Application Research of Grey Information Renewal Model GM(1,1) in Water Demand Forecasting

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***Abstract* —This paper researches the application of grey system theory in urban water demand forecasting. Forecasting accuracy for data of recent years is high while error exists for medium and long-term water demand prediction when using conventional GM (1,1) model. Aimed at the flaws of the conventional GM (1,1) model, the grey information renewal model GM (1,1) has been established for water demand forecasting and applied in the water demand forecasting of Xilingol League area from 2015 to 2020, which shows that the prediction result is reasonable and reliable with less error and higher precision. This model, with advantages of simple and practical application and high accuracy, provides a new method for water demand forecasting.**

Keywords - urban water demand; grey information renewal model; grey forecasting

I. INTRODUCTION

Water is the foundational natural resource, strategic economic resources and public social resource and is one of the important controlling factors of the ecology and environment. In the process of China's urbanization, the development of water resource is increasing. With the limited urban water supply, the water shortage and water cut-off events occurred from time to time and contradiction between water supply and demand is increasingly significant. According to statistics, about 670 cities in China, more than half have varying degrees of water shortage problem, among which more than 110 cities have severe water shortage. Nine of the 14 open coastal cities in China has severe water shortages and the most serious ones are Beijing, tianjin, Qingdao and Dalian etc..

In terms of the region of relatively lack of water resources, the sustainable development and utilization of water resources is critical and the premise of sustainable utilization is to reasonable allocate the water resources. Water resources allocation is a multi-stage, multi-level, multi-objective and multi-agent decision problem [1], also is the process of risk decision-making and in-advance-decision-making. Analysis on balance between supply and demand is the basis of reasonable allocation of water resources. And the analysis process involves two aspects of water supply prediction and water demand forecasting. Under the condition of highly development of economy and society and increasing water demand, to provide accurate and reasonable water demand forecasting [2], is significant to the coordinated development of social economy and environment.

In recent years, main methods for urban water demand forecasting are quota method (or called index method) and regression analysis method [3~4]. Although the forecasting result by these simple and intuitive conventional methods has certain reference value, as many factors involved in water demand forecasting and insufficient consideration of these factors in conventional methods, errors exist in the results of prediction accuracy. The grey theory recently has been applied in areas such as agriculture and meteorology etc. with higher prediction precision. Therefore to introduce grey theory for urban water demand forecasting is considered.

Grey system is "small sample" of "partial known information and partial unknown information" and uncertain system of "poor information". Through generation and development of "part" of the known information, it will realize a precise description and understanding of the real world. Urban water demand forecasting relates to the factors such as population, economic structure, water reservation level and the local water resources endowment, is a complex multi-level and multi-factor system including known and unknown information, which conforms to the characteristics of the grey system. According to the grey system theory, research of internal factors and relationship between water system is not needed. The various factors influencing the water demand will be considered as grey amount related to time and changed in a certain range, useful information will be driven from its own data, and a model will be established to find and reveal the potential rules of systematic water demand. And this model will be applied for future urban water demand forecasting.

Currently when using grey model to predict, the most commonly is the conventional GM (1, 1) model [5]. The modeling sequence of this only considers all the data in the past of the real time t=n, the influence of some of future disturbance factors on the system is less, which leads to accuracy of only 1~2 data after t=n is higher when using the model to predict but the prediction error of the model will be higher for future data. In order to make up this deficiency, the grey information renewal GM (1,1) model has been introduced. With the growing data values, early data share gradually reduces and appears relatively minor, so information renewal model arises to add a new information while delete one of the earliest old information, thus maintaining total data and the sequence equant [6]. When grey information renewal model been adopted to forecast, a value will be provided by GM (1, 1) model established through a known sequence, then the predicted value will be added to the known sequence and one of the early data will be removed at the same time to maintain the sequence equant. Another GM (1, 1) model will be built according to the new sequence to predict the next data, and the data will be added to the sequence and one of the early data will be removed at the same time, thus caused information renewal, which predict individually, replace in turn and at last get the forecasting value of the planning target year.

II. GREY MODEL THEORY

A. Conventional GM (1,1) model

Conventional GM (1, 1) model is one of the most commonly used models in grey system. It is the model consisted of a first order differential equation including single variables. Through the generation and processing of raw data, a whole systematic principle will be sought based on original data accumulation generation. A model will be established to find and reveal the potential rules of systematic water demand, which changes in a index trend way. Through establishing the index differential equation, a forecasting model can be achieved. Modeling steps for basic GM (1, 1) model of are as follows:

(1) Taking irregular, random and obviously pendular existing raw data of urban water supply $x^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right\}$ as the first-order accumulative calculation [7~8], accumulative sequence $x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right\}$ will be generated; the newly generated data sequence is monotonic increasing sequence, which increased the regularity of the original data sequence.

