State Estimation of Thermal Systems with Multiple Operation Modes

Shota Sasaki, Kentaro Hirata†, Yoichiro Masui

Graduate School of Natural Science and Technology
Okayama University
3-1-1 Tsushima-Naka, Kita-Ku, Okayama 700-8530, JAPAN
Email: {shotas@s, kent@sys, yoichiro@s}.okayama-u.ac.jp

†: corresponding author

Masahiro Samei, Akihiro Kawasaki, and Yasuharu Kawarasaki
Platform Development Center,
Imaging Engine Development Division,
RICOH Company, Ltd.
13-1 Himemuro-Cho, Ikeda, Osaka 563-8501, JAPAN
Email: {masahiro.samei, akihiro.kawasaki, yasuharu.kawarasaki}@nts.ricoh.co.jp

Abstract — The state estimation of thermal systems with multiple operation modes is considered. Based on a description as hybrid dynamical systems, we reduce it into an observer design problem for switched linear systems. In this design, a tradeoff between fast response and robustness must be taken into account in addition to a guarantee of stability as a switched system. For this purpose, we introduce LMIs with common Lyapunov function to place the closed-loop poles of the observer for each subsystem in the prescribed region in the complex plane. The effectiveness of proposed method is verified through numerical simulations with experimental data set.

Keywords - fusing process; hybrid dynamical systems; LMI; switching observer; common Lyapunov function

I. INTRODUCTION

In the fusing process of laser printers [1], the toner particles are melted by heat and pressure to be bonded to the paper. Thus the printing quality heavily relies on the surface temperature of the fuser roller at the time of paper feeding. To achieve good printing quality and cost performance simultaneously, it is required to obtain the temperature information with sufficient precision at a reduced cost for sensors. One solution is to take “soft sensor” approach [2], [3], [4], [5], [6], that is a kind of model-based approach.

In recent years, Model-Based Design or Model-Based Development (MBD) is widely used in various systems design, especially, in industries. In control systems design, MBD approach obtains mathematical or numerical models of the plant and the controller first. Then through the numerical simulations with them, the design specifications are evaluated. Although such model-based viewpoint is relatively common in control engineering, recent trend of MBD tries to extend the usage of virtual plants and introduce it to the fields where such approaches have never been attempted due to the difficulty of the modeling [7], [8], [9], [10].

Since MBD does not need the conventional prototyping and experiments, it can speed up the development time and reduce the development cost. Flexibility against changes is another advantage of MBD. The total system is divided into the components conceptually. If change of parts is needed, then one can replace the involved components virtually and quickly without worrying about physical restrictions. As described above, MBD has a great advantage over conventional way of development, and therefore, many industries start to employ it [11]. JMAAB [12] is a leading example of an organized activity by Japanese automotive industries to promote MBD.

In this study, we attempt to introduce this model-based approach in the development of the control system for the fuser units of laser printers mentioned earlier. Since various types of fuser unit are used in the products lineup, it is quite beneficial to obtain a “standard” model of them to be used in the future development. The first phase is to derive a mathematical model for the process of our interest and the second phase is to use it for the estimation of internal temperatures without precise measurements.

Our soft sensor is realized by a state estimator (observer) for Finite-Dimensional Linear Time-Invariant (FDLTI) systems that approximate the actual thermal phenomenon possibly infinite-dimensional and nonlinear. To deal with the multiple operation modes of the fuser unit, this soft sensor technique is extended to the case with multiple models.

In the following sections, the change of plant dynamics due to the operation conditions is represented by switching between linear systems and an observer for these systems is designed. In this design process, a tradeoff between fast response and robustness must be taken into account in addition to a guarantee of stability as a switched system. For this purpose, we introduce Linear Matrix Inequalities (LMIs) with common Lyapunov function to place the closed-loop poles of the observer for each subsystem in the prescribed region in the complex plane. The effectiveness of proposed method is verified through numerical simulations with experimental data set.
II. TARGET SYSTEM

A schematic diagram of a typical fuser unit in laser printers is shown in Fig. 1. As mentioned above, the requirement for the fusing process is to melt the toner particles by heat and pressure.

Figure 1. Schematic diagram of fuser unit

Two rollers in the figure play this role. One is the fuser roller which has a heater inside to give sufficient temperature to the surface and another is the pressure roller to push the paper going through the gap against the fuser roller. If the temperature of the contact surface with the paper is too high, then it induces paper deformation and excessive energy consumption. Conversely, if it is too low, then the melting of the toner particles becomes insufficient and it causes bad printing quality. Thus controlling the heater output according the precise temperature information is crucial. For this purpose, usually a temperature sensor is equipped aside of the fuser roller. However, from the cost viewpoint, the performance of the sensor should be limited.

