A Study on the Reliability of Online Upgrade System for Vehicle ECU using Markov Models

Anyu Cheng, Liangbo Xiong*, Min Xiang, Yao Wang

School of Automation, Chongqing University of Posts and Telecommunications, Chongqing 400065, China

***Abstract* — In order to solve the problem that online upgrade system for vehicle ECU (Electronic Control Unit) has poor reliability and stability, this paper presents a reliability assessment model based on Markov mathematical model. According to the practical operation of online upgrade system, this paper divided the work state of the system into eight kinds such: normal running state, equipment failure state, hardware failure state, and so on, and builts an assessment model of system reliability combined with Markov mathematical model. In our study we used the method of frequency and duration to analyze the reliability of online upgrade system, and calculated the probability of each steady state by Laplace transformation and its properties. Based on this, the available rate, unavailable rate, failure frequency, mean time between failures, mean time to repair and other series of reliability indices of the system were caculated. The results show that this model has quantitative assessment of the reliability of the online upgrade system, and has strong practical significance.**

Keywords: Markov Model; Online Upgrade system; Reliability

I. INTRODUCTION

In order to meet consumers' growing demand for the function of car, online upgrade system for vehicle ECU has become more and more important. For a long time, most researches focused on how to implement an online upgrade system, in order to solve the problem that the upgrade of vehicle ECU is inconvenient [1,2,3]. The research is broad and the technology is relatively mature. Now in our country and abroad, the reliability and stability research of online upgrade system for vehicle ECU is still in initial stage. The information about this technology can be rarely found. Today, some scholars put forward by using the method of probability theory to analyze online upgrade system from the aspects of possible failure, and predict the reliability of the system. But this method is too simple, and doesn't consider the situation that online upgrade system can transform between each state and has a large influence on the system reliability. So today's research about the reliability of online upgrade system is too simple and not comprehensive enough. Most importantly, it's lack of detail theory research.

Markov mathematical model has simple structure, easy calculation process, and low requirement for the mathematical tools. So when a system is in a certain state during the use process, Markov model can be used to describe the probability of the system. The process is very convenient and has good practical application [4]. Nowadays, Markov mathematical model has been widely used in financial, industrial, network, power electronics and other fields, which can be used to analyze the reliability of different complex systems [5,6,7,8]. In view of the all kinds of state of online upgrade system is in the work process. This paper first combines Markov model with online upgrade system, and establishes the mathematical model to analyze each state probability and reliability index of online upgrade system. This model provides a strong theoretical support to analyze the state of online upgrade system and formulates a reasonable download strategy.

This paper is organized as follows: Section 2 introduces the basic theory of Markov model. Section 3 describes the combination between Markov model and online upgrade system. Section 4 establishes the reliability assessment model of online upgrade system basing on Markov model. Section 5 gets the detail data by experiment and verifies the reliability assessment model. Section 6 summarizes and prospects the full text.

II. MARKOV PROCESS

A. The Basic Thought of Markov Process

Markov process with continuous time has no memory. The state space is discrete and the time is continuous. Comparing with the Markov process with discrete time, the state change moment of Markov process with continuous time is arbitrary, and the value is continuous. The definition is as follows [9].

Take the stochastic process $\left\{X_t, t \in T=\left[0,\infty\right)\right\}$ on the set of non-negative integer ϕ, if all the moments $t_1 < t_2 < \cdots < t_n \left(n \ge 1\right)$ of T and the non-negative integer i_k, $k = 1,2,\cdots n$ satisfy the relationship of $P\left(X_{t_k} = i_k, 1 \le k \le n\right) > 0$. There has the following define

$$P\left(X_{t_n} = i_n \middle| X_{t_k} = i_k, 1 \le k \le n-1\right) = P\left(X_{t_n} = i_n \middle| X_{t_{n-1}} = i_{n-1}\right) \quad (1)$$

we call $\left\{X_t, t \in T\right\}$ is a Markov process with continuous time [10]. For the Markov process which has been given, the following relationship exists

$$P\{X(t+\Delta t)=j \mid X(t)=i\}=\lambda_{ij}\Delta t+\mathrm{O}(\Delta t) \tag{2}$$

