Universal Method on Analysis of Degree of Freedom and Working Space and Singularity of Mechanism

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***Abstract* — Pure translation and pure rotation moving elements obtained from the analysis of sub moving elements in fixed coordinate system, the equivalent space motion element is obtained by means of coordinate transformation, decomposition and synthesis carried out to find the equivalent moving elements of each joint, also the equivalent main translation moving elements and equivalent vice translation moving elements and equivalent rotation moving element obtained then. Planar six dimensional topological line map constructed to express the set of moving elements corresponding to six line, different calculation and analysis methods and rules presented to serial mechanism and parallel mechanism and hybrid mechanism, constitutes and number of degree of freedom obtained. Equivalent moving elements set of joints corresponding to the spatial moving capability of joints on mechanism, spatial moving capability of joint or branch or mechanism can be found from the map, calculation and analysis rules given to the moving elements set on six lines, characteristics of distribution and volume and type of working space obtained as well. Complementary relationship existed between the working space and singularity, complement set of working space elements set on the planar six dimensional topological map is singularity elements set, corresponding calculation and analysis rules given, the characteristics of distribution and type of singularity obtained as well.**

Key words - Mechanism, Degree of freedom, Working space, Singularity, Universal method

I. INTRODUCTION

Several dozens of formulas and methods have been put forward from the past one hundred and fifty years of Ci Bbe Scherf on the research of the degree of freedom of mechanism[1], from the principle of the method, which can be divided into five categories, the disadvantages of these methods are not suitable for all mechanism and solve all kinds of complex problems. Huang et al.[2] proposed method based on the anti spiral theory, to redefine the public constraint with the anti-screw, and gives a unified and effective modified G-K formula[3], which can solve the problem proposed by Gogu[4]. The generally used methods for working space of the mechanism is still numerical method and analytical method[5]. The core algorithm of numerical method is the boundary of working space is constrained by the nature of the boundary. Make use of inverse solution of position and K-T condition search boundary point set[6]. The grid method, Jocobi method, Carlo Monte method and the optimization method[7], these algorithms generally rely on the inverse solution of position, the basic idea of the analytical method on the parallel mechanism is disassembled into several SOCS, using the envelope surface theory to solve the single open chain spatial boundary, then using surface intersection in the whole workspace boundary[8]. In order to determine the working space of parallel mechanism, the positive solution of its position is usually required[9]. The study of singular configuration is the basis of reasonably determining the effective working space, performance of isotropic index and operability and flexibility, etc.[10]. Gosselin[11] proposed an analysis method based on the input and output speed of the mechanism, Grassman line geometry method, Jacobi matrix analysis method[12], Hunt proposed singularity analysis of parallel mechanism based on the theory of the screw theory[13], Huang et al.[2] proposed the kinematical theory in singularity based on the spiral theory.

All of the above theories and methods have its own characteristics, and have made important contributions to the study of the mechanism. There are some limitations in the scope of adaptation to the object of these theories and methods. Some conclusions are simple and intuitive, but not specific in its composition. This work proposes a new approach to the analysis of degree of freedom and working space and singularity, which defined as FWS analysis here, it is universal for all kinds of mechanism, display the number and constitute of degree of freedom intuitively and specifically, distribution and volume and type of working space and singularity as well, this method provide a solid and powerful scientific method and basic tool for the design and application of mechanism, still provide a theoretical basis for the follow-up software and automated analysis.

II.MAPPING OF MOVING ELEMENT AND TOPOLOGICAL ANALYSIS

The end point of one single kinematic chain has a certain motion representation in each joint coordinate system, by means of homogeneous coordinate transformation, coordination of end point of kinematic

chain can be transformed to coordination in the fixed base coordinate system, as shown in Expression (1), moving elements represented by points mapped from the space active coordinate system to the fixed coordinate system.

The homogeneous coordinate transformation, as shown in Expression (1), it can solve many applications such as translation and rotation and scaling, through the addition of translation matrix, or through the multiplication of rotation matrix and scaling matrix, effective mapping of points from a coordinate system of two-dimensional or three-dimensional or even higher dimensional space to another coordinate system.