(2) Establish differential equation for accumulative generation sequence:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u$$

among which: a and u is undetermined coefficients

Its discretization induces the matrix form $Y = XB$. In the formula,

$$Y = \begin{bmatrix} x_1^{(0)}(2) \\ x_1^{(0)}(3) \\ \vdots \\ x_1^{(0)}(N) \end{bmatrix}$$

$$B = \begin{bmatrix} a \\ u \end{bmatrix},$$

$$X = \begin{bmatrix} -\frac{1}{2}\left[x_1^{(1)}(1) + x_1^{(1)}(2)\right] & 1 \\ -\frac{1}{2}\left[x_1^{(1)}(2) + x_1^{(1)}(3)\right] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}\left[x_1^{(1)}(N-1) + x_1^{(1)}(N)\right] & 1 \end{bmatrix}$$

(3) To obtain undetermined coefficients *a* and *u*, least square method can be used to solve above equation, which will obtain:

$$\hat{B} = (X^T X)^{-1}(X^T Y) = \begin{bmatrix} \hat{a} \\ \hat{u} \end{bmatrix}$$

the solution of differential equation is :

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{u}{a})e^{-ak} + \frac{u}{a}$$

the simulation value of $x^{(0)}(k)$ is :

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k).$$

B. Forecasting theory of grey information renewal GM (1,1) model

Initial conditions for the grey prediction model is $x^{(0)}(1)$, the latest predicted information is not been fully applied in the model, thus leading to higher prediction accuracy is only for few recent data and prediction error exists for medium and long-term prediction.

As per principle of equivalent dimensional addition, grey information renewal model adds new data information $x^{(0)}(n+1)$ in original data sequence while deleting the oldest information $x^{(0)}(1)$, the sequence is replaced by $x^{(0)} = \left\{x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(n+1)\right\}$. GM (1, 1) model will be established through repeated above steps. Thus to repeat and replace in turn until the prediction goal completed.

Data number of original data sequence in grey information renewal model should not be less than 4. In practical application, data will be selected for modeling according to simulation accuracy requirement while not all the data in the original data sequence will be involved.

Generally, some or all of the data, including the origin $x^{(0)}(n)$ will be selected to constitute GM (1, 1) model group and the simulation accuracy of each model in the group will be compared. Then tmodel with higher simulation precision will be chosen as the foundation model and corresponding dimension of its modeling data sequence will be determined.

C. Accuracy check

Usually a posteriori-variance-test will be adopted to check the precision of the model [9~10], and the accuracy will be assessed jointly by indicators of mean square error ratio C and minor error probability p.

Set S_1^2, S_2^2 as the mean square error (MSE) of the original sequence $x^{(0)}$ and residual sequence $\varepsilon^{(0)}$, and $\hat{x}^{(0)}$ is GM model simulation sequence, the posterior differential ratio calculation formula is as follows:

$$\begin{cases} \bar{x} = \frac{1}{n}\sum_{k=1}^{n} x^{(0)}(k) \\ S_1^2 = \frac{1}{n}\sum_{k=1}^{n}(x^{(0)}(k)-\bar{x})^2 \\ \bar{\varepsilon} = \frac{1}{n}\sum_{k=1}^{n}\varepsilon^{(0)}(k) \\ \varepsilon^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}(k) \\ S_2^2 = \frac{1}{n}\sum_{k=1}^{n}(\varepsilon^{(0)}(k)-\bar{\varepsilon})^2 \end{cases} \Rightarrow \begin{cases} C = \frac{S_2}{S_1} \\ p = P\left\{\left|\varepsilon(i)-\bar{\varepsilon}\right| < 0.6745 S_1\right\} \end{cases}$$

According to above formula, for index of the mean square error ratio C and small error probability p, index C required by calculation accuracy should be as small as possible and the precision of model forecasting will be higher with higher index p, the model accuracy level equals Max { the level of C, the level of p } . Standard for model accuracy grading [11] is shown in table I.

TABLE I. GREY MODEL FORECASTING ACCURACY GRADING TABLE

Grade	MSE ratio *C*	Small error probability *p*
Grade 1(good)	≤0.35	≥0.95
Grade 2 (qualified)	0.35＜*C*≤0.50	0.80≤*p*＜0.95
Grade 3 (barely qualified)	0.50＜*C*≤0.65	0.70≤*p*＜0.80
Grade 4 (unqualified)	＞0.65	＜0.70

D Model modification

When the model accuracy grade is low, it can be modified through the method of residual sequence modeling, so as to improve the precision of model [12].

Taking sequence $x^{(1)}(k)$ generated by the original model and its simulation valuation $\hat{x}^{(1)}(k)$ as difference, the number for the residual sequence $\varepsilon^{(0)}$ will be generated as $\{\varepsilon^{(0)}(i), \varepsilon^{(0)}(i+1), \cdots, \varepsilon^{(0)}(n)\}$ .Then to set up GM(1,1) model for $\varepsilon^{(0)}$, getting $\varepsilon^{(1)}(k+1) = (\varepsilon'^{(0)}(1) - \frac{u'}{a'})e^{-a'k} + \frac{u'}{a'}$.