Figure 2. Experimental Apparatus

Fig. 2 is a photo of our experimental apparatus and Fig. 3 illustrates an example of the sensor measurement in our experiment; from intellectual property viewpoint, the scale for each axis is omitted when we describe the real data from the commercial product. The true value of the temperature is measured by an additional thermocouple. One can observe that there exists a certain amount of time delay in the sensor dynamics. Our objective here is to estimate “un-delayed” temperature from “delayed” sensor output by using an observer designed for a linearized model of this process.

Figure 3. Experimental data (sensor output and true value)

III. MODEL-BASED APPROACH

To begin with, we verify the advantage of the model based approach thorough a numerical simulation with a low order system. Suppose that the thermal system in Fig. 1 is modeled by a series connection of the plant and sensor dynamics as shown in Fig. 4.

Figure 4. Thermal system with sensing delay.

A. Preliminary result

Suppose that a lumped parameter approximation with single point temperature is valid for the target thermal process, i.e., the thermal dynamics can be modeled as

\[ \dot{\theta} = \frac{R_1 \Delta \theta}{C_1} + u \]

where \(\theta\), \(\Delta \theta\), \(C\), \(R\) and \(u\) denote the target temperature, the atmospheric temperature, the heat capacity, the heat resistance and the heat flow, respectively. Then the deviation from the equilibrium temperature

\[ \Delta \theta = \theta - \theta_e \]

follows the linear dynamics

\[ \frac{d}{dt} \Delta \theta = -R \dot{\theta} + u, \quad (1) \]

Also suppose that the sensing delay is described by a first-order lag (rather than a pure time-delay), i.e.,

\[ y + y = \Delta \theta, \quad (2) \]
where \( y \) is the sensor output and \( T \) is the time constant. Fig. 5 shows an example of the step response of (1) and (2) with \( C = 2.5 \), \( R = 1.1 \).

Our mission is to estimate the blue line data from the green ones.

In the model-based approach, we utilize both (1) and (2) to estimate \( \Delta \theta \), while we only use (2) (or equivalently, the value of the time constant \( T \)) in the model-free approach\(^2\). To be specific, we derive a state space model of (1) and (2) and estimate the internal state by an observer using the input and output information of this system in the former approach. The corresponding block diagram is given in Fig. 6 where \( n \) denotes the additive measurement noise. Since \( \Delta \theta \) is an internal state, this gives a reconstruction process of \( \Delta \theta \) from \( y \). Let \( G_2(s) \) be the transfer function from \( \Delta \theta \) to \( y \). From (2), it is given by

\[
G_2(s) = \frac{1}{Ts + 1}.
\]

In contrast, we try to design an approximate inverse of \( G_2(s) \) in the latter approach since it is impossible to obtain a precise inverse of a strictly proper transfer function \( G_2 \). Let \( G_3(s) \) be

\[
G_3(s) = \frac{Ts + 1}{Ts + 1}.
\]

When \( \sigma_0 < T \), it is expected that \( G_3 \) well approximates \( G_2^{-1} \) at low frequency range. The reconstruction process is shown in Fig. 7; Strictly speaking, it might be an abuse of the terminology since the model (2) is used. However, we regard it as a model-free approach since the thermal process model which plays a central role in the current phenomenon is not used.

Let us compare these two approaches through numerical simulations under the assumption that the sensor output is contaminated by an additive white Gaussian noise \( n \) with variance \( \sigma^2 \). Fig. 8 shows the results of the estimation of
both approaches. The parameter $T_D$ is chosen as 0.2. When the delay length is relatively short and the noise level is mild ($T = 1$ and $\sigma^2 = 0.5$), the estimation result seems to be satisfactory in both cases (Fig. 8-a). For larger time delay ($T = 5$), the estimate of the model-free approach becomes noisy, while the behavior of the model-based approach remains almost the same (Fig. 8-b). When the noise level is increased to $\sigma^2 = 2.5$ from Fig. 8-a, the result of the model-free estimation deteriorates even for the case $T = 1$ (Fig. 8-c).