The equivalent mapping results of arbitrary line segment in three-dimensional space is still a line segment or a point, the decomposition of the translation moving element in the three-dimensional fixed coordinate system realized then, as shown in Expression (2), the coordinates region is the mapping results of corresponding segment, the set of one-dimensional mapping contains only the coordinates region. Any circle can be seen as a rotation moving element of a rotation joint around determined axis, an ellipse or circle or line segment is obtained by projecting the circle to three planes of the three-dimensional coordinates, the ellipse can be divided into four segments of the arc again, as shown in Expression (3). In the three plane, the ellipse or circle or line segment has a corresponding projection line or point on the three axis, decomposition of arbitrary rotation moving element in three-dimensional fixed coordinate system realized as well, as shown in Fig. (1) to Fig. (2) and Expression (4), the rotation angle range of the corresponding plane curve (ellipse or circle) around three-dimensional Cartesian coordinate system is the mapping results, the coordinates region of the projection line segment or point in the corresponding plane of the plane curve (ellipse or circle) or line segment or point is the mapping results as well, the set of one-dimensional mapping include region of coordinates and region of angles.

$$P_0 = {}^1_0T \cdot {}^2_1T \cdot {}^3_2T \cdots\cdots {}^m_{m-1}T \cdot P_m \tag{1}$$

$$\{P_n\} \to \{P_x \cup P_y \cup P_z\}_{main} \tag{2}$$

$$\{R_O\}^e \to \{R_{c_1}(\alpha_1)\}^c \cup \{R_{c_2}(\alpha_2)\}^c \cup \{R_{b_1}(\alpha_3)\}^c \cup \{R_{b_2}(\alpha_4)\}^c \tag{3}$$

$$\{R_n\} \to \{R_x \cup R_y \cup R_z\} \cup \{P_x \cup P_y \cup P_z\}_{vice} \tag{4}$$

In Expression (2), P_x , P_y , P_z is the corresponding mapping results of decomposed sub moving element of arbitrary translation moving element P_n in three dimensional Cartesian coordinate system respectively, which defined as the main translation moving element, expressed with $\{\}_{main}$. In Expression (2), $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ is the angle region corresponding to these moving interval. In Expression (4), R_x, R_y, R_z is the corresponding mapping results of decomposed sub rotation moving element of arbitrary rotation moving element R_n in three dimensional Cartesian coordinate system respectively, P_x, P_y, P_z is the corresponding mapping results of decomposed sub translation moving element of arbitrary rotation moving element R_n in three dimensional Cartesian coordinate system respectively, which defined as the vice translation moving element, expressed with $\{\}_{vice}$.

$$\begin{aligned}
&\{{}^0P_z\} \cup \{{}^1R_y\} \cup \{{}^2R_x \cup {}^2R_z\} \cup \{{}^mP_y\} \to \\
&\{{}^0P_z\} \cup \{{}^0R_y\} \cup \{{}^0P_x({}^1R_y) \cup {}^0P_y({}^1R_y) \cup {}^0P_z({}^1R_y)\} \\
&\cup \{{}^0R_x \cup {}^0R_z\} + \{{}^0P_x({}^2R_x) \cup {}^0P_y({}^2R_x) \cup {}^0P_z({}^2R_x)\} \\
&\cup \{{}^0P_x({}^2R_z) \cup {}^0P_y({}^2R_z) \cup {}^0P_z({}^2R_z)\} \cup \{{}^0P_y\} \\
&= \{{}^0R_x \cup {}^0R_y \cup {}^0R_z\} \cup \{{}^0P_y \cup {}^0P_z\}_{main} \cup \{{}^0P_x \cup {}^0P_y \cup {}^0P_z\}_{vice}
\end{aligned} \tag{5}$$

Take the serial branch as the example as shown in Fig. (1), moving elements in each coordinate system mapping transformed to the fixed coordinate system, we get the equivalent moving elements set, synthesize the equivalent mapping results from above.