Therefore, the modified model can be obtained as follows:

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{u}{a})e^{-ak} + \frac{u}{a} + \delta(k-i)\left[(\varepsilon'^{(0)}(1) - \frac{u'}{a'})e^{-a'k} + \frac{u'}{a'}\right]$$

Among which: $\delta(k-i) = \begin{cases} 1 & k \ge i \\ 0 & k < i \end{cases}$, $k = i, i+1, \cdots, n$

III. EXAMPLES OF APPLICATION

Xilingol League area is located in the middle of Inner Mongolia plateau and on the edge of low hills to the west of the Greater Hinggan Mountains. Abundant mineral resources exist within the territory of the League and mainly are oil and coal. In recent years with the exploration of coal resources, development of coal-electricity base and coal chemical projects, economic and social development is rapid and tension between supply and demand of water resources is increasing, which has become the main factor of restricting economic and social development. Water consumption from 2004 to 2013 of Xilingol League area is shown in table II.

TABLE II. WATER CONSUMPTİON FOR 2004~2013 XİNLİNGOL LEAGUE (00,000 M^3)

Year	2004	2005	2006	2007	2008
Water consumption	20432	21828	26339	25878	32110
Year	2009	2010	2011	2012	2013
Water consumption	30587	37920	38829	38081	36901

According to the principle of grey information renewal forecasting, water consumption data from 2004 ~ 2013 (10 d) and from 2006 to 2013 (8 d) respectively was taken to establish the grey information renewal model GM (1, 1). To compare the conventional GM (1, 1) model and grey model, the conventional GM (1, 1) model has been established at the same time. According to the accuracy test formula, the accuracy calculation under different dimensions is shown in table III..

TABLE III. ACCURACY TEST TABLE

Item	Type	Years	Dimensions	Aver. relative error	MSE ratio C	Small error probability p
Model □	conventional	2004~2013	10	1.46%	0.362	1
	modified			1.38%	0.289	1
Model □	conventional	2006~2013	8	0.86%	0.351	1
	modified			0.34%	0.135	1

According to the model accuracy grading standard, the prediction accuracy for modified grey information renewal GM (1, 1) model is "grade 1" and the average relative error and mean square error ratio C is less than conventional GM (1, 1) model, which means that grey GM (1, 1) model has the advantages of high prediction accuracy and good simulation effect. Through contrast of the two grey information renewal GM (1, 1) models, it is observed that the models are also "grade 1"; however through comparing the average relative error and the mean square error ratio, simulation accuracy of model II is higher than model I. Therefore, model II will be chosen as the base model for water demand forecasting. The simulation values of the base model are shown in table IV..

TABLE IV. ACCURACY TABLE (00,000M^3)

Year	2006	2007	2008	2009
Actual value	26339	25878	32110	30587
Simulation value	26339	25918.14	32050.51	30659.21
Year	2010	2011	2012	2013
Actual value	37920	38829	38081	36901
Simulation value	37842.95	38708.04	38312.88	37146.01

Based on above grey information renewal model, water demand of Xilingol League area from 2015 to 2020 has been forecasted and the results are shown in table VI..

TABLE V. WATER DEMAND FORECASTING（00,000M^3）

Year	2015	2016	2017
Simulation value	37413.84	37726.6	38007.74
Year	2018	2019	2020
Simulation value	38300.22	38610.75	38906.98

The table V shows that forecasted water demand in 2020 is 389.0698 million m3, and compared with local forecasted value 396.8 million m3, the error is only 1.9%. Thus the model can be used for water demand prediction in the region.

From the forecasted values, it is observed that water demand in this region in 2015 ~ 2020 will be presented in a slow growth trend. Therefore, suggestion was made to strengthen water consumption planning, adjust and optimize the industrial structure, develop water-saving economy and continue to develop industry of less-water-consumption and high-water-consumption-efficiency.

The result provides certain reference value for a comprehensive understanding of the region's water resources situation. The forecasted value can be applied to reasonably allocate water resources or adjust the industrial structure, so as to realize the sustainable development and utilization of water resources in the region.

V. CONCLUSION

In the process of researching urban water demand forecasting problem, this paper introduces the grey system theory. Aimed at the problems existing in the conventional GM (1, 1) model, the grey system theory is introduced to establish the grey information renewal GM (1, 1) water demand forecasting model. As this model considered perturbation factors in the process of system development, supplemented the new information and removed the old information as time pass by, thus it can reflect the current characteristics and is more practical than conventional GM (1, 1) model. Also this model was applied to predict water demand of Xilinhot city and the results show that accuracy and simulation effect of this model is better than conventional GM (1, 1) model, which reflects actual change characteristics of the future water demand and is suitable for urban water demand prediction in a medium and long term.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

ACKNOWLEDGMENT

This work is supported by grants from the Chinese National Natural Science Foundation under contract (No.51409290), Inner Mongolia Water Resources scientific and technological project.

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