In this formula, $X(t)$ is the state of system at the time of t. $X(t+\Delta t)$ is the state of system at the time of $t+\Delta t$. P is the probability of the system after time Δt and transfer to the state j in the condition that at the time of t the system is in state i. λ_{ij} is the transition probability, which refers to the probability of the state transferred from i to j in unit time. The value of λ_{ij} is constant. $\mathrm{O}(\Delta t)$ is the probability of the state transition occurs two or more times during the time of Δt [11]. When Δt is small enough, we have the following relationship

$$P\{X(t+\Delta t)=j \mid X(t)=i\}=\lambda_{ij}\Delta t \tag{3}$$

According to the properties of the homogeneous Markov process, the probability of the state transferred from i to j in unit time is as follows

$$P\{X(t+\Delta t)=i \mid X(t)=i\}=1-\sum_{j=1,\, j\neq i}^{n}\lambda_{ij}\Delta t \tag{4}$$

so the matrix form of transition probability λ_{ij} is

$$p(\Delta t)=\begin{bmatrix} 1-\sum_{k=2}^{n}\lambda_{1k}\Delta t & \lambda_{12}\Delta t & \cdots & \lambda_{1n}\Delta t \\ \lambda_{21}\Delta t & 1-\sum_{k=1,k\neq 2}^{n}\lambda_{2k}\Delta t & \cdots & \lambda_{2n}\Delta t \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{n1}\Delta t & \lambda_{n2}\Delta t & \cdots & 1-\sum_{k=1}^{n-1}\lambda_{nk}\Delta t \end{bmatrix} \tag{5}$$

B. Solve the Probability of Steady State

According to the C-K equation of Markov process, there has the following relationship

$$P(t+\Delta t)=P(t)*P(\Delta t) \tag{6}$$

$$\frac{P(t+\Delta t)-P(t)}{\Delta t}=\frac{P(\Delta t-I)}{\Delta t}P(t) \tag{7}$$

which solve the limit of both sides

$$\lim_{\Delta t\to 0}\frac{P(t+\Delta t)-P(t)}{\Delta t}=\lim_{\Delta t\to 0}\frac{P(\Delta t)-I}{\Delta t}P(t) \tag{8}$$

basing on the definition of the reciprocal, the following relationship exists

$$P'(t)=\lim_{\Delta t\to 0}\frac{P(\Delta t)-I}{\Delta t}P(t)=QP(t) \tag{9}$$

in this formula, Q is called density matrix. Combined with formula (5), the density matrix Q is as follows

$$Q=\begin{bmatrix} -\sum_{k=2}^{n}\lambda_{1k} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & -\sum_{k=1,k\neq 2}^{n}\lambda_{2k} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & -\sum_{k=1}^{n-1}\lambda_{nk} \end{bmatrix} \tag{10}$$

Adopting the method of solution the differential equation in modern control theory, and transferring the differential equation from time domain to frequency domain [12], the equation of Laplace transformation on formula (9) can be got as follows

$$\mathrm{s}P(s)-P(0)=QP(s) \tag{11}$$

$$P(s)=(sI-Q)^{-1}P(0) \tag{12}$$

then do Laplace inverse transformation on formula (12) and get the result as follows

$$P(t)=L^{-1}\left[(sI-Q)\right]P(0) \tag{13}$$

In this formula, $P(t)$ is the column matrix which is composed of all state probability when the system is in the steady state at the time of t. $P(0)$ is the initial matrix. The specifics are as follows

$$P(t)=\begin{bmatrix} P_1(t) \\ P_2(t) \\ \vdots \\ P_n(t) \end{bmatrix} \qquad P(0)=\begin{bmatrix} P_1(0) \\ P_2(0) \\ \vdots \\ P_n(0) \end{bmatrix} \tag{14}$$

