that the aisles has 1 meters wide and is a typical narrow aisles layout. The picker can be a person, or an automatic picking. In narrow aisles, only one picker can be accommodated to picking and passing. When picker picks goods, he will have a horizontal or vertical move; need to have a certain space to remove the goods, so in the aisles pickers can't pass each other.

(3) The person follows the path of the S-type path strategy, which means that the picker make the picking only in one direction in the aisles.

(4) In the actual distribution center, there are two rows of shelves in each aisle, in order to simplify the problem, suppose that the picking is not associated with any of the two shelves in the aisles.

(5) The states of the pickers in the picking zone is picking, walking or congestion.

(6) The probability that the picker picks at each picking face is p, The probability of not picking is q. If one picker picks $I(I \le n)$ goods in average, the probability of one goods picked on a certain picking face is $p = I/n$ in a single picking system.

(7)Assuming that there are k pickers, and the average congestion rate of the pickers is $b_{1:1}(k)$, and $0 \le b_{1:1}(k) \le 1$. In this paper, we choose 2 pickers, that is, $k = 2$.

C. State Description

Denote states of two pickers during *t* period is S_i^t, and subscript *i* shows possible types of states. We also respectively positions of two pickers in terms is N_1 and N_2, and the distance between two pickers in terms of d_{12}^t when *t* period ends. So we have,

$$d_{12}^t = (n + N_1 \text{-} N_2) \mod n . \tag{1}$$

And the specific expression of S_i^t are $d_{pp}, d_{pw}, d_{wp}, d_{ww}, (d = 1 \ldots n-1)$, which respectively expresses the distance and state between two pickers when *t* period ends (here omitted superscript *t*), and the first and second subscript respectively expresses the state of picker 1 and picker 2. For example, when $d_{12} = 1$, 1_{pp} means the distance between two pickers is 1 and both of them are in picking states. Besides, 1_{pw} is congestion state, because picker 1 is picking while picker 2 is walking, but the distance between them is 1 so picker 2 cannot pass through picker 1 in narrow aisles. $(n-1)_{wp}$ is also a congestion state. Once congestion occurs, the downstream picker must wait for the upstream picker to finish work. In this picking system, all states between two pickers are belows,

$$S = \{1_{pp}, 1_{pw}, 1_{wp}, 1_{ww}, 2_{pp}, 2_{pw}, 2_{wp}, 2_{ww}, \cdots, \\ (n-1)_{pp}, (n-1)_{pw}, (n-1)_{wp}, (n-1)_{ww}\} \tag{2}$$

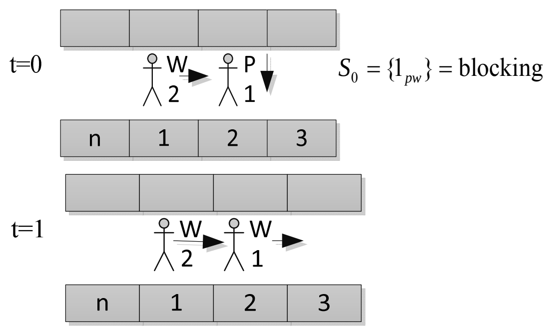
where 1_{pw} and $(n-1)_{wp}$ are congestion states . The interval of distance between two pickers d_{12}^t is $0 \le d_{12}^t \le n-1$. Each distance has four states, and the four states of different distances at present may turn to the other states of distances in the next period. So it is crucial important to get state transition matrix to analyze congestion rates.

IV. MARKOV MODEL UNDER PICKING AND WALKING SPEED RATIO IS 1:1

Identifying the discrete Markov process when the system is congestion, that is to say, looking for a random variable with no aftereffect characteristics, which is the first point of constructing a Markov model. Secondly, analyze the status transition of the picker distance; analyze the status transition change of the picker distance. Correctly describing the state transfer matrix, obtaining the steady state distribution, is the key to the study. In this paper, we study the traditional shelf layout have n picking faces, and the situation of two pickers are picking at the same time in the aisles. In the single item picking system, when picking and walking speed ratio is 1:1, that means the required time of picking one item from a picking face, is equal to required time of walking through a picking face.