By rewriting

$$\Delta \theta_{(2)} = G_2(2) y(2)$$

into

$$\Delta \theta = \frac{1 - T/T_D}{T/T_D + 1} y + \frac{T}{T_D} y_0$$

one can see that the coefficient of the direct feed-through term is proportional to $T$.

Figure 9. Gain characteristics of model-free compensation

This is the reason of the anti-noise vulnerability of the model-free approach. Let us explain it in detail from a frequency response viewpoint. Fig. 9 depicts the gain characteristics of $G_1$, $G_3$, and $G_1G_3$, respectively. The black line is the curve for $T = 1$ and the red line is for $T = 5$.

Since the perfect reconstruction:

$$G_1G_2 = 1$$

is impossible, all we can expect is to achieve the unity gain until the frequency $1/T_D$ beyond the original bandwidth $1/T$. For this purpose, the phase lead compensator $G_2$ is connected in series to lift up the gain between $[1/T, 1/T_D]$ at the same rate of decay of $G_2$ over $1/T$. Since the rate is constant, larger lift up range (longer delay) inevitably results in larger high frequency gain. It is regarded as a performance limitation of the model-free approach and one must rely on the model-based approach to overcome it when we try to employ cheaper sensors with longer time-constants.

B. Modeling and estimation of fusing Process

In this section, we extend the aforementioned technique to the modeling and estimation of the real fusing process. Although the thermal process is a distributed parameter system in nature, we employ a lumped parameter approximation here from a practical viewpoint. Also we prefer to use a gray-box model rather than a black-box model to examine the meaning of physical parameters to be used in our model.

Let us consider $N$ representative temperature points in the fusing unit. Let $\theta_i$ be the temperature of the fusing roller to be estimated. As in the previous example, the deviation $\Delta \theta_i$ from the atmospheric temperature $\theta_0$ is used to construct a system of ordinary linear differential equations

$$C_i \Delta \theta_i = \sum_{j \in N_i} R_{ij} (\Delta \theta_j - \Delta \theta_i) - R_{il} \Delta \theta_i + u_i$$

where $C_i$, $R_{ij}$, and $u_i$ are the heat capacity, the heat resistance against the atmosphere and the heat flow for the $i$-th component, respectively. Also $N_i$ is the node set describing the neighborhood of node $i$ and $R_{ij}$ is the heat resistance between the nodes $i$ and $j$. Together with

$$T_i = y$$

we constitute a model for the target process.

By using the experimental data from the fusing process of actual laser printer, the model-based approach described above is tested. We first choose the number of the temperature representative points $N$ by trial and error, and then identified the physical parameters in (4). The experiments for the data acquisition were carried out in 8 typical operating conditions (cold or hot start, intermittent or continuous paper feeding and so on).

Figure 10. Estimation result with experimental data
Fig. 10 is an example of the estimation result after the data fitting. In spite of the extensive parameter fitting, the precision of estimation is not enough. This motivates us to extend the model-based approach further. Based on the observation that the process has several operation modes even in one operating condition, we employ a switching model representation among multiple modes (switched linear systems with known switching instance). Since it contains both continuous and discrete state transitions, it is regarded as a hybrid dynamical system.

IV. EXTENSION TO THE CASE WITH MULTIPLE MODES

A. Modeling as switched linear systems

The fuser unit of the laser printer experiences several operation modes, such as startup, roller rotation, paper feeding, wait and so on. Since each mode has its own boundary condition as a thermal system, it is reasonable to switch the models describing them according to the modes. We assume that the thermal system has \( M \) distinct modes as in Fig. 11.

Further assumptions are 1) each mode can be approximated by an FDLTI system, 2) the mode transition can be detected from the operation data, and 3) the sensor dynamics is unchanged and given by the first order lag with unit DC gain.

Then, the target process is represented by a switched linear system with known switching timing. Each subsystem corresponding to the mode variable \( k = 1, 2, \ldots, M \) is given by

\[
\begin{align*}
\dot{x}_k(t) &= A_k x_k(t) + B_k u(t), \\
y(t) &= C_k x_k(t).
\end{align*}
\]

For the current system of interest, the \( C \) matrices can be taken to be common \( C_1 = C_2 = \ldots = C_M \). After the parameter identification by the gray box modeling using the optimization toolbox of Matlab, we obtain the state space model. It is verified that the pairs \((A_k, C_k)\) for \( k = 1, 2, \ldots, M \) are observable.