III. THE APPLICATION OF MARKOV MODEL IN THE ONLINE UPGRADE SYSTEM

A. The Features of Online Upgrade System

Online upgrade system is a repairable system. When the system has some failures, it can be repaired by finding out and maintaining the point of failures, until the system can run normally again. All the process can be considered as a stochastic process. During the practical work, the state of online upgrade system can transfer from normal to abnormal. After the system has been repaired, the state can transfer from abnormal to normal. So, this kind of probability of system state shifting from one to another is random. In the process of Markov model, the research objects are the states of repairable system and the conversion between the all states. The probability of system move from one state to another has no relationship with the history state of the system, and the future state is uncertain [13]. This process is similar with the work process of online upgrade system, so the Markov model can be used to analyze the reliability of online upgrade system. When we are going to use Markov process to build the reliability evaluation model, the following assumptions should be considered [14]:

1. The system can only be either in normal operation state or in fault state, and a failure of a unit of the system is independent of other units.

2. In the system, the state transition probability among each unit is a constant. It means that the happen time of events should obey exponential distribution.

3. The transition among the states of the system can be done at any time. The state transition probability is only with the current state of the system, and has nothing to do with the time.

4. During the time of Δt, the probability of the state transition occurs two or more times is 0. It means that during the time of Δt, the times of a unit has failure or repair is not more than one time.

5. Don't consider the condition that the failure can't be repaired, and all the repairs are effective.

B. *The Division of the State of Online Upgrade System*

The work state of online upgrade system can be divided into two states: normal and abnormal. Normal state is that the system can carry out online upgrade normally, and the abnormal state is that the system can't work normally. According to the different reasons which lead to the system abnormal, the abnormal state can be divided into seven kinds, which are download tool failure, equipment failure, software failure, communication failure, download file failure, operation failure and hardware failure. The communication failure is that because of the abnormal of bus communication, the online upgrade for a particular ECU can't be operated normally. The file failure is that the format or the size of download file doesn't conform to the requirements. The operation failure is that the failure is caused by improper operation during the process of downloading. During the work process of online upgrade system, the state model is shown in Fig. 1.

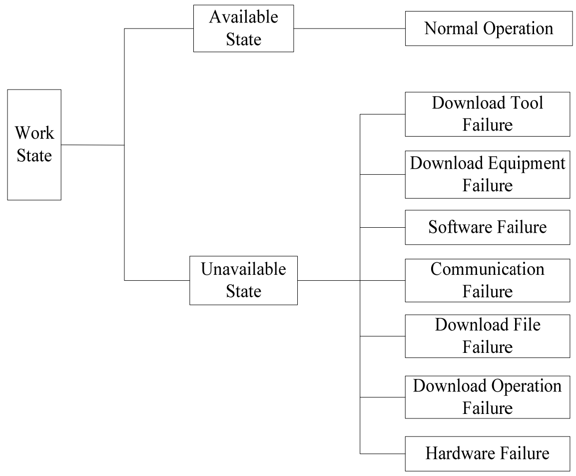

Fig. 1. State Model of Online Upgrade System

IV. BUILD THE RELIABILITY MODEL OF ONLINE UPGRADE SYSTEM BASED ON MARKOV MODEL

A. *Build the Markov Model of Online Upgrade System*

According to the state model of online upgrade system, the failure rate of the system can be set as λ_i, and the repair rate of system can be set as μ_i. Which means that the probability of system transferring from normal state to each other abnormal state is λ_i, and the probability of system transferring from each other abnormal state to normal states is μ_i. The system will stop running when it is in abnormal state, so the transition probability between each abnormal state is 0. From above we can get the state transition diagram of online upgrade system as shown in Fig. 2.

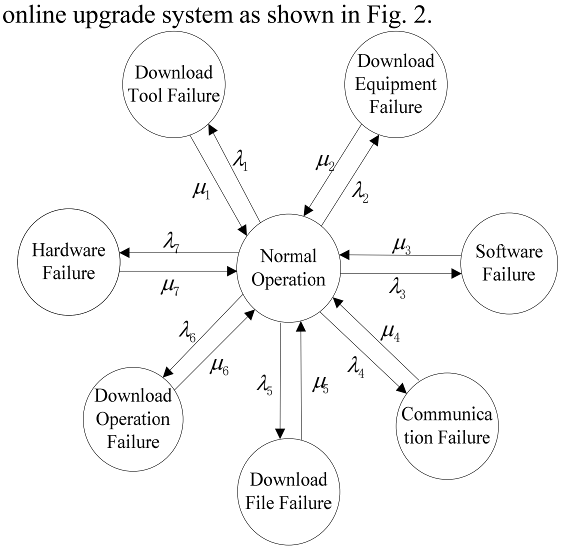

Fig. 2. State Transition Diagram of Online Upgrade System

In this system, the failure rate and repair rate are constant, and the state transition process is a homogeneous Markov process. Within the time of Δt, basing on formula (5), the state transition matrix of system is