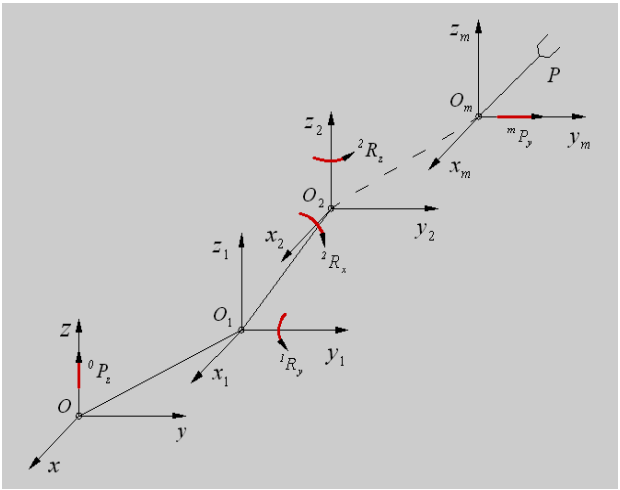

Fig.1 Moving element on serial branch

We can find the equivalent mapping results of all moving elements in the serial branch as shown in Fig. (2), which include the equivalent mapping set of main

translation moving element $\{{}^0P_y \cup {}^0P_z\}_{main}$ and equivalent mapping set of vice translation moving element $\{{}^0P_x \cup {}^0P_y \cup {}^0P_z\}_{vice}$ and equivalent mapping set of the rotation moving element $\{{}^0R_x \cup {}^0R_y \cup {}^0R_z\}$.

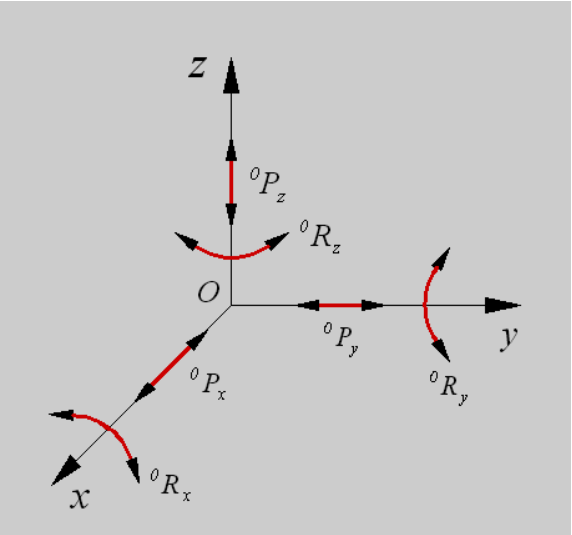

Fig.2 Equivalent mapping results of Fig.1.

III. FWS ANALYSIS

Based on the basic principle and method of graph theory and topology, three topological expressing line for translation moving element set corresponding to the axis in fixed coordinates system constructed, the equivalent mapping main moving elements on branch represented by low and thin horizontal region lines. Three topological expressing line for rotation moving element set corresponding to the axis in fixed coordinates system constructed on the right of the above mentioned three lines, the equivalent mapping rotation moving elements on branch represented by low and thin horizontal region lines, the equivalent mapping vice moving elements on branch represented by high and thick horizontal region lines on the left three lines.

A. Analysis on Degree of Freedom

The moving capability of serial branch is the overlying of that of each joint, union operation carried out with set region on six topological expressing line respectively, it is clearly whether a non empty set or non single point set exist on the six topological expressing line respectively, the number of topological expressing line with non empty set or non single point set is the number of DOF of this serial branch, as well as its constitute can be seen clearly. $\{{}^0R_x \cup {}^0R_y \cup {}^0R_z\} + \{{}^0P_y \cup {}^0P_z\}_{main}$ is the definite or main DOF, $\{{}^0P_x \cup {}^0P_y \cup {}^0P_z\}_{vice}$ is the indefinite or following DOF, thus found the DOF of this mechanism is five, when the mechanism locate in special configuration, that is $\{{}^0P_x\}_{vice} \neq \Phi$, the indefinite DOF is three, there will be six for the DOF, as shown in Fig. (3).