A. State Transition Matrix

We analysis the case that the status transition of the two pickers from $t = 0$ to $t = 1$. Fig. 3 shows the state transfer from $t = 0$ with state 1_{pw}, where 1_{pw} is a congested state, till the next period $t = 1$, picker 1 will walk with the probability of 1. So the state can only transfer to one situation, that is 1_{ww}.



$$S_1 = \{1_{ww}\}$$
$$p\{S_1 | S_0\} = p\{1_{ww} | 1_{pw}\} = 1$$

Fig.3 State Transfer to *t*=1 When *t*=0 State is 1_{pw}

Fig.4 denotes the state transfer from *t*=0 with state 1_{ww}. 1_{ww} is non congestion state. At this time, two pickers will

DOI 10.5013/IJSSST.a.17.30.14

ISSN: 1473-804x online, 1473-8031 print

both walk, and at the next period $t=1$, there may be four kinds of states occur, respectively, $\{1_{pp}, 1_{pw}, 1_{wp}, 1_{ww}\}$. The corresponding transfer probability is p^2, pq, pq, q^2.

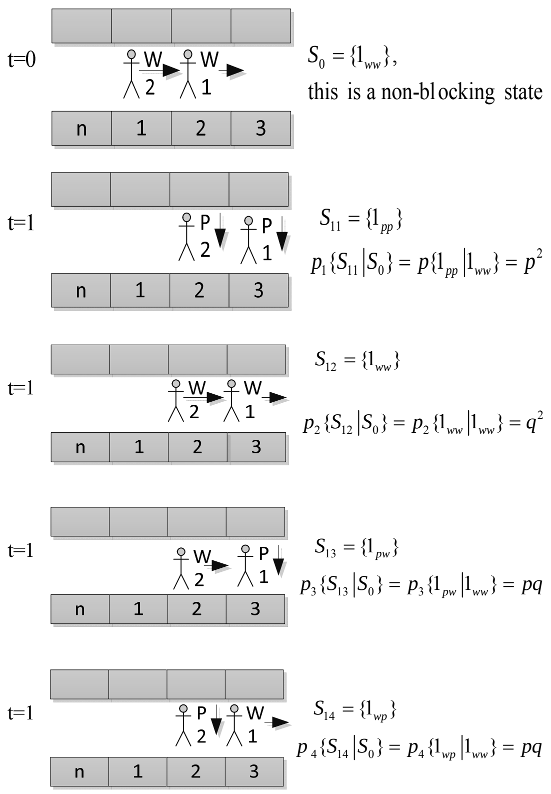


Fig.4 State transfer to *t*=1when *t*=0 state is 1_{ww}

Order by analogy, Fig. 5 represents the state transition situation.

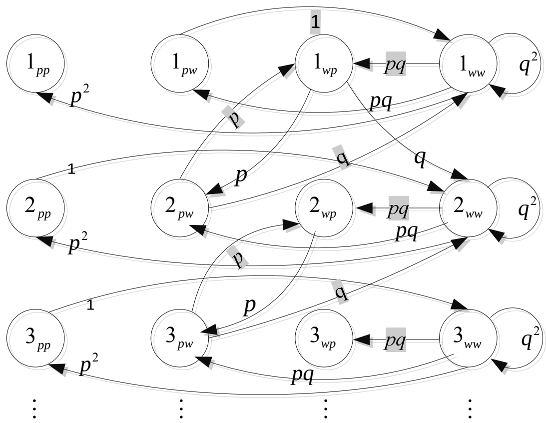


Fig.5 Markov Chain State Transfer of Single Item Picking System with Narrow Aisles

When the status is **Error! No bookmark name given.**, the distance between two pickers is one picking face. At the moment, picker 1 and 2 are picking. Due to the hypothesis of a pick face only pick one item, then, in the next period both two pickers will walk and their distance is still one pick face, namely the transition probability to state **Error! No bookmark name given.** is 1.

When the state is **Error! No bookmark name given.**, the two pickers are walking. Then, at the next time, both two pickers may be picking at next pick face so that state transfer to **Error! No bookmark name given.** and transition probability is**Error! No bookmark name given.**. The state transfers to **Error! No bookmark name given.** if picker 2 still walks with transition probability**Error! No bookmark name given.**. If picker 1 choose to continue walk and picker 2 is picking then state transfers to **Error! No bookmark name given.** with transition probability**Error! No bookmark name given.**. Besides, picker 2 may also choose walk, then it transfers to state **Error! No bookmark name given.** and the transition probability is q^2.