$$p(\Delta t)=\begin{bmatrix} 1-\sum_{i=1}^{7}\lambda_i\Delta t & \lambda_1\Delta t & \lambda_2\Delta t & \lambda_3\Delta t & \lambda_4\Delta t & \lambda_5\Delta t & \lambda_6\Delta t & \lambda_7\Delta t \\ \mu_1\Delta t & 1-\mu_1\Delta t & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_2\Delta t & 0 & 1-\mu_2\Delta t & 0 & 0 & 0 & 0 & 0 \\ \mu_3\Delta t & 0 & 0 & 1-\mu_3\Delta t & 0 & 0 & 0 & 0 \\ \mu_4\Delta t & 0 & 0 & 0 & 1-\mu_4\Delta t & 0 & 0 & 0 \\ \mu_5\Delta t & 0 & 0 & 0 & 0 & 1-\mu_5\Delta t & 0 & 0 \\ \mu_6\Delta t & 0 & 0 & 0 & 0 & 0 & 1-\mu_6\Delta t & 0 \\ \mu_7\Delta t & 0 & 0 & 0 & 0 & 0 & 0 & 1-\mu_7\Delta t \end{bmatrix} \quad (15)$$

and the transfer density matrix is

$$Q=\begin{bmatrix} -\sum_{i=1}^{7}\lambda_i & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 & \lambda_7 \\ \mu_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ \mu_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 \\ \mu_4 & 0 & 0 & 0 & -\mu_4 & 0 & 0 & 0 \\ \mu_5 & 0 & 0 & 0 & 0 & -\mu_5 & 0 & 0 \\ \mu_6 & 0 & 0 & 0 & 0 & 0 & -\mu_6 & 0 \\ \mu_7 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_7 \end{bmatrix} \quad (16)$$

Because of the system is in running state under the normal condition, and the probability is 1, so the initialization matrix of the system is

$$P(0)=\begin{bmatrix} p_0(0) \\ p_1(0) \\ p_2(0) \\ p_3(0) \\ p_4(0) \\ p_5(0) \\ p_6(0) \\ p_7(0) \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

According to formula (9), there exists the following relationship between steady sate probability $p(t)$ and transfer density matrix Q

$$P'(t)=QP(t) \quad (18)$$

which is equal to the following equation

$$\begin{cases} p_0'(t)=-\sum_{i=1}^{7}\lambda_i gp_0(t)+\sum_{i=1}^{7}\lambda_i gp_i(t) \\ p_i'(t)=\mu_i gp_0(t)-\mu_i gp_i(t) \end{cases} \quad (19)$$

basing on the Laplace transformation on formula (19) and the initialization matrix of the system, there exists the following relationship

$$\begin{cases} P_0(s)-1=-\sum_{i=1}^{7}\lambda_i gP_0(s)+\sum_{i=1}^{7}\lambda_i gP_i(s) \\ P_i(s)=\mu_i gP_0(s)-\mu_i gP_i(s) \end{cases} \quad (20)$$

solving formula (20), the following results can be got

$$\begin{cases} P_0(s)=\dfrac{1}{s+s\sum_{i=1}^{7}\dfrac{\lambda_i}{s+\mu_i}} \\ P_i(s)=\dfrac{\lambda_i}{s+\mu_i}\times\dfrac{1}{s+s\sum_{i=1}^{7}\dfrac{\lambda_i}{s+\mu_i}} \end{cases} \quad (21)$$