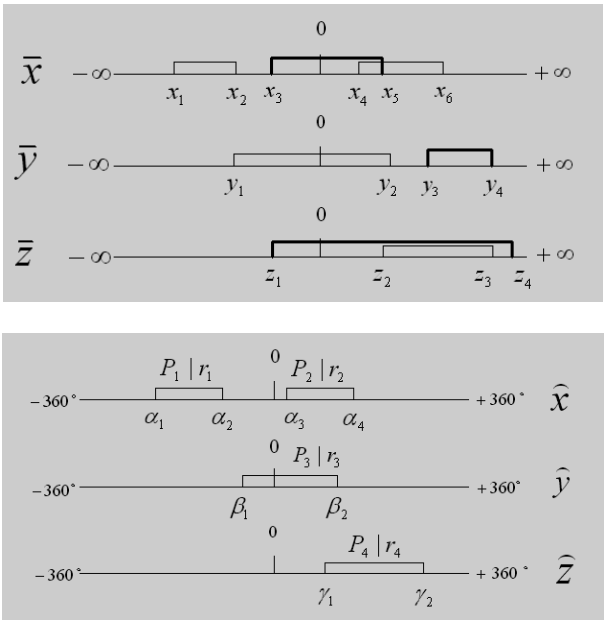

Fig.3 Topological analysis of DOF of mechanism of Fig. (1).

The formula for DOF can be obtained as following:

$$F = F(\bar{x}) + F(\bar{y}) + F(\bar{z}) + F(\hat{x}) + F(\hat{y}) + F(\hat{z}) \qquad (6)$$

Among that, $F, F(\bar{x}), F(\bar{y}), F(\bar{z}), F(\hat{x}), F(\hat{y}), F(\hat{z})$ is the DOF and index along the axis x, y, z and index around the axis x, y, z.

When $\{\bar{x}\} = \{\overline{x_1 x_2}\} + \{\overline{x_3 x_6}\} \neq \Phi$ and it is non single point set, $F(\bar{x})$=1, else $F(\bar{x})$=0.

When $\{\bar{y}\} = \{\overline{y_1 y_2}\} + \{\overline{y_3 y_4}\} \neq \Phi$ and it is non single point set, $F(\bar{y})$=1, else $F(\bar{y})$=0.

When $\{\bar{z}\} = \{\overline{z_1 z_4}\} \neq \Phi$ and it is non single point set, $F(\bar{z})$=1, else $F(\bar{z})$=0.

When $\{\hat{x}\} = \{\overline{\alpha_1 \alpha_2}\} + \{\overline{\alpha_3 \alpha_4}\} \neq \Phi$ and it is non single point set, $F(\hat{x})$=1, else $F(\hat{x})$=0.

When $\{\bar{y}\}=\{\overline{\beta_1\beta_2}\}\neq\Phi$ and it is non single point set, $F(\bar{y})$=1, else $F(\bar{y})$=0.

When $\{\bar{z}\}=\{\overline{\gamma_1\gamma_2}\}\neq\Phi$ and it is non single point set, $F(\bar{z})$=1, else $F(\bar{z})$=0.

The DOF of mechanism corresponding to Fig. (3) can be found

$$\begin{aligned} F &= F(\bar{x})+F(\bar{y})+F(\bar{z})+F(\hat{x})+F(\hat{y})+F(\hat{z}) \\ &= 1+1+1+1+1+1 \\ &= 6 \end{aligned} \tag{7}$$

The parallel mechanism is composed of a series of serial branch, according to the general theory and method of analysis on spatial moving capability of mechanism, intersection adopted to the spatial moving capability of each branch in parallel mechanism, thus we get the DOF. Make use of the analysis results on DOF obtained from the serial branch, intersection adopted to the moving element set on corresponding six topological expressing line between serial branches in parallel mechanism, the DOF obtained then.