B. Pole placement within a prescribed region via LMI

At the state estimation of the switched systems, the final estimation error at one mode becomes the initial error of the subsequent mode. In this sense, the observer gain should be large enough to guarantee a certain level of error convergence within the dwell time. However, at the same time, too large gain must be avoided from the anti-noise performance viewpoint related to the signal quantization effect at the implementation stage. Thus the gain selection tends to become a severer issue when we consider the situation with multiple modes.

\[a = (\alpha_1 + \alpha_2)/2, \quad b = (\alpha_1 - \alpha_2)/2, \quad c = \epsilon p, \quad d = (\epsilon p - \epsilon^2)p.
\]

Given the matrices \( A \in \mathbb{R}^{n \times n} \) and \( C \in \mathbb{R}^{m \times n} \), an output feedback \( A + LC \) to achieve the pole placement inside the ellipsoidal region can be designed by the following theorem.

**Theorem 1:** If the following LMI has a solution \( P \in \mathbb{R}^{n \times n} \) and \( \gamma \in \mathbb{R}^{n \times n} \), then the eigenvalues of \( A + LC \)
are placed inside the prescribed ellipsoidal region mentioned above by the gain $L = F^{-1} Y$.

$$\Phi(A, C, F, Y) = \begin{bmatrix} \theta_{21} & \theta_{22} \\ \theta_{11} & \theta_{12} \end{bmatrix} < 0,$$ \quad (6)

where

$$
\begin{align*}
\theta_{11} &= a^T P_1 + a b P_2 + c P_3 + d P, \\
\theta_{12} &= A^T P A + A^T Y C + C^T Y^T A, \\
\theta_{22} &= A^T Y D + B(P A + Y C), \\
\theta_{21} &= -a Y B + b(A^T P + C^T Y^T), \\
\theta_{12} &= -P.
\end{align*}
$$

Proof: Although the proof is rather straightforward from the standard manipulations on LMI, the derivation process is shown below for the sake of completeness.

![Figure 14. Ellipsoidal region](image)

From [14], the eigenvalues of $A$ are inside the prescribed ellipsoidal region if and only if

$$\begin{bmatrix} c(A X + X A^T) + d X & a A X + b X \end{bmatrix} < 0, \quad (9)$$

has a symmetric solution $X$. By taking the Schur complement, (9) is equivalent to

$$
(AX + XA^T) + CX < 0, \quad (10)
$$

$$
X > 0. \quad (11)
$$

Congruence transformation of (10) by $P = X^{-1}$ yields

$$
(AX + XA^T) + CX < 0, \quad (10')
$$

$$
X > 0. \quad (11')
$$

Let $A_2 = A + LC$ and $Y = PL$. Then (12) is equivalent to

$$
(AX + XA^T) + CX < 0, \quad (13)
$$

By taking the Schur complement of (13), we obtain (8).

C. Stability as switched systems

Stability is one of the central issues of switched linear systems theory [15]. An interesting fact is that, under arbitrary switching, Hurwitz stability of each subsystem does not necessarily mean the stability of switched system as a whole.

Example: Let $z \in \mathbb{R}$ and $\xi \in \{0, 1\}$. We consider the following damped harmonic oscillator:

$$
\ddot{z} + 2\xi\omega_n \dot{z} + \omega_n^2 z = 0.
$$

Suppose that $\xi = 1$. Under the initial condition $z(0) = z_0$, $\dot{z}(0) = 0$, the solution is given by

$$
z(t) = e^{-\omega_n t} \sin \left( \sqrt{1 - \xi^2 \omega_n^2} \right) z_0,
$$

and

$$
\dot{z}(t) = e^{-\omega_n t} \cos \left( \sqrt{1 - \xi^2 \omega_n^2} \right) z_0.
$$

If we take $x_1 = z$ and $x_2 = \dot{z}$, then they satisfy the following equation:

$$
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right)^T \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} < 0.
$$

Thus the corresponding phase portraits for $\omega_n \geq 1$ and $\omega_n < 1$ are given by Fig. 15(a) and (b), respectively. Let us denote the system corresponding to the phase portrait Fig. 15(a) by $\xi(0) = A_1 \xi(0)$, $\xi(t) = [e^{A_1 t}]$. Similarly, $\xi(0) = A_2 \xi(0)$ corresponds to the phase portrait Fig. 15(b). Then obviously both $A_1$ and $A_2$ are Hurwitz.

![Figure 15. Phase portraits of two subsystems](image)

We regard $\xi(0) = A_i \xi(0)$, $i = 1, 2$ as two subsystems of a switched linear system. Now we consider the following two switching policies:

1) When the state is in the 1st and 3rd quadrants, it follows the dynamics described by the subsystem $i = 1$. Otherwise it follows the dynamics for $i = 2$.