According to the final value theorem of Laplace transformation, if time function $f(t)$ has Laplace transformation $F(s)$, and the limit $\lim_{t\to\infty} f(t)$ exists. There has the following relationship

$$f(\infty)=\lim_{t\to\infty} f(t)=\lim_{s\to 0} sF(s) \quad (22)$$

$f(\infty)$ is the specific value of time function $f(t)$ in the steady state. According to the formula (22), we can get the probability of the system in steady state as follows

$$\begin{cases} p_0(t)=\lim_{s\to 0} s\times\dfrac{1}{s+s\sum_{i=1}^{7}\dfrac{\lambda_i}{s+\mu_i}}=\dfrac{1}{1+\sum_{i=1}^{7}\dfrac{\lambda_i}{\mu_i}} \\ p_i(t)=\lim_{s\to 0} s\times\dfrac{\lambda_i}{s+\mu_i}\times\dfrac{1}{s+s\sum_{i=1}^{7}\dfrac{\lambda_i}{s+\mu_i}}=\dfrac{\lambda_i}{\mu_i}\times\dfrac{1}{1+\sum_{i=1}^{7}\dfrac{\lambda_i}{\mu_i}} \end{cases} \quad (23)$$

B. Build the Reliability Assessment Model of Online Upgrade System

The method of frequency and duration time can be used to analyze the reliability of online upgrade system [15]. In the state i, the frequency of system can be set as f_i. The state probability can be set as p_i, and the transition probability can be set as λ_{ij}. The frequency f_i is the expected frequency that per unit time stays in the state i when the online upgrade system is in the steady state. According to the definition, the frequency of system in the state i should be equal to the sum of all transition probabilities

$$f_i = \sum_{i \neq j} p_i \lambda_{ij} = p_i \sum_{i \neq j} \lambda_{ij} \tag{24}$$

The duration time of system t_i is the average duration time under the condition of system in the state i. According to the relationship between frequency and time, the calculate formula can be got as follows

$$t_i = \frac{p_i}{f_i} = \frac{1}{\sum_{i \neq j} \lambda_{ij}} \tag{25}$$

The state model of online upgrade system indicates that the system is a whole system. Each state can be regarded as each component of the system, one failure of the components can lead to paralysis of the whole system. So the online upgrade system can be regarded as a series system, and the reliability of the system can be analyzed by the method of series system. The detail reliability indexes are as shown in Table 1.

TABLE 1 THE RELIABILITY INDEXES OF ONLINE UPGRADE SYSTEM

Reliability indexes	Description
Available rate A_s	The probability of system run normally
Unavailable rate U_s	The probability of system run abnormally
Failure frequency v_s	The number of system having failure in unit time
Mean time between failures t_{fs}	The time interval between repairation and first failure of the system
Mean time to repair t_{rs}	The average time to repair the system

From the formula (23), there has $p_i(t) = p_0(t)\frac{\lambda_i}{\mu_i}$, which means that at the time of t, the probability of system transformation from available to unavailable state is $\frac{\lambda_i}{\mu_i}$. The unavailable rate of system is the sum of all probabilities that system transformation from available to all unavailable states. So the unavailable rate is

$$U_s = p_0(t)\sum_{i=1}^{7} \frac{\lambda_i}{\mu_i} \tag{26}$$

and the available rate of system is

$$A_s = 1 - U_s \tag{27}$$

and the failure frequency of system is

$$v_s = \prod_{i=1}^{7} A_i \sum_{i=1}^{7} \lambda_i \tag{28}$$

We set the mean time between failures as t_{fs} and set the mean time to repair as t_{rs}. Because there exist the following relationship between the failure period and failure frequency of the system

$$T = \frac{1}{v_s} \tag{29}$$

so the mean time between failures and the mean time to repair can be calculated as follows

$$t_{fs} = A_s \times T = \frac{A_s}{v_s} \tag{30}$$

$$t_{rs} = U_s \times T = \frac{U_s}{v_s} \tag{31}$$

V. EXAMPLE ANALYSIS

The online upgrade system has been tested on the instrument of one car. After a long time continuous testing, for all the abnormal states, Table 2 shows the failure rate λ_i and mean time to repair t_{rs} of each part of the system.