For example，the circumstances of states transfer from $\{1_{pp}, 1_{pw}, 1_{wp}, 1_{ww}\}$ at $t=1$ to state $t=2$ as follows:

$$A_1 = \begin{array}{c} \\ 1_{pp} \\ 1_{pw} \\ 1_{wp} \\ 1_{ww} \end{array} \begin{array}{c} \begin{array}{cccc} 1_{pp} & 1_{pw} & 1_{wp} & 1_{ww} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ p^2 & pq & pq & q^2 \end{bmatrix} \end{array}, \tag{3}$$

$$B = \begin{array}{c} \\ 1_{pp} \\ 1_{pw} \\ 1_{wp} \\ 1_{ww} \end{array} \begin{array}{c} \begin{array}{cccc} 2_{pp} & 2_{pw} & 2_{wp} & 2_{ww} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & p & 0 & q \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}. \tag{4}$$

The circumstances of states transfer from $\{2_{pp}, 2_{pw}, 2_{wp}, 2_{ww}\}$ to various states in the next period as follows:

$$C=\begin{array}{c} \\ 2_{pp} \\ 2_{pw} \\ 2_{wp} \\ 2_{ww}\end{array}\begin{array}{c} \begin{array}{cccc}1_{pp} & 1_{pw} & 1_{wp} & 1_{ww}\end{array} \\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & p & q\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\end{array}, \quad (5)$$

$$A=\begin{array}{c} \\ 2_{pp} \\ 2_{pw} \\ 2_{wp} \\ 2_{ww}\end{array}\begin{array}{c} \begin{array}{cccc}2_{pp} & 2_{pw} & 2_{wp} & 2_{ww}\end{array} \\ \begin{bmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ p^2 & pq & pq & q^2\end{bmatrix}\end{array}, \quad (6)$$

$$B=\begin{array}{c} \\ 2_{pp} \\ 2_{pw} \\ 2_{wp} \\ 2_{ww}\end{array}\begin{array}{c} \begin{array}{cccc}3_{pp} & 3_{pw} & 3_{wp} & 3_{ww}\end{array} \\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & p & 0 & q\\ 0 & 0 & 0 & 0\end{bmatrix}\end{array}. \quad (7)$$

The transformation matrix from $\{(n-1)_{pp},(n-1)_{pw},(n-1)_{wp},(n-1)_{ww}\}$ to $\{(n-1)_{pp},(n-1)_{pw},(n-1)_{wp},(n-1)_{ww}\}$ is

$$A_{n-1}=\begin{array}{c} \\ 1_{pp} \\ 1_{pw} \\ 1_{wp} \\ 1_{ww}\end{array}\begin{array}{c} \begin{array}{cccc}1_{pp} & 1_{pw} & 1_{wp} & 1_{ww}\end{array} \\ \begin{bmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ p^2 & pq & pq & q^2\end{bmatrix}\end{array}. \quad (8)$$

Other state transition matrices are consistent with A, B, C matrices. Thus, we can obtain the total state transition matrix as below.

$$T=\begin{bmatrix}A_1 & B & 0 & \cdots & 0\\ C & A & B & \ddots & \vdots\\ 0 & \ddots & \ddots & \ddots & 0\\ \vdots & \ddots & C & A & B\\ 0 & \cdots & \cdots & C & A_{n-1}\end{bmatrix}_{(n-1)\times(n-1)}. \quad (9)$$

T is an $(n-1)\times(n-1)$ matrix.

B. Stationary Distribution and Congestion Rate

Seeking stationary distribution Z of state transition matrix *T*, namely solving equation $ZT=Z$, the solution is

$$Z=[\overbrace{p^2,p-p^2,p,1}^{d=1},\overbrace{p^2,p,p,1}^{d=2},\cdots,\overbrace{p^2,p,p,1}^{d=n-2},\overbrace{p^2,p,p-p^2,1}^{d=n-1}]. \quad (10)$$

Markov stationary distribution density of the process is determined by the second-order norm of $\|Z\|$. Because the congestion status is 1_{pw} and $(n-1)_{wp}$, each pickers' average congestion rate is

$$\begin{aligned} b_{1:1}(k=2) &= \frac{z_b}{k\sum_j z_j}\\ &= \frac{2(p-p^2)/2}{(n-3)(p^2+2p+1)+2(p^2+2p+1-p^2)} \quad (11)\\ &= \frac{p-p^2}{(n-3)p^2+(2n-2)p+(n-1)}. \end{aligned}$$

From the above equation, is a decreasing function with respect to n. To get the maximum value, seeking first order partial derivative of p for equation (2) and let the first order derivative is zero, that is

$$\begin{aligned} & b_{1:1}'(k=2)\\ &= \frac{(5-3n)p^2+(2-2n)p+(n-1)}{\left[(n-3)p^2+(2n-2)p+(n-1)\right]^2} \quad (12)\\ &= 0. \end{aligned}$$

