Mechanics Analysis of String Unit in XYZ Direction under Hydraulic Conditions

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***Abstract* - The paper presents a general method to calculate fluid force and moment of arbitrary units (line unit or curve unit) of a rod string within a hydraulic environment. Rod string which used in oil and gas wells has working conditions of long and narrow 3D wellbore which is filled with fluid within a hydraulic environment. We compare the calculation results of the model with the ANSYS analysis results verify correctness. The results show that this model has enough precision for the X Y Z direction force and moment of the string element.**

Keywords: Static fluid; ANSYS analysis; Mechanical analysis; String unit

I. INTRODUCTION

In deep well ,in order to accurate analysis the rod string mechanical characteristics, discrete rod string and mechanics analysis of the discrete unit structure must be implemented first in Figs.(1) and (2).The liquid force on the string unit is a very important effect under hydraulic conditions. No matter in statics analysis or dynamic analysis, it is actually a calculating parameter.

However, On mechanical analysis of string unit in the drill string system dynamics [1-4], Mechanics of acidification and fracturing string [5-6], Mechanical analysis of Injection string [7-11], Dynamic mechanical analysis of sucker rod [12-15] and so on, all the researches only take into account the vertical (Z) force (buoyancy) of fluid effect and ignore the X Y direction force and moment. Due to the upper and lower section of the string unit are not exposed in fluids, then the fluid force and moment acts on the string both in the Z direction and X Y direction.

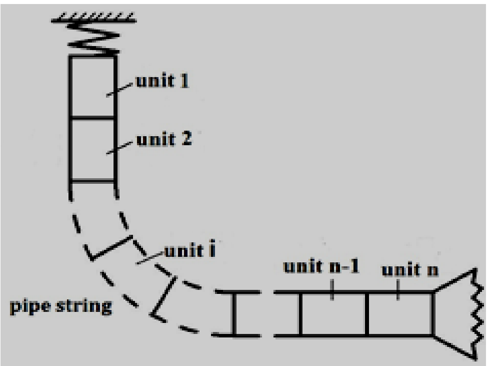


Fig. (1). Discrete Model Unit Structure.

This paper demonstrates a computational method can be applied as a rational means to analyze the fluid force on string unit in XYZ direction under hydraulic conditions. Compared the calculation results of the mode with the ANSYS analysis results to make sure the method is correct.

II. METHODS AND INSUFFICIENCY OF EXISTING MODELS CONSIDERING FLUID FORCE

A. The Existing Model Considering Fluid Force of String Unit

The references below have had impact on the fluid force (buoyancy) on the string unit in Z direction in petroleum engineering. The definition of the neutral point in a tubular is fundamental for the buoyancy discussion and Lubinski (1974) [16] definitions are much used. In 1980 Goins published two articles [17-18] (Goins 1980a and 1980b) about this issue. In the first article Goins (1980a) showed that “buoyancy is equal to the weight of the displaced fluid” is not always true. The second article (Goins 1980b) shows calculation procedures and numerical. Examples that have been widely used because of its clarity. Patillo and Randall (1980) [19] performed a review of buoyancy models, and identified the confusion in industry with respect to identifying the neutral point of a tubular. They conclude that the force-area method must be used with caution, and they provide considerable insight into buoyancy effects. Aadnoy and Kaarstad [20] reviewed the fundamental principles as applied to petroleum engineering. Mitchell [21] presented a fundamental analysis of buoyancy where he included curved boreholes and effects of dynamic flow inside and outside of a pipe. Recently Aadnoy and Kaarstad [22] reviewed the generalized Archimedes’ principle as applied to petroleum engineering in torque and drag analysis.

In above-cited references, the following equations are valid both for vertical and deviated boreholes to considering the submerged weight of a wellbore tubular in the Z direction.

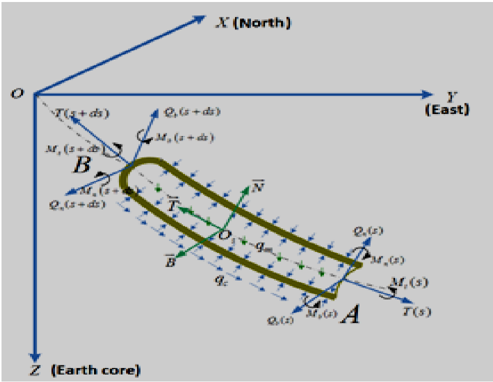
$$k_f = 1 - \rho_m / \rho_s \tag{1}$$

Where ρ_m is the fluid density, ρ_s is the string material density.

Thus, the fluid force on the string units is equal to the buoyancy force of pipe string units in the Z direction shown as Fig.(2).

$$F = \rho_m