TABLE 2 THE FAILURE RATE AND MEAN TIME TO REPAIR OF EACH PART OF ONLINE UPGRADE SYSTEM

	Failure rate λ_i (times/h*100)	Mean time to repair t_{rs} (h/times)
Download tool	67.8571	0.0896
Download equipment	2.3862	0.4987
Software failure	3.9661	0.4362
Communication failure	21.1697	0.1573
Download files	2.1501	0.2372
Operation failure	75.5231	0.0478
Hardware failure	1.3349	0.5019

The transition probability between each abnormal state of online upgrade system is 0. It means that the state of the system is just transferred from normal state to one of abnormal states, or transferred from one of abnormal states to normal state. According to the relationship $t_i = \frac{1}{\sum_{i \neq j} \lambda_{ij}}$, the repair rate of each abnormal state of online upgrade system can be calculated as shown in Table 3.

TABLE 3 THE REPAIR RATE OF EACH PART OF ONLINE UPGRADE SYSTEM

	Repair rate μ_i (times/h)
Download tool	11.1607
Download equipment	2.0052
Software failure	2.2925
Communication failure	6.3573
Download files	4.2159
Operation failure	20.9205
Hardware failure	1.9924

Putting the above data into formula (23), we can get the probability of the system in normal state as follows

$$p_0(t) = 0.8539$$

This result indicates that when system is in the steady state, the probability of the system in normal state is 85.39%, and the other probabilities of the system are as shown in Table 4.

TABLE 4 THE PROBABILITIES OF ONLINE UPGRADE SYSTEM IN EACH STEADY STATE

	Steady state probability $p_i(t)$ (%)
Normal operation	85.39
Download tool	5.19
Download equipment	1.02
Software failure	1.48
Communication failure	2.84
Download files	0.43
Operation failure	3.08
Hardware failure	0.57

Table 4 shows that in all the unavailable states, the probabilities of download files and hardware failure are much lower than others. The reason is that the download files and hardware themselves have very low failure rate, and the results are in accord with the practical. Download tool failure has the highest probability in all unavailable states, because the process of online upgrade can be easily affected by download tool. So improving the stability of the download tool is the key factor to improve the success rate of online upgrade system.

Putting the experimental data into the system reliability assessment model, and according to the formula (24) and (25), the frequency and average duration time of each state can be calculated when the system is in the steady state. The specific values are as shown in Table 5.

TABLE 5 EACH STATE FREQUENCY AND AVERAGE DURATION TIME OF ONLINE UPGRADE SYSTEM

	Each state frequency f_i (times/h)	Average duration time t_i (h)
Normal operation	1.4891	0.5838
Download tool	0.5792	0.0896
Download equipment	0.0205	0.4987
Software failure	0.0339	0.4362
Communication failure	0.1805	0.1573
Download files	0.0181	0.2372
Operation failure	0.6444	0.0478
Hardware failure	0.0114	0.5019

According to the formula (26) to formula (31), the available rate A_s, unavailable rate U_s, failure frequency ν_s, mean time between failures t_{fs} and mean time to repair t_{rs} can be calculated. The specific values are as shown in Table 6.

TABLE 6 ALL THE RELIABILITY PARAMETERS OF ONLINE UPGRADE SYSTEM

Name	Value
Available rate A_s (%)	85.38
Unavailable rate U_s (%)	14.62
Failure frequency ν_s (times/h)	1.4889
Mean time between failures t_{fs} (h)	0.5734
Mean time to repair t_{rs} (h)	0.0982

Comparing the data in Table 5 and Table 6, the results show that when the system is in normal operation, the mean time between failures t_{fs} is nearly equal to the average duration time t_0, and the available rate A_s is nearly equal to the probability $p_0(t)$. The results proved that the mathematical model which has been established in this paper is correct.

VI. CONCLUSION AND FUTURE WORK

Basing on the analysis of Markov mathematical model deeply, this paper divided the real work state of online upgrade system for vehicle ECU and combined all states with Markov model, then built the reliability assessment model of the system. When the system is in stable condition, this model can well reflect the probability of each system state, and can calculate a series of reliability indexes, which are available rate, unavailable rate, failure frequency, mean time between failures, mean time to repair and so on. All the reliability indexes can quantitative assessment the system. The calculation is simple and has strong operability. This model can provide a strong theoretical support to analyze the work state of online upgrade system and solve the problems that current online upgrade system for vehicle ECU has poor reliability and stability.