The hybrid mechanism is composed of serial branch and parallel mechanism, it can be seen as the union of moving capability of two serial branches, in order to found the DOF of hybrid mechanism, the first step is to found six topological expressing line of moving elements set of parallel mechanism, the second step is to found six topological expressing line of moving elements set of serial branch, the third step is to adopt union operation between the corresponding six topological expressing line of moving elements set of the aforementioned results in last two step, from the result of third step, we can found the number and constitute of DOF easily.

B. Analysis on Working Space

In-depth analysis of the working space is an important basis for the practical application of mechanism after the degree of freedom is determined. The working space of the mechanism is the working area of the output end of the mechanism, It is an important index to measure the performance of mechanism. It is relatively easy for the analysis of working space of serial mechanism, but it is a very complicated problem to solve the problem of the working space of parallel mechanism and hybrid mechanism.

Based on the analysis of the previous part of this paper and the existing theory of the mechanism analysis, joint moving element set corresponding to the spatial moving capability of moving joint of mechanism, we can get the spatial moving capability of joint or branch or mechanism from the planar six-dimensional topological analysis as shown in Fig. (3), spatial moving capability set on six topological expressing lines can be obtained as follows.

$$W(\bar{x})=\{\bar{x}\}=\{\overline{x_1x_2}\}+\{\overline{x_3x_6}\} \tag{8}$$

$$W(\bar{y})=\{\bar{y}\}=\{\overline{y_1y_2}\}+\{\overline{y_3y_4}\} \tag{9}$$

$$W(\bar{z})=\{\bar{z}\}=\{\overline{z_1z_4}\} \tag{10}$$

$$W(\hat{x})=\{\hat{x}\}=\{\overline{\alpha_1\alpha_2}\}+\{\overline{\alpha_3\alpha_4}\} \tag{11}$$

$$W(\hat{y})=\{\hat{y}\}=\{\overline{\beta_1\beta_2}\} \tag{12}$$

$$W(\hat{z})=\{\hat{z}\}=\{\overline{\gamma_1\gamma_2}\} \tag{13}$$

From Expression (8) to (13), the spatial moving capability set of mechanism can be known,

$$\begin{aligned} W &= W(\bar{x})+W(\bar{y})+W(\bar{z})+W(\hat{x})+W(\hat{y})+W(\hat{z}) \\ &= \{\overline{x_1x_2}\}+\{\overline{x_3x_6}\}+\{\overline{y_1y_2}\}+\{\overline{y_3y_4}\}+\{\overline{z_1z_4}\} \\ &\quad +\{\overline{\alpha_1\alpha_2}\}+\{\overline{\alpha_3\alpha_4}\}+\{\overline{\beta_1\beta_2}\}+\{\overline{\gamma_1\gamma_2}\} \end{aligned} \tag{14}$$

In iew of the sets of elements on the six dimensional topological line, corresponding operation and analysis rules are given as follows.

In order to found the value of spatial moving capability range, namely volume, following form can be known from the planar six dimensional topological graph, the spatial geometry and the theory of calculus.

(1) Points set, $V=\bigcup_{i=1}^{m}\{P_i\}=\{P_1\cup P_2\cup P_3\cdots\cdots\cup P_m\}$,

each element in the set is a single point coordinate element.

(2) Lines set, $V=\bigcup_{j=1}^{n}\{L_j\}=\{L_1\cup L_2\cup L_3\cdots\cdots\cup L_n\}$,

each element in the set is an interval element.

(3) Planes set, $V=\bigcup_{k=1}^{r}\{PL_k\}=\{PL_1\cup PL_2\cup PL_3\cdots\cdots\cup PL_r\}$,

each element in the set is a spatial surface area element.

(4) Solids set, $V=\bigcup_{h=1}^{s}\{S_h\}=\{S_1\cup S_2\cup S_3\cdots\cdots\cup S_s\}$,

each element in the set is a region element of the spatial geometry.

(5) Set of any two kinds of points, lines, surfaces, and solids, each element in the set contains any two elements of the preceding four item, that is:

$V=\{P\}+\{L\}/\{P\}+\{PL\}/\{P\}+\{S\}/\{L\}+\{PL\}/\{L\}+\{S\}/\{PL\}+\{S\}$.

(6) Set of any three kinds of points, lines, surfaces, and solids, each element in the set contains any three elements of the preceding four item:

$$V = \{P\}+\{L\}+\{PL\}/\{P\}+\{L\}+\{S\}/\{P\}+\{PL\}+\{S\}/\{L\}+\{PL\}+\{S\}\cdot$$

(7) Set of four kinds of points, lines, surfaces, and solids, each element in the set contains four elements of the preceding four item, $V = \{P\}+\{L\}+\{PL\}+\{S\}\cdot$

Specific number of points, lines, surfaces, solids can be obtained from the planar six dimensional topological graph of FWS analysis of mechanism, here defined as FWS analysis graph, the length of the line, the area of the surface, the volume of the solid can be obtained through integral operation on related elements on FWS analysis graph.

According to the physical manifestation and spatial distribution of working space, it can be used to analyze the distribution and type of working space in addition to the volume, improve the analysis results
Of working space characteristics deeply.

C. Analysis on Singularity

Special distribution and relationship between the working space and the interior belongs to the analysis on singularity, singularity is the inherent nature of mechanism, it has a serious impact on the performance of mechanism. When the mechanism is in some particular position, the input members of the mechanism lost control over the output members, singular position should be avoided in the design and application of mechanism. The study of singular configuration is the basis of reasonable determination of effective working space, making the performance index of isotropic, operability, flexibility and so on.

Based on the foregoing analysis and the existing theory on mechanism analysis, complementary relationship exist between the working space and the singularity of mechanism. Complement set of spatial moving capability set is singularity elements set on FWS analysis graph, the blank area between the range of spatial moving capability is unreachable space, in the translation domain and rotation domain, other regions except the reachable domain is singularity domain, complementary relationship exists between these two kinds of domain, there will be:

$$S = \Omega - W = \overline{W} \tag{15}$$

unreachable spatial domain of joint or branch or mechanism which can be obtained from FWS analysis graph, as shown in Fig. (3). The singularity elements set on six topological expressing lines can be written as:

$$S = S(\overline{x})+S(\overline{y})+S(\overline{z})+S(\bar{x})+S(\bar{y})+S(\bar{z})\,. \tag{16}$$

The type of singularity can be found from the research on the distribution characteristics of translation domain and(or) rotation domain, distribution characteristics of main translation and vice translation, distribution characteristics of translation domain and rotation domain along one axis, distribution characteristics of translation domain or rotation domain between different axis, quantity and interval and distribution and size and level of each kind of domain, the research on the spatial moving capability of mechanism of great significance.

IV. CONCLUSION

(1) It is feasible to analyze the FWS of mechanism jointly based on the equivalent mapping of spatial moving elements, and proposed a method of spatial equivalent mapping based on decomposition and synthesis of moving element, equivalent main translation moving element set, equivalent vice translation moving element set, equivalent rotation moving element set can be expressed on planar six-dimensional topological expressing line with interval.

(2) The DOF can be obtained through different operation with the analysis results displayed in planar six-dimensional topological expressing line with interval, union operation adopted for serial mechanism, intersection operation adopted between branches for parallel mechanism, union operation adopted between serial branch and parallel mechanism for hybrid mechanism, the number and constitute can be seen from the last analysis and operation results on six topological expressing line.

(3) The characteristics of constitute and volume and distribution and type can be obtained from the physical manifestation of moving element set on topological expressing line, special integral operation adopted to found the volume.

(4) Complementary relationship exists between the working space and the singularity space of mechanism, characteristics of constitute and type can be obtained from the quantity and interval and distribution and size and level of each kind of domain on FWS analysis graph.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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