An S[4]PR Class Petri Net Supervisor for Manufacturing System

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***Abstract* - Petri Nets are a family of formalisms that allow the effective modelling of manufacturing systems. The experimental approach method is provided to ensure formality in elementary siphons to distinguish deadlock conditions for a class in terms of generalized Petri nets, namely S4PR. A siphon is a special structural object of a Petri net and plays an important role in synthesizing a live Petri net controller for Flexible Manufacturing Systems FMS. An FMS corresponds to a class of concurrent systems called Resource Allocation Systems (RASs). The application of Petri Nets (PNs) in RASs is an active research field devoted to defining and exploiting different subclasses of Petri Nets allowing the modeling of the widest set of RAS. A monitor is added for a derived minimal siphon such that it is max-controlled if it is elementary with respect to the siphons that have been derived. Simulation of PN for the structural analysis and reachability graph analysis are employed to synthesize and control FMS. An example is used to illustrate this method.**

Keywords – Simulation, Siphons, Petri net, S4PR, FMS

I. INTRODUCTION

An automated manufacturing system is a computer controlled system consisting of finite resources such as machines, automatic guided vehicles (AGVs), robots, buffers, and can be supply process different types of parts. Typically an FMS consist of several machines to process concurrently different types of raw parts processed in the system production sequence competed sharing a limited number of resources consequently the problems of blocking, conflict and deadlock may occur in the system. Deadlock is especially situations regarded as an important issue derives it can disable operations of the entire or a part of manufacturing systems [13], [19], [20] - [26], [36]. The design a structurally efficiently operates an automated manufacturing system and to make the perfect usage of system resources, it is necessary to coordinate and control developed for the shared resources.

A Resource Allocation Systems (RAS) in FMS consist of a set of processes that share competitively a finite set of resources on FMS [14], [10]−[15], [21]−[28], [31]−[36]. The reaction competition for resources between the processes can cause deadlock problems. Deadlock problems that can appear in the systems designed with the approach appropriation system resources which can be analyzed and solved in an efficient mannerism as by mathematical utilization of models such as PN. By using PNs approach to the various systems can be modeled, including FMS. The formulation of these systems under this approach, they have led to several subclasses [1]−[4] of Petri nets models with characterization specification results of depending on a structural analysis. One of the overall large classes of these subclasses nets with the theoretical solid develop of the structural analysis in an S[4]PR net.

A fast deadlock detection approach is particularly fabulous proposed by Chu and Xie [7]. They could be syntheses an algorithm based on mixed integer programming (MIP), and the unmarked siphons structurally are bounded nets connects to deadlocks. They developed the potential siphons for the analysis of ordinary Petri nets and proposed a mathematical programming to approach and a mixed integer programming accomplishment is checking Petri nets and structurally bounded Petri nets, and developed the complete siphon enumeration of a plant model. The mixed integer programming (MIP)-based method is more efficient than the traditional complete siphon enumeration approach to computes a maximal emptied siphon.

The major successes of a (system of simple sequential processes with resources S^3PR) were an early work by Ezpeleta et al. [12]. They have developed a design method of characterized the liveness in terms of siphons and propose a deadlock prevention approaches that adds monitors for strict minimal siphons (SMS), and from the emptied siphons can be generated. A Petri net model has a powerful tool for describing and analysis, the behavior of FMS was developed to control deadlock prevention in the class of Petri nets namely (S[3]PR). There are many deadlock control policies are proposed based on siphons [1]−[3], [19], [21]−[25], [35].

In [4] the authors describe a deadlock prevention and avoidance methods based on the structure theory of Petri nets for a class called S[4]R (Systems of Sequential Systems with Shared Resources). They proposed S[4]R, which are a generalization of S[3]PR nets, to extend S[3]PR and Production Petri Nets (PPNs) nets to model systems that can use not only alternative resources, as in S[3]PR nets, but they can also utilize more than one resource simultaneously. Furthermore, an S[4]PR (Simple Systems of Simple Sequential Processes with Resources) is equivalent to an S[4]R [4]. The benefits of

DOI 10.5013/IJSSST.a.17.33.31

ISSN: 1473-804x online, 1473-8031 print

the deadlock prevention policy, by means of adding a control place (monitor) for each siphon to remain marked for all reachable markings. Furthermore, the author [4] studies a deadlock prevention and deadlock avoidance policy to control the reinforcement S^4R nets using a resource allocation policy based on the unsafe marking concept. The resource allocation decision can be structured computation the controller depended on siphons. Recent researchers [10], [15]−[17], [21]−[28] are developed a novel approach also suffers from such problems as computational complexity [10], [11], [36], behavior permissiveness, and structural complexity.

The structural object of siphons is extensively used to characterize and analyze deadlock situations in FMS that are modeled on Petri net (PN) [13]. The generalized PN is considerably more complex than that in an ordinary one on account of a marked siphon might not be sufficient in the absence of dead transitions. An iterative method to deal with the deadlocks in S^4PR nets is developed in [6], [13],[15]−[17], [23], [24], [27], [28], [31]−[34] . The iteration, it typically uses an integer programming to find out approaching an insufficiently marked siphon that can cause dead transitions and then designs control places for the siphon by solving an integer programming problem.

Recently, there exists an increase in Petri net models for modelling of RAS in the states of FMS, that is, lead to some of the syntactical limitations of the S^3PR class [1]−[3],[10],[19], [24], [25], [35]−[36]. The classes of an S^4PR is solicitude the modeling of concurrent cyclic sequential processes sharing common resources, where an operation places can use simultaneously multiple resources of different types. Classes of an S^4PR net model [6], [15]−[17], [26], [28], [31-34] generalizes the earlier, that allowing multiple simultaneous allocations of resources before the process. One of the formal, recent FMS representations is a Petri net model, the classical model for both pictorial and a formal description of the systems where concurrent processes are performed. The production Petri nets S^3PR in [12], Linear S^3PR (L-S^3PR) [11], S^4R [4], S^4PR [6], [27, [28], [33], **extended** the capabilities of S^3PR to be S^4PR models beyond Sequential RAS (ES^3PR) [21], [22], and **weighted** S^3PR (systems of simple sequential processes with resources) WS^3PR [9], [26] etc. Chao [8] proposed deadlock control approaches from S^3PGR^2 (systems of simple sequential processes with general resources necessity) experience in incorrect or restricted liveness of the system is depending on max-controlled siphons. Shih et al. [30] proposed a sequence of control in S^3PMR.

Petri nets have an appealing graphical representation of a powerful mathematical formulation and is suitable tool for the modelling of FMSs with parallel and concurrent activities. Thus, it has generated intense interest in many researchers [1]−[28], and [30]−[36]. Petri nets are a formalism that allows the modeling of systems involving concurrency, resource sharing, conflict, which allows to the effectiveness of the correctness requirement are performed on the qualitative properties of the net modeling. Qualitative properties of Petri nets (such as siphons, liveness, deadlock, conflict, and safeness) are useful in discrete manufacturing applications. The formalism employed to ensure the desired pattern of the flow control, particularly the deadlock avoidance of FMS. In this work, the monitors are added to the plant model such that the elementary siphons in S^4PR are all max-controlled and no insufficiently marked siphon is generated.

The remainder of this paper is organized as follows: section II provides the preliminaries and notations of Petri nets and S^4PR nets which has the focus of this work. A deadlock prevention policy for an S^4PR is developed in Section III. In Section IV, an S^4PR example is proposed as a method to show our application efficiency and simulation PN of the proposed method. Section V concludes the paper.

II. PRELIMINARIES

Petri nets have been mainly used for studies in computer operating system behavior, but recently used to model and visualize behaviors comprising concurrency, sequential processes with resource sharing found in FMSs.

A. Petri Net

A generalized Petri net [1], [19], [21] is a four-tuple $G = (P, T, E, W)$, where P and T are finite, non-empty and disjoint sets. P is a set of places and T is a set of transitions with $P \cup T \neq \emptyset$, and $P \cap T = \emptyset$. $E \subseteq (P \times T) \cup (T \times P)$ is called a flow relation of the net, represented by arcs with arrows from places to transitions or from transitions to places. $W : (P \times T) \cup (T \times P) \rightarrow Z^+$, where $Z^+ = \{0,1,2,...\}$ is a mapping that assigns a weight to an arc: $W(x, y) > 0$ if $(x, y) \in E$, and $W(x, y) = 0$ otherwise, where $(x, y) \in (P \cup T) \cup (T \cup P)$. A net is said to be ordinary, denoted by $G = (P, T, E)$, if $\forall (x, y) \in E$, $W(x, y) = 1$. A net $G = (P, T, E, W)$ is pure (self-loop free) if $W(x, y) > 0$ implies $W(x, y) = 0$, $\forall (x, y) \in (T \times P)$. A pure net $G = (P, T, E, W)$ can be alternatively represented by its incidence matrix $[C]$ that is a $|P|$ x $|T|$ integer matrix with $[C](p,t) = W(t, p) - W(p,t)$.

Given a net $G = (P, T, E, W)$, and a node $x \in P \cup T$, The preset of x is defined as ${}^\bullet x = \{y \in P \cup T \mid (y, x) \in E$. While the post-set of a node $x \in P \cup T$ is defined $x^\bullet = \{y \in P \cup T \mid (x, y) \in E$. This notation can be extended to a set of nodes as follows: given $x \in P \cup T$,

${}^{\bullet}X = \cup_{x\in X} {}^{\bullet}x$, and $X^{\bullet} = \cup_{x\in X} x^{\bullet}$. One can also define ${}^{\bullet\bullet}x = \cup_{y\in {}^{\bullet}x} {}^{\bullet}y$, and $x^{\bullet\bullet} = \cup_{y\in x^{\bullet}} y^{\bullet}$. Given a place p, $\max\{W(p,t) \mid t \in p^{\bullet}\}$ is denoted by $\max_{p^{\bullet}}$. A Petri net $G = (P,T,E)$ is said to be a state machine if $\forall t \in T, |{}^{\bullet}t| = |t^{\bullet}| = 1$. G is said to be a **marked graph** if $\forall p \in P, |{}^{\bullet}p| = |p^{\bullet}| = 1$.

A marking M of a net G is a mapping from $P \rightarrow Z^{+}$, where $Z^{+} = \{0, 1, 2, \dots\}$. $M(p)$ denote the number of tokens contained in place p. Place p is marked by M iff $M(p) > 0$. A subset $S \subseteq P$ is marked at M if at least one place in S is marked at M. $M(S)$ is denotes the sum of tokens contained in S at marking M, where $M(S) = \sum_{p\in S} M(p)$. (G, M_0) is called a net system or marked net. For economy of space, we use $\sum_{p\in P} M(p)p$ to denote vector M. For instance, M = $(7, 3, 0, 2, 0, 4)^T$ in a net with seven places p_1–p_7 can be rewritten as: $M_0 = (7p_1 + 3p_2 + 2p_4 + 4p_5)$. A place p is marked at M if $M(p) > 0$.

Transition $t \in T$ is enabled at M if $(\forall p \in {}^{\bullet}t)$, $M(p) \geq W(p,t)$. this fact is denoted as $M[t\rangle$; when fired, this gives a new marking M' such that $\forall p \in P$, $M'(p) = M_0(p) - W(p,t) + W(t,p)$, which is denoted as $M[t\rangle M'$. Marking M' is said to immediately reachable from M. Marking M'' is said to be reachable from M if there exists a sequence of transitions $\sigma = \{t_1, t_2, \dots, t_k\}$ and markings $M_1, M_2, \dots$, M_n such that $M_0[t_1\rangle M_1[t_2\rangle M_2, \dots$, $M_{n-1}[t_k\rangle M'$ holds. The set of markings reachable from M in net G is called the reachability set of Petri net (G, M) and denoted as: $R(G, M)$. Transition $t \in T$ is live at M_0 if $\forall M \in R(G, M_0)$, $\exists M' \in R(G, M_0)$, $M'[t\rangle$ holds, t is friable under M'. A transition $t \in T$ is **dead** under M_0 if $\nexists$ $M \in R(G, M_0)$, where t is firable. A marking $M \in$ $R(G, M_0)$ is a (total) deadlock iff $t \in T$, t is dead.

(G, M) is bounded if $\exists k \in \mathrm{N} \setminus \{0\}$, $\forall M \in R(G, M_0)$, $\forall p \in P, M(p) \leq k$ holds. (G, M) is said to be reversible, if for each marking $\forall M \in R(G, M_0)$, M_0 is reachable from M. A marking M_0 is said to be a home state, if for each marking $M \in R(G, M_0)$, M' is reachable from M. Reversibility is a special case of the home state property, i. e. if the home state $M' = M_0$, then the net is reversible.

A $P(T)$-vector is a column vector $I : P \rightarrow Z (J : T \rightarrow Z)$ indexed by $P(T)$, where Z is the set of integers. We denote a column vector whose entries equal 0 or (1) by $0(1)$. I^T and $[G]^T$ are their transposed versions. $I(J)$ is a $P(T)$-invariant if $I \neq 0 (J \neq 0)$ and $[G]^T \cdot I = 0, ([G] \cdot J = 0)$.
$\| I \| = \{p \in P \mid I(p) \neq 0\}$ $(\| J \| = \{t \in T \mid J(t) \neq 0\})$ is called the support of $I(J)$, where $\| I \|^{+} = \{p \in P \mid I(p) > 0\}$ $(\| J \|^{+} = (t \in T \mid J(t) > 0\})$ is called the positive support while $\| I \|^{-} = \{p \in P \mid I(p) < 0\} (\| J \|^{-} = \{t \in T \mid J(t) < 0\})$ is called the negative support. A $P(T)$-invariant is minimal if it contains no $P(T)$-invariant as a proper subset. $P(T)$-invariant is said to be a $P(T)$-semiflow if none of its components is negative. An invariant is called minimal when its support is not a strict superset of the support of any other, and the greatest common divisor of its elements is one. If I is a P-invariant of net (G, M_0), then $\forall M \in R(G, M_0)$, and $I^T . M = I^T . M_0$. In addition, $P(T)$-invariant) of a net $G = (G, M_0)$ is a non-negative row integer $|P|$-vector x (resp., $|T|$-vector y) satisfying the equation $x^T \cdot C = 0$, $(resp.\ C \cdot y^T = 0)$, where C is the incidence matrix of G. A non-zero integer vectors $y \neq 0, (resp\ x \neq 0)$.

A nonempty set $S \subseteq P$ is a siphon if ${}^{\bullet}S \subseteq S^{\bullet}$ and $Q \subseteq P$ is a trap if $Q^{\bullet} \subseteq {}^{\bullet}Q$). A siphon is minimal if there is no siphon contained in S as a proper subset. A minimal siphon containing no support of any trap is a strict minimal one.

A siphon S is said to be invariant- controlled by P-invariant I if $I^T M_0 > 0$, and $\| I \|^{+} \subseteq S$. A siphon S is said to be max-marked at $M \in R(G, M_0)$ *if* $\exists p \in S$ such that $M(p) \geq \max_{p^{\bullet}}$, where:

$\max_{p\bullet} = \max\{W(p,t) \mid t \in p^{\bullet}\}$. A siphon S is said to be max-controlled if it is max-marked at any reachable marking. (G, M_0) is satisfies the maximal controlled-siphon (cs)-property (maximal (cs)-property, for short) if each minimal siphon of G is max-controlled. Max-controlled siphons can always a marked sufficiently to allow firing a transition once at least. Barkaoui et al. [5] present that a Petri net is deadlock-free if it satisfies the maximal (cs)-property. An invariant-controlled siphon is a special case of a max-controlled one.

Definition 1. Let P be the set of places of a net G, I be a P-vector, and siphon $S \subseteq P$ be a subset of places of net G. $I \setminus S$ is defined to be $\sum_{p \in P\setminus S} I(p)p$. **For instance**, $I = \{p_2 + 4p_3 + 3p_4 + 5p_5\}$ is a P-vector and the siphon $S = \{2p_3, p_5\}$ in some net. Then, we have $I \setminus S = p_2 + 3p_4$. Where $I \bigcap S \neq \emptyset$, it is means that $\exists p \in S, I(p) \neq \emptyset$.

__Lemma__ **1.** Consider Petri net $G = (G, M_0)$ with siphon $S \subseteq P$ that is empty under the initial marking $M_0 \in G^{|p|}$. Then $\sum_{p\in S} M(p) = 0$ holds for all elements $M \in R(G, M_0)$.

B. S^4PR net

The definitions of a class of generalized Petri nets namely S[4]PR is given in this section, see [6, 28]. An S[4]PR net bargains with the modelling of concurrently cyclic sequential processes sharing common resources. Every resource place of an S[4]PR net is structurally implicit. In fact, all rows of the incidence matrix in the aforementioned non-negative linear.

Definition 5. Let $I_m = \{1, 2, ..., m\}$ be a finite set of indices. An S^4PR is a generalized, self-loop free net. $G = \text{O}_{i=1}^{n}\ G_i = (P, T, E, W)$, where

1) $G_i = (P_A^i \cup \{P_i^0\} \cup P_R^i, T_i, W_i), i \in I_m$.

2) $P = (P_A \cup P^0 \cup P_R)$ is a partition of such that

(2.1) $P = \bigcup_{i\in I_m} P_A^i,\ P_A^i \neq \emptyset\, and\ P_A^i \cap P_A^j = \emptyset$, $\forall i \neq j\,(i, j \in I_m)$;

(2.2) $P_R = \bigcup_{i\in I_m} P_R^i = \{r_1, r_2, ..., r_n\}, n \in N^+$, where $i = \{1, 2, ..., n\}$;

(2.3) $P_i^0 = \bigcup_{i\in I_m} \{P_i^0\}$;

(2.4) the elements in P^0, P_A and P_R are called idle, operation and resource places, respectively, and

(2.5) the output transitions of an **idle place** are called source transitions.

3) $T = \bigcup_{j=1}^{n} T_i$, is called the set of transitions, where $T_i \neq \emptyset,\ and T_i \cap T_j = \emptyset\ for all\ i \neq j, where (i, j \in I_m)$.

4) $\forall i \in I_m$, the subset G_i generated by $P_A^i \bigcup \{P_i^0\} \bigcup T_i$ is a strongly connected state machine such that every G_i contains p_i^0.

5) $\forall r \in P_R$, there exists a unique minimal P-invariant $I_R \in G^{|p|}$, such that $\{r\} = \| I_r \| \bigcap P_R$, $P^0 \bigcap \| I_r \| = \emptyset,\ P_A \| I_R \| \neq \emptyset\, and\ \ I_r(r) = 1$;

6) $P_A = \bigcup_{r\in P_R} (\| I_r \|\} \setminus \{r\})$.

7) G is a strongly connected net.

Definition 7 : A well-marked S^4PR net (G, M) is a marked Petri net $G = (P, T, E, W)$ with initial marking M_0 such that $\forall p \in P_A, M_0(p) = 0$; $\forall r \in P_R, M_0(r) \geq \max_{p\in\|Ir\|} I_r(p)$; and $\forall P_i^0 \in P^0$, $M_0(P_i^0) \geq 1$.

III. DEADLOCK PREVENTION FOR S[4]PR

Petri net models with the special classes are introduced that allow recognizing resource allocation events are used to synchronize processes that have to share a set of reusable system resources. In this section, based on the MIP approach discussed, we develop deadlock prevention method of an S[4]PR net. The following notations and properties of generalized Petri nets are from [5]. The control places p, we denote: $\max_{t\in P\bullet} \{W(p, t)\} by \max_{p\bullet}$.

Definition 4 [5]: Let (G, M_0) be a marked net and S be a siphon of G. A siphon S is said to be max-marked at a marking $M \in R(G, M_0)\ if\ \exists p \in S$ such that $M(p) \geq \max_{p\bullet}$.

Definition 5 [5]: Let (G, M_0) be a marked net and S be a siphon of G. A siphon S is said to be max-controlled if S is max-marked at any reachable marking.

DOI 10.5013/IJSSST.a.17.33.31

ISSN: 1473-804x online, 1473-8031 print

Definition 6 [5]: A net (G,M_0) is said to be satisfying the max **cs-property** (**controlled-siphon property**, for short) if each minimal siphon of G is max-controlled.

Definition 7 [23]: Let (G,M_0) be a marked S^4PR net. Let S be a siphon of G. Then, $Th(S) = \| I_{S_R} \| \setminus S_i$ is the set of **"thieves"** of S, i.e. the set of process places of the net that use resources of the siphon and do not belong to that siphon.

Definition 8 [31]: Let (G,M_0) be a marked S^4PR net. Let S be a siphon of G. Then, $Th(S) = \| H_{S_R} \| \setminus S_i$ is the set of **"thieves"** of S. Where $Th(S)$ is used to show the relation among these two sets.

Definition 9 [34]: Let S be a strict minimal siphon in an S^4PR plant net model $(G_{\mu 0}, M_{\mu 0})$, where $G_{\mu 0} = \text{O}_{i-1}^n G_i = (P^0 \cup P_A \cup P_R, T, E_{\mu 0}, W_{\mu 0})$. Let $\{\alpha,\beta,\dots,\gamma\} \subseteq \{1,2,\dots,n\}$ such that $\forall i \in \{\alpha,\beta,\dots,\gamma\},\ Th(S) \cap P_{A_i} \neq \emptyset\ and$ $\forall j \in \{1,2,\dots,n\} \setminus \{\alpha,\beta,\dots,\gamma\},\ Th(S) \cap P_{A_i} \neq \emptyset.$ For S, a non-negative P-vector K_S is **constructed**. For example, the computation of K_S for siphon S in an S^4PR, if we have S a siphon with $Th(S) = 5p_3 + 4p_7 + p_8 + 3p_9$. Then we have $K_S(p_3) = 5$, $K_S(p_7) = 4$, $K_S(p_8) = 1$, $K_S(p_9) = 3$.

Proposition 1 [21]: Let S be a strict minimal siphon in a marked S^4PR plant net model $(G_{\mu 0}, M_{\mu 0})$, where $G_{\mu 0} = (P^0 \bigcup P_S \bigcup P_R, T, E_{\mu 0}, W_{\mu 0})$. **Construct** K_S for S depending on Definition 9. A monitor V_S is added to $(G_{\mu 0}, M_{\mu 0})$ by the enforcement that $g_S = K_S + V_S$ is a P-invariant of the resultant net system $(G_{\mu 1}, M_{\mu 1})$, where $G_{\mu 1} = (P^0 \bigcup P_S \bigcup P_R \bigcup \{V_S\}, T, E_{\mu 1}, W_{\mu 1})$;

$$\forall p \in P^0 \bigcup P_S \bigcup P_R,\ M(p_{\mu 1}) = M_{\mu 0}(p).$$

$Let\ h_S = \sum_{r \in S_R} I_r - g_s$, and

$M_{\mu 1}(V_S) = M_{\mu 0}(S) - \xi_S (\xi_S \in Z^+ \setminus \{0\}).$ Then S is a max-controlled if $\xi_S > \sum_{p \in S} h_S(p)(\max_{p\bullet} - 1).$

Let r be a **resource place**, S be a strict minimal siphon (SMS), and $H(r) = I_r \setminus \{r\}$ in an S^4PR net. We define $Th(S) = \sum_{r \in S_R} H(r) \setminus S$. Obviously, we can find out that $Th(S) \subseteq P_S$ is true. Let us use $\sum_{p \in S} h(p)p$ to denote $Th(S)$. It is indicate that siphon S loses $h(p)$ token(s) if the number of tokens in p increases by a unit. Note that each **resource place** r corresponds to a minimal P-semiflow I_r. We have observed three minimal P-semiflows that are associated with resources.

Proposition 2 [5]: Let (G,M_0) be a marked net and S be a siphon of G. S is max-controlled if there exists a P-invariant I such that $\forall p \in (\| I \|^- \bigcap S),\ \max_{p\bullet} = 1$, $\| I \|^+ \subseteq S$, $\sum_{p \in P} I(p) M_0(p) >$ $\sum_{p \in S} I(p)(\max_{p\bullet} - 1)$, then S is max-controlled.

A siphon S and a marking $M \in RM(G,M_0)$ is verifying the properties will be said to be a bad siphon and a $S-$deadlocked marking, respectively. The following notation is used to bad siphon S, such as: $\forall p \in P_S, I_{S_R}(p) = \sum_{r \in S_R} I_r(p)$. A P-semiflow that I_{S_R} is the total amount of **resource** units belonging to siphon S (i.e. in S_R) used by each active process in p.

Definition 10: Let $G = (P_A \cup P^0 \cup P_R, T, E, W)$ be an S^4PR net. Let **resource** $r \in P_R$. The **holders** of resource r are defined as the difference of two multi-sets I_r and $r : H(r) = I_r - r$. According the above definitions, let S is a siphon in an S^4PR. $S_R = S \cap P_R$ is denotes the set of resource places in S. As a multi-set, we define **"thieves"** of S as: $Th(S) = \sum_{r \in S_R} H(r) - \sum_{r \in S_R} I_r(p)p$. For a places $p \in Th(S)$ implies $Th(S) \neq 0$. We can see that $Th(S) \subseteq P_A$ is true (real). Let $\sum_{p \in Th(S)} h(p)p$ denote $Th(S)$. While $h(p)$ indicate that siphon S loses $h(p)$ token(s) if the number of tokens in p is increases by one. The definition above can be extended in the natural way to sets of resources.

Definition 11: Let S be a strict minimal siphon in an S^4PR plant net model $(G_{\mu 0}, M_{\mu 0})$, where $G_{\mu 0} = \text{O}_{i=1}^n G_i = (P^0 \bigcup P_S \bigcup P_R, T, E_{\mu 0}, W_{\mu 0})$. Let

$\{\alpha,\beta,\dots,\gamma\} \subseteq I_n\,(I_n = \{1,2,\dots,n\})$ such that $\forall i \in \{\alpha,\beta,\dots,\gamma\}$, $Th(S)\bigcap P_{S_1} = \varnothing$. For S, a non-negative P-vector k_S is **constructed** as follows:

Step 1. $\forall p \in P_S \bigcup P^0 \bigcup P_R, k_S(p) := 0;$

Step 2. $\forall p \in Th(S), k_S(p) := Th_S(p)$, where $Th(S) = \sum_{p\in Th(S)} Th_S(p)p;$

Step 3. $\forall i \in \{\alpha,\beta,\dots,\gamma\}$, let $p_S \in Th(S)\bigcap P_{S_i}$ be such a place that $\forall p_t \in SP(p_\mu, p_i^0)$, $p_\mu \in p_S^{\bullet\bullet}, p_t \notin Th(S)$. We assume that there are m such places, $p_S^1, p_S^2,\dots, p_S^m$. Certainly, we have $\forall p_S^i$, let $p_v \in SP((p_i^0, p_S^i)$ be such a place that $Th_S(p_v) \ge Th_S(p_w), \forall p_w \in SP(p_i^0, p_S^i)$. For all element $\forall p_v, \forall p_x \in SP(p_i^0, p_v)$, $\forall p_y \in \bigcap_{i=1}^{m} SP(p_i^0, p_S^i)$, $k_S(p_y) := Th_S(p_z^i)$, where $p_z^i \in Th(S)\bigcap P_{S_i}$, if $\nexists\ p \in Th(S)\bigcap P_{S_i}$, $Th_S(p) \ge Th_S(p_z^i)$.

The modeling of the system dynamics is complete of an initial marking must be provided with the net. Tokens are resided in place, while the reachable marking can have different meanings: A place $(p \in P_S)$ is containing token will model an active process (a part being operated) while states are modeled by means of place p (the part is at the state represented by this node). The buffering capacity of resources r is representing model by tokens in place $r \in P_R$ (buffering capacity will be used to represent either capacity or availability). Markings need to represent states that have a physical information flow (distributed system). In this concept, the **acceptable initial markings** are defined and will be considered according to the requirement systems.

Theorem 1 : Let (G, M_0) be a well initially marked S^4PR. G is live if every siphon in G is max-controlled.

IV. PETRI NET MODELING OF FMS RECOGNIZED AS RAS

In this section, we are studying an example of flexible manufacturing cells is employed as an example. The development of an efficient deadlock prevention policy of S4PR is used elementary siphons where are a more general class of PNs. We will be used to show how to utilize elementary siphons PN in a class of S4PR to analyze RAS to liveness and deadlock in FMS.

Example. Let us consider of a flexible manufacturing cell layout design and performance evaluation function is shown in Fig. 1(a) which has been studied in the literature [32], albeit with different modified structured net where is used S4PR.

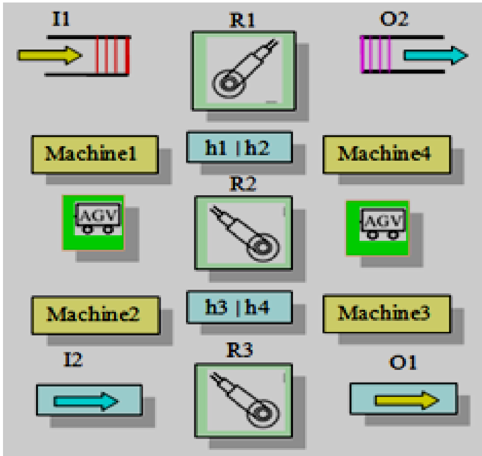


Fig. 1(a). An FMS layout

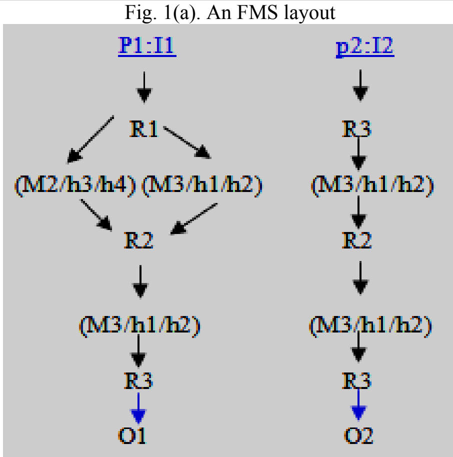


Fig. 1. (b) Production cycles.

This cell consists of four different machines, $M1, M2$, $M3$ and $M4$, and three robots $(R1-R3)$; each one can hold a product at a time) to produced two types of parts. The four machines, where $M1$ and $M4$ can process two parts, while $M2$ and $M3$ can process three parts concurrently). In the state, we consider that there are two parts type's jobs $J1-J2$'s can be produced in this system, and parts to enter during input/output conveyor $I1/O1$ and $I2/O2$.

A Robot R1 is loading parts in conveyor I1 to machines M1 and M3 and unloads parts of machine M3 to conveyor O2. Machines M1 and M3 can use tools H1 and H3. The load and unload machines $M1-M4$ can be performed by robot R2. Machines M2 and M4 can use tools H2 and H4. Unload machines M2 is performed by Robot R3, to load exit conveyor O1 and can load machine M4 from conveyor I2.

Tow types of parts must be processed, where a part of type1 arrives the system by conveyor I1, and leave it by conveyor O1. Machines M1 or M3 can be processed first. While Machine M1 needs to get tools H1 to achievement its work and M3 need to occupy tool H1 and H3. Machine M2 has a part to be processed which is needed to use tools H2 and H4 (arrival to operations of production routing $J1$.

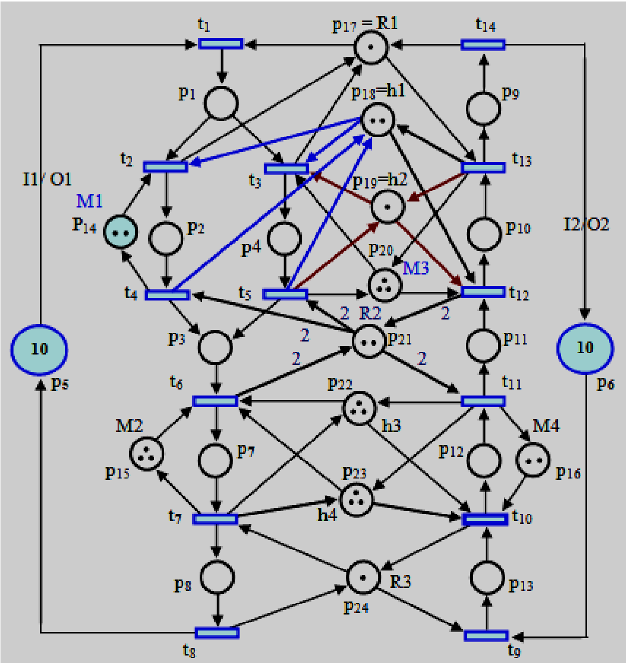


Fig. 2. Petri net model for an $S^4PR(G_{\mu 0}, M_{\mu 0})$

This cell produced two products types, i.e. part-1 and part-2 (p1, and p2), is performed to show how to model and control of an FMS using methodologies Petri net implemented throughout this example. There are two part product types, namely P1, and P2, to be performed. For these raw product types the production cycles are as shown in Figure 1(b), belonging to the S^{4}PR class. They perform two types of parts must be processed achieved to their own production routings that are depicted in Figure 1(b). An S^{4}PR (Simple Systems of Simple Sequential Processes with Resources) is showing in Fig. 2 belong to generalized Petri nets have competitive processes of the Resource Allocation Systems RAS modeling, and design of FMS. Figure 2, shows an example of a Petri net which models an FMS problem that was designed using the Petri tool application. The Places (p_1– p_7) represent the operation of R1, M2, R2 and M3 respectively, for production cycle's part type-P1, while production cycles part type-P2 (p_{13} – p_9) are performed on the operation place of M3, R2 and M2 respectively. The Petri net is illustrating an example of and by this place specification that uses a multi-set of resources at a production routing. Marking is acceptable tokens in Fig. 2, where is token $M_0(p_5) = M_0(p_6) = 10$, places $M_0(p_{17}) = M_0(p_{19}) = M_0(p_{24}) = 1$, places $M_0(p_{14}) = M_0(p_{16}) = M_0(p_{18}) = M_0(p_{21}) = 2$. While places $M_0(p_{15}) = M_0(p_{20}) = M_0(p_{22}) = 3$, and others places are zero. Places $p_{17}, p_{18} - p_{24}$, denote the shared resources R1, H1, H2, M3, R2, H3, H4, and R3 respectively. In the system, there are two H1, one H2, three H3 and three H4 tools. The process idle places are: P^0 = {p_5, p_6}, and the resource places are: {$p_{14} - p_{24}$}. The places p_{14}, p_{15} and p_{16} is denoted the resources M1, M2, and M4 respectively. The PN is bounded, not live and not reversible but repetitive so that it can be made live. The initial marking of PN can be denoted as: M_0 = {$10p_5 + 10p_6 + 2p_{14} + 3p_{15} + 2p_{16} + p_{17} + 2p_{18} + p_{19} + 3p_{20} + 2p_{21} + 3p_{22} + p_{24}$}. We are assumed that the weight of arcs: $W(p_{21}, t_4) = W(p_{21}, t_5) = W(p_{21}, t_{11}) = W(t_6, p_{21}) = W(t_{12}, p_{21}) = 2$, in order to take specifications of weight arcs to PN. A net is a format to the generalized PN.

The net shown in Figure 2 is pure and bounded and there exist dead reachable states and not live. The modeling and simulation of FMS are used in our example with Petri net Toolbox with Matlab [29]. There are eleven resources in this system leading to eleven minimal P-invariants, we would like mention as follow: $I_1 = p_2 + p_{14}$, where $M_0(p_{14}) = 2$, $I_2 = p_7 + p_{15}$, where $M_0(p_{15}) = 3$, $I_3 = p_{12} + p_{16}$, where $M_0(p_{16}) = 2$, $I_4 = p_1 + p_9 + p_{17}$, where $M_0(p_{17}) = 1$, $I_5 = p_2 + p_4 + p_{10} + p_{18}$, where $M_0(p_{18}) = 2$, $I_6 = p_4 + p_{10} + p_{19}$, where $M_0(p_{19}) = 1$, $I_7 = p_4 + p_{10} + p_{20}$, where $M_0(p_{20}) = 3$, $I_8 = 2p_3 + 2p_{11} + p_{21}$, where $M_0(p_{21}) = 2$, $I_9 = p_7 + p_{12} + p_{22}$, where $M_0(p_{22}) = 3$, $I_{10} = p_7 + p_{12} + p_{23}$, where $M_0(p_{23}) = 3$, $I_{11} = p_8 + p_{13} + p_{24}$, where $M_0(p_{24}) = 1$.

Simulation and structural analysis of the behavioral properties of Petri net model use the PN-tool with MATLAB [29], starts with coverability tree keys. We can see the original net system has (1330) reachable states with initial marking, among which there are (18)-deadlock states. In order to solve the deadlock problem, one of the most widely is exercised the Petri nets tool to design a controller to avoid deadlock. We can find out minimal siphons of Petri net shown in Fig. 2. The net system of FMS is an S^{4}PR, and contains deadlocks. Analysis structured of PN, there is 44 strict minimal siphon SMS as shown below. Among the set of siphons from S_1–S_3 are elementary siphon and S_4–S_{44} which are referred to the element dependent ones are marked by *.

S_1 = {p_3, p_{10}, p_{18}, p_{21}}, S_2 = {p_8, p_{12}, p_{23}, p_{24}},
S_3 = {p_3, p_{10}, p_{20}, p_{21}}, S_4* = {p_2, p_4, p_9, p_{17}, p_{18}},
S_5* = {p_7, p_{11}, p_{21}, p_{23}}, S_6* = {p_8, p_{12}, p_{22}, p_{24}},
S_7* = {p_3, p_{10}, p_{19}, p_{21}}, S_8*= {p_7, p_{11}, p_{21}, p_{22}},
S_9* = {p_8, p_9, p_{14}, p_{17}, p_{20}, p_{21}, p_{23}, p_{24}},
S_{10}* = {p_8, p_9, p_{14}, p_{17}, p_{19}, p_{21}, p_{22}, p_{24}},
S_{11}* = {p_8, p_9, p_{14}, p_{17}, p_{19}, p_{21}, p_{23}, p_{24}},
S_{12}* = {p_8, p_9, p_{14}, p_{17}, p_{20}, p_{21}, p_{22}, p_{24}},
S_{13}* = {p_8, p_{10}, p_{20}, p_{21}, p_{23}, p_{24}}, S_{14}* = {p_8, p_{10}, p_{20}, p_{21}, p_{22}, p_{24}}, S_{15}* = {p_8, p_{10}, p_{19}, p_{21}, p_{23}, p_{24}},
S_{16}* = {p_8, p_{10}, p_{19}, p_{21}, p_{22}, p_{24}}, S_{17}* = {p_8, p_9, p_{17}, p_{18}, p_{21}, p_{23}, p_{24}}, S_{18}* = {p_8, p_9, p_{17}, p_{18}, p_{21}, p_{22}, p_{24}},

S_{19}* = {p_8, p_{10}, p_{18}, p_{21}, p_{23}, p_{24}},
S_{20}* = {p_8, p_{10}, p_{18}, p_{21}, p_{22}, p_{24}},
S_{21}* ={p_8, p_{11}, p_{21}, p_{23}, p_{24}}, S_{22}* $=\{p_8, p_{11}, p_{21}, p_{22}, p_{24}\}$, {$p_8$, p_{11}, p_{21}, p_{22}, p_{24}}, S_{23}* = {p_7, p_9, p_{14}, p_{17}, p_{20}, p_{21}, p_{23}},
S_{24}* = {p_7, p_9, p_{14}, p_{17}, p_{20}, p_{21}, p_{22}},
S_{25}* = $\{p_8, p_9, p_{14}, p_{15}, p_{16}, p_{17}, p_{20}, p_{21}, p_{24}\}$,
S_{26}* = {p_3, p_9, p_{14}, p_{17}, p_{20}, p_{21}}, S_{27}* = {p_7, p_9, p_{14}, p_{17}, p_{19}, p_{21}, p_{23}}, S_{28}* = {p_7, p_9, p_{14}, p_{17}, p_{19}, p_{21}, p_{22}},
S_{29}* = {p_8, p_9, p_{14}, p_{15}, p_{16}, p_{17}, p_{19}, p_{21}, p_{24}},
S_{30}* = {p_3, p_9, p_{14}, p_{17}, p_{19}, p_{21}}, S_{31}* = {p_7, p_9, p_{17}, p_{18}, p_{21}, p_{23}}, S_{32}* = {p_7, p_9, p_{17}, p_{18}, p_{21}, p_{22}}, S_{33}* = {p_8, p_9, p_{15}, p_{16}, p_{17}, p_{18}, p_{21}, p_{24}}, S_{34}* = {p_3, p_9, p_{17}, p_{18}, p_{21}},
S_{35}* = {p_7, p_{10}, p_{18}, p_{21}, p_{23}}, S_{36}* = {p_7, p_{10}, p_{18}, p_{21}, p_{22}},
S_{37}* = {p_8, p_{10}, p_{15}, p_{16}, p_{18}, p_{21}, p_{24}},
S_{38}* = {p_8, p_{10}, p_{15}, p_{16}, p_{19}, p_{21}, p_{24}},
S_{39}* = {p_8, p_{10}, p_{15}, p_{16}, p_{20}, p_{21}, p_{24}},
S_{40}* = {p_8, p_{11}, p_{15}, p_{16}, p_{21}, p_{24}},
S_{41}* = {p_7, p_{10}, p_{20}, p_{21}, p_{23}}, S_{42}* = {p_7, p_{10}, p_{19}, p_{21}, p_{23}},
S_{43}* = {p_7, p_{10}, p_{20}, p_{21}, p_{22}}, S_{44}* = {p_7, p_{10}, p_{19}, p_{21}, p_{22}}.

In Fig. 2, there are eleven resources exist in this system leading to eleven minimal P-semi-flows in table:

TABLE 1

Eleven **resources :** (P-Invariant equations)	The **holders** of resource *r*: $H(r) = I_r \setminus \{r\}$
$I_{p_{14}}$ = $p_2 + p_{14}$,	H(p_{14}) = {$p_2 + p_{14}$}\ p_{14} = p_2 ;
$I_{p_{15}}$ = $p_7 + p_{15}$,	H(p_{15}) = {$p_7 + p_{15}$}\ p_{15} = p_7 ;
$I_{p_{16}}$ = $p_{12} + p_{16}$,	H(p_{16}) = p_{12} ;
$I_{p_{17}}$ = $p_1 + p_9 + p_{17}$,	H(p_{17}) = $p_1 + p_9$;
$I_{p_{18}}$ = $p_2 + p_4 + p_{10} + p_{18}$	H(p_{18}) = $p_2 + p_4 + p_{10}$;
$I_{p_{19}}$ = $p_4 + p_{10} + p_{19}$,	H(p_{19}) = $p_4 + p_{10}$;
$I_{p_{20}}$ = $p_4 + p_{10} + p_{20}$,	H(p_{20}) = $p_4 + p_{10}$;
$I_{p_{21}}$ = $2p_3 + 2p_{11} + p_{21}$,	H(p_{21}) = $2p_3 + 2p_{11}$;
$I_{p_{22}}$ = $p_7 + p_{12} + p_{22}$,	H(p_{22}) = $p_7 + p_{12}$;
$I_{p_{23}}$ = $p_7 + p_{12} + p_{23}$,	H(p_{23}) = $p_7 + p_{12}$;
$I_{p_{24}}$ = $p_8 + p_{13} + p_{24}$,	H(p_{24}) = $p_8 + p_{13}$.

Definition 10. A Petri net is a four-tuple $G=(P,T,E,W)$ is ordinary, denoted as $G=(P,T,E)$, if $\forall e \in E, W(e)=1$.

While PN is said to be a generalized net iff $\forall e \in E$, $W(e)>1$.

Having applied the Proposition 1, in order to make all elementary siphons has a max-controlled. Our experiment is checking three siphons and added three monitors (or control places) VS_1−VS_3 are satisfying the max cs-property.

The complementary sets of siphons S_1 – S_3 are computed as:

1)- For Siphon S_1 = {p_3, p_{10}, p_{18}, p_{21}}, is contain two resource places (i.e. p_{18}, p_{21}).

Let $l_1 = I_{p_{18}} + I_{p_{21}} = \sum_{r \in S_{1R}} I_{p_{18}} + I_{p_{21}}$,

l_1 = {$p_2 + 2p_3 + p_4 + p_{10} + 2p_{11} + p_{18} + p_{21}$};

Th(S_1) = [H(p_{18}) + H(p_{21})] \ S_1 ;

$Th(S_1)$ = $p_2 + p_4 + 2p_{11}$, where applied definition 1.

k_{S_1} = $p_1 + p_2 + p_4 + 2p_{11} + 2p_{12} + 2p_{13}$;

$g_{S_1} = k_{S_1} + VS_1$,

g_{S_1} = {$p_1 + p_2 + p_4 + 2p_{11} + 2p_{12} + 2p_{13} + VS_1$}; Clearly,

$h_{S_1} = l_1 - g_{S_1}$.

h_{S_1} = $2p_3 + p_{10} + p_{18} + p_{21} - VS_1 - p_1 - 2p_{12} - 2p_{13}$; is a P-invariant. Noticing that we say that S_1 is max-controlled by the P-invariant h_{S_1}. $M_{\mu 1}(VS_1) = M_0(S_1) - \sum_{p \in S_1} (\max_{p\bullet} - 1) - 1 = (2+2-1) - 1 = 2$.

2)- For Siphon S_2 = {p_8, p_{12}, p_{23}, p_{24}} is contain two resource places (i.e. p_{23}, p_{24}).

Let $l_2 = \sum_{r \in S_{2R}} I_{p_{23}} + I_{p_{24}}$,

l_2 = {$p_7 + p_8 + p_{12} + p_{13} + p_{23} + p_{24}$};

$Th(S_2)$ = [H(p_{23}) + H(p_{24})] \ S_2,

$Th(S_2)$ = $p_7 + p_{13}$.

k_{S_2} = $p_1 + 2p_{10} + 2p_{11} + 2p_{12} + 2p_{13}$; and

$g_{S_2} = k_{S_2} + VS_2$,

g_{S_2} = {$p_1 + 2p_{10} + 2p_{11} + 2p_{12} + 2p_{13} + VS_2$} ; and

$h_{S_2} = l_2 - g_{S_2}$,

h_{S_2} = $p_7 + p_8 + p_{23} + p_{24} - p_1 - 2p_{10} - 2p_{11} - p_{12} - p_{13} - VS_2$.

We say that S_2 is max-controlled by the P-invariant h_{S_2}.

$M_{\mu 1}(VS_2) = M_0(S_2) - \sum_{p \in S_2} (\max_{p\bullet} - 1) - 1 = 3 - 1 = 2.$

3)- For Siphon S_3 = {p_3, p_{10}, p_{20}, p_{21}}, is contained two resources (i.e. p_{20}, p_{21}).

Let $l_3 = \sum_{r \in S_{3R}} I_{p_{20}} + I_{p_{21}}$,

$l_3 = \{2p_3 + p_4 + p_{10} + 2p_{11} + p_{20} + p_{21}\}$,

$Th(S_3) = p_4 + 2p_{11}$

$k_{S_3} = 2p_1 + 2p_2 + 2p_3 + 2p_4 + 2p_7 + p_{13}$

$g_{S_3} = k_{S_3} + VS_3$,

$g_{S_3} = 2p_1 + 2p_2 + 2p_3 + 2p_4 + 2p_7 + p_{13} + VS_3$;

$h_{S_3} = l_3 - g_{S_3}$,

$h_{S_3} = p_{10} + 2p_{11} + p_{20} + p_{21} - 2p_1 - 2p_2 - p_4 - 2p_7 - p_{13} - VS_3$;

$M_{\mu 1}(VS_3) = M_0(S_3) - \sum_{p \in S_3} (\max_{p\bullet} - 1) - 1 =$

$(3+2) - 2 - 1 = 2.$

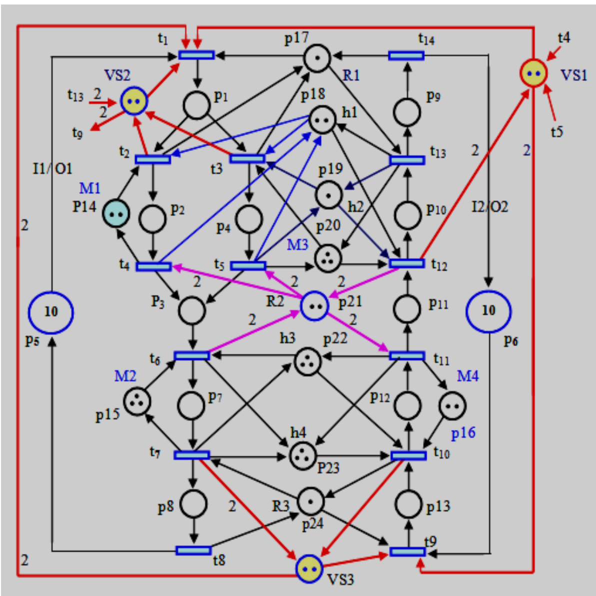
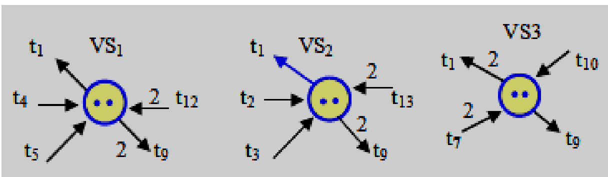


Figure 3. Liveness Petri net supervisor $(G_{\mu 1}, M_{\mu 1})$ with Three monitors (or Control places).



Three Control places

If we are adding two control places VS_1–VS_2, the Petri net is live and has (183) reachable marking states with initial marking. When added three control place (VS_1–VS_3) the reachability graph is live and obtain (21) reachable states with initial marking as shown in Fig. 4. We can apply proposition 1, 2, and definitions 9, 10, and 11 to adding monitors or control places (VS_1–VS_3) to the net of Fig. 3, the Petri net is live. A Petri nets are suitable to describe the sequential and parallel execution of assignments to or without synchronization; it is possible to define loops and the conditional execution of tasks in FMS. Conditional transitions are required at every region in the Petri net where two or more activated transitions shares the same input place and compete for the token that are located to that place. Petri nets possess special characteristics that can be defined mathematically and are used to analyze and control of FMS. Terms like conflict, confusion, contact, liveness and deadlock are well-defined properties of Petri nets that may be helpful when optimizing FMS. The reachability graph [18] analysis is an important technique for deadlock control, which always suffers from a state explosion problem since it requires generating all or a part of reachable markings.

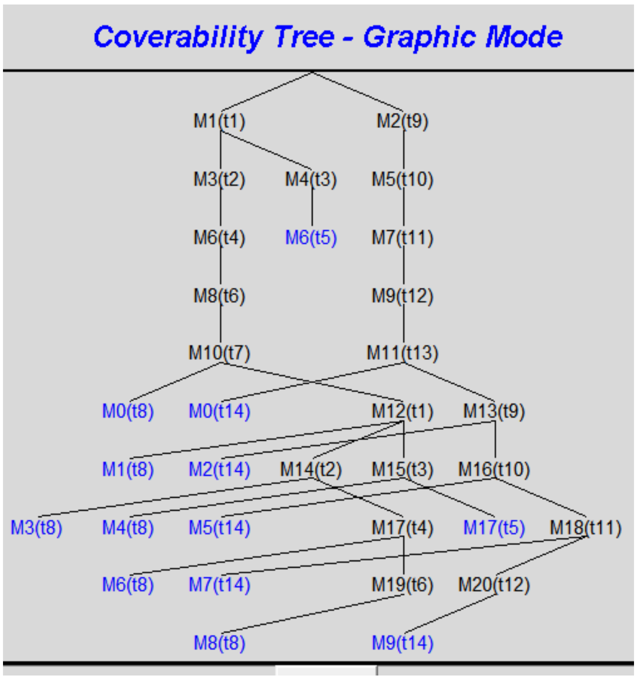


Fig. 4. shows reachability graph of Fig. 3, with monitors (VS_1–VS_3)

Our experimental approaches computation of this work is used Petri net toolbox with MATLAB and tested on a Pentium computer with 512 MB of RAM, Intel(R) Core(TM) i5-3337U CPU@ 1.80GHz, and system type is 64-bit, with 4Gb of RAM, under Windows 7 Ultimate operating system.

The experimental approaches based elementary siphons of the structural analysis of the Petri net toolbox with MATLAB [29] are given, depicting the monitors of Figure 4, that can be application to make reconfigurable so that Petri nets is suitable for FMSs with dynamic structures. A

DOI 10.5013/IJSSST.a.17.33.31

ISSN: 1473-804x online, 1473-8031 print

simulation of Petri net models is the execution of a model of RAS in FMS, represented by a computer program that gives structure information in the location of deadlock on the net. The popular software is training and research where the installation Petri Net Toolbox in MATLAB, which can be to draw Petri net $V.2.3$ software's with the exploitation of Graphical User Interfaces (GUIs), depended on this prevalent software.

The performance results show that is our proposed method reduces the reachability graph of a class of an S^4PR net for modeling, simulation, and control FMS complexity and it will help managers to understand the changing behaviors of FMS. Adding the controller places (monitors) to the net of Figure 4 is live and obtained to M_{20} reachable states marking so that the Petri net is deadlock-freeness. A siphon is generating even though the monitors are added. The reachability graph has reduced to (21) states with initial marking shown in Figure 4. This example gives as a Petri nets tool dynamic siphon are used for the purpose of control of the network, which is representing the system such as FMS. The visualization of Petri net simulation is helped us to structure analysis of Petri nets, and the MATLAB software can show the PN manner of the simulation analytical structure of siphons are correct results. The resulting models can be visualized using Petri net with MATLAB toolbox.

V. CONCLUSION

The use of the maximal control siphon (cs) property to control a generalized Petri net is still an open problem requiring further study. Also, a sufficient and necessary siphon control condition for S^4R, and S^4 PR family remains open. A deadlock prevention policy of FMSs was proposed in this paper, which can obtain a live Petri net controller based on a controllable siphon basis. Petri net models of FMS provide the possibility to consider integrating the planning phase to drive different elements of resources, by frequently changing assortment production which requires concurrent, random order processing of various workpiece types. Through efficient experimental studies we used a class of an S^4PR based siphons property of Petri net in order to obtain the liveness of FMS. Petri net based siphon is used to detect the deadlock in RAS, that determines the properties of Petri net and superior bounds for the minimum achievable range to maximize productivity and other variables. Based on Petri nets to deal with the deadlock problem with a class of Petri nets, namely S^4PR, we developed a novel and effective deadlock prevention policy. By explicitly controlling elementary siphons via adding monitors, a liveness enforcing controlled system can be determined in the reachability graph which can fully reflect the behavior of a system. The extension of the proposed class of an S^4PR net in our future work will include developing deadlock control policies of more general Petri nets. A structural non-liveness of Petri net condition is realizing in the local structure of siphons which contains no trap. The cs property highlights the conditions of the markings since it provides an explanation of the features of liveness property.

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