2) When the state is in the 1st and 3rd quadrants, it follows the dynamics described by the subsystem $i = 2$. Otherwise it follows the dynamics for $i = 1$.

Under the policy 1), the trajectory of the switched system will behave as Fig. 16(i) and it will converge to the origin. In contrast, under the policy 2), the trajectory of the switched system will undergo the changes shown in Fig. 16(ii). Since the distance from the origin is monotonically increasing, this switched system will diverge.

From a practical viewpoint, to maintain the stability even in the worst case scenario is important. Therefore, we add a guarantee of stability as switching systems via common Lyapunov solution for LMIs. By using a common
for each mode, one can make the energy function (Lyapunov function) $V(x) = x^TPx$ continuous at the time of switching. Since the Hurwitz stability of each mode guarantees monotonous decrease of $V(x)$ during the dwell time, this continuity ensures the stability as a switching system against arbitrary switching.

Specifically, we solve a feasibility problem for the following pair of LMI conditions

$$\Phi(A_i, C_i, P_i, Y_i) < 0, \quad i = 1, 2, ..., M$$ (14)

to find the matrix variables $P_i, Y_i$. If it is feasible, then the observer gain for each mode is derived as $L_i = P^{-1}Y_i$ ($i = 1, 2, ..., M$).

Figure 16. Trajectories of switched system under different switching policies

V. NUMERICAL SIMULATION

In this section, the effectiveness of the proposed design method is verified by numerical simulations. The data used here is the measurement from our experimental apparatus.

The parameters to specify the ellipsoidal region for the pole placement are chosen as $r_1 = 2, r_2 = 3, c_1 = 3$. The feasibility problem with LMIs (14) is solved to obtain $P_1, P_2, ..., P_M$. By using a standard solver, we can find a solution. It is verified that all poles are placed inside the prescribed region successfully. The estimation result of the undelayed temperature by the switching observer is given in Fig. 17.

Figure 17. Estimation with switching observer

Compared to the result shown in Fig. 10, the precision of the proposed method considering multiple operation modes is better than that of the previous method where total dynamics is modeled by only one FDLTI system.

Finally, the estimation results for 8 operating conditions are evaluated. Fig. 18 shows the average error with the standard deviation bar for 8 data sets when the multiple modes are considered. On the other hand, Fig. 19 illustrates the result for the single model case. The improvement of the estimation precision is obvious. Note that we do not use different models and physical parameters for each data set and the multiple mode transitions occur within most data sets.

Table I summarizes the quantitative changes of the accuracy of the proposed method relative to the single model case. The average estimation error is denoted by $\bar{\epsilon}$ and the corresponding standard deviation is denoted by $\sigma_{\epsilon}$.

<table>
<thead>
<tr>
<th>Data set</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_1/\epsilon_2$</td>
<td>0.964</td>
<td>1.29</td>
<td>0.598</td>
<td>0.726</td>
<td>0.744</td>
</tr>
<tr>
<td>$\epsilon_1/\epsilon_2$</td>
<td>1.052</td>
<td>1.144</td>
<td>0.457</td>
<td>0.236</td>
<td>0.412</td>
</tr>
<tr>
<td>Data set</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>$\epsilon_3/\epsilon_4$</td>
<td>0.846</td>
<td>0.846</td>
<td>0.865</td>
<td>0.752</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_3/\epsilon_4$</td>
<td>0.0318</td>
<td>0.201</td>
<td>0.201</td>
<td>0.568</td>
<td></td>
</tr>
</tbody>
</table>

Table I. Relative improvement of estimation precision

The subscripts $m$ and $s$ stand for the case of the multiple and single mode model. One can confirm that the estimation accuracy is improving in general except for the data sets No. 1 and 2 that have less mode transition than others.

VI. CONCLUSIONS

To introduce a model-based soft sensor technique to the temperature measurement of the fusing process of laser printers, a state estimation method for thermal systems with multiple operation modes is proposed.
By using the modeling as linear switched systems, the estimation problem is reduced to a switching observer design. In the design process, a tradeoff between fast response and anti-noise performance is taken into account via LMI-based pole placement within the prescribed region. Also the common Lyapunov function approach is employed to guarantee the stability as a switched system. The effectiveness of the proposed method is verified through numerical simulations with experimental data.

REFERENCES