When $n=2$, we can draw that picking probability p is -2.4142 or 0.4142 making the above equation hold. And the congestion rate achieves maximum when p is 0.4142 because of $0\le p\le 1$. Take second derivative of the formula (3), which is less than 0, the maximum value is achieved at this time. Therefore, congestion rate reach maximum, when

$$p=\frac{n-1-\sqrt{4n^2-10n+6}}{5-3n}. \quad (13)$$

C. Factors Analysis of Affecting Congestion Rate

From formula (11) can be known, with the increase of n, the picking probability decreases when the congestion rate $b_{1:1}(k=2)$ gets maximum value. When there is an infinite picking face, that is $n\to\infty$. Picking probability density is 0.3333 when the congestion rate is the highest. $n=2$ and $n\to\infty$ are two extremes cases of n values. This limits the critical range of maximum picking probability p when the congestion is generated.

When take different n values, the relationship between picking probability p and congestion rate b is shown in Fig. 6. That selects the cases when the storage area has four kinds of situation of 2, 10, 20, 50 picking faces, respectively. In the graphics, they are denoted with different curves. This figure shows the situation of the change of congestion rate when the size of the picking region changed. From the congestion rate formula (2) can be known, with the increase of n, the picking congestion rate decreases.

With the change of n, the picking congestion rate reaches highest when $0.3333 \le p \le 0.4142$. It can be seen from Fig. 6, when the picking region is fixed, with the increase of picking density, the average congestion rate increases to a maximum value and then decrease. When the picking density p is certain, the average congestion rate b decreases with picking face number n increasing. When there are two pickers, the average congestion rate of picking is very small. Therefore, when the picking and walking speed equal, and one can only pick one item at a pick face, the congestion in the picking system is not the key issue.

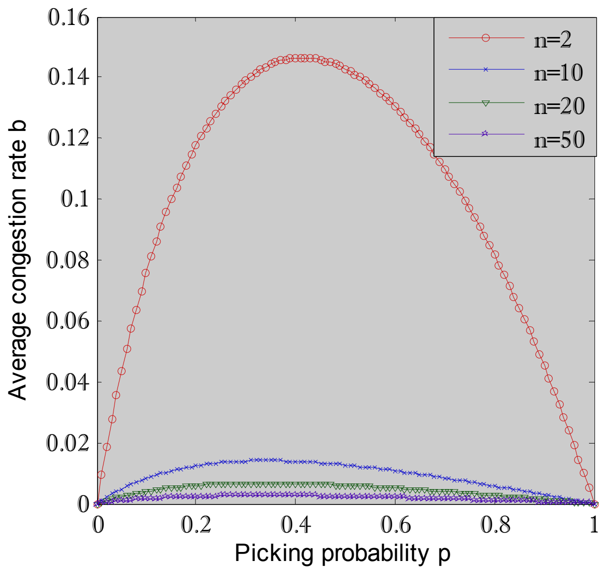


Fig.6 The Relationship of b、n、p Under Equal Picking and Walking Speed of Single Item Picking System

V. CONCLUSIONS

In this paper, we study the congestion factors affecting the picking efficiency. In the simplest conditions, the picking congestion model is established, That is, in the traditional layout narrow aisles system, with equal picking and walking speed, and once only pick one item at a pick face, in this case, we analyze the model. Markov chain method is used to construct the model, deducing a step transition probability matrix, then obtaining the stationary distribution, calculating the analytical expression of the average congestion rate b with picking probability p and the pick faces number n. The relational graph between the picking probability p and the average congestion rate b is obtained. When the picking and walking speed ratio is $1:1$, there exists a picking probability values which makes maximum congestion rate. The overall congestion rate value is small, which indicates that the congestion problem is not serious, which will not affect the operation efficiency.

In the following study, we will continue to model and study the complex picking system of the more than two pickers, multiple items picking at a pick face, and other speed ratio of picking and walking. Theoretically speaking, our research develops the application of stochastic process theory in storage and picking, lays the foundation for the following study, to provide decision making basis for the efficiency management and optimal control of the picking system, and provide new ideas and methods for the supply chain management theory and the basic operation technology.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

ACKNOWLEDGMENT

The study is supported by the National Nature Science Foundation of China “Research on the warehouse picking system blocking influence factors and combined control strategy” (No. 71501015) and the National Nature Science Foundation of China (No. 71371033), and Beijing Key Laboratory of Intelligent Logistics System, (BZ0211), and the project of Beijing Wuzi University Yunhe River scholar, high level cultivation project of Beijing Wuzi University (No.0541502703), and Beijing Intelligent Logistics System Collaborative Innovation Center.

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