This paper just divided the work state of online upgrade system into eight kinds. But in the practical work, the system may be affected by other factors, and the system may work on the other unknown states. So the reliability indexes which were calculated by this model may exist some inaccurate. In the future, deep research can be proceed, such as through real car test to ensure all affecting factors, and make the result to be more accurate.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

ACKNOWLEDGMENT

This research is funded by Chongqing Research Program of Basic Research and Frontier Technology (No. cstc2015jcyjA40007)

REFERENCES

[1] Xu Y, Wang R G, Cheng A Y, et al. “Design of Online Upgrade System for the Software of Vehicle ECU based on CAN-Bus,”. International Journal of Advancements in Computing Technology, vol. 5, no. 1, pp. 79-87, 2013.

[2] Tan T Q, Tang H H, Zhou Y L. “Design and Implementation of Bootloader for Vehicle Control Unit Based on Can Bus,” Proceedings of the FISITA 2012 World Automotive Congress. Springer Berlin Heidelberg, pp. 447-457, 2013.

[3] Yan Jie, Xu Xiaosu, Wang Lihui, et al. “Design of Secondary Bootloader for Embedded System based on DSP,” International Journal of Computer Science Issues, vol. 10, no. 1, pp. 544-548, 2013.

[4] Wang Y, Han X, Ding Y, et al. “Markov Chain-Based Rapid Assessment on Operational Reliability of Power Grid,” Power System Technology, vol. 37, no. 2, pp. 405-410,2013.

[5] Lisnianski A, Elmakias D, Laredo D, et al. “A multi-state Markov model for a short-term reliability analysis of a power generating unit,” Reliability Engineering & System Safety, vol. 98, no. 1, pp. 1-6, 2012.

[6] Dhople S V, Domínguez-García A D. “A Parametric Uncertainty Analysis Method for Markov Reliability and Reward Models,” Reliability IEEE Transactions on, vol. 61, no. 3, pp. 634-648, 2012.

[7] Ahadi A, Ghadimi N, Mirabbasi D. “An analytical methodology for assessment of smart monitoring impact on future electric power distribution system reliability,” Complexity, vol. 21, no. 1 , pp. 99–113, 2015.

[8] Zhou Kuanjiu, Wang Xiaolong, Hou Gang, et al. “Software Reliability Test Based on Markov Usage Model,” Journal of Software, vol. 7, no. 9, pp 2061-2068, 2012.

[9] Yu M, He Z Y, Qian Q Q. “Reliability Analysis of Combined Hardware/Software System Based on Markov Process,” Acta Electronica Sinica, vol. 38, no. 2 , pp. 473-479, 2010.

[10] Zhang G P, Wang X K, Deng N. “Research of Networked Control System Based on Markov Process,” Software Guide, vol. 1, pp. 116-118, 2010.

[11] Xu D Y, Dong Q X, Zhang F, et al. “Reliability Analysis of Repairable System with Component Series,” Journal of Nanchang University, vol. 37, no. 5, pp. 422-427, 2013.

[12] Wang Z Q, Wang D Z. “Reliability Evaluation of Accelerograph System Based on Markov Process,” Aviation Precision Manufacturing Technology, vol. 3, pp. 55-57, 2013.

[13] Yi J, Luo X, Ao J X, et al. “Software Fault Classification Prediction Model Based on Markov Chain,” Journal of University of Chinese Academy of Sciences, vol. 30, no. 4, pp. 562-567, 2013.

[14] Gao Z, Lei J H, Zhang D, et al. “Application of Markov Model to Analyze the Reliability of the Landing Gear,” Automation & Instrumentation, vol. 5, pp. 146-149, 2014.

[15] Liao R J, Xiao Z N, Gong J, et al. “Markov Model for Reliability Assessment of Power Transformers,” High Voltage Engineering, vol. 36, no. 2, pp. 322-328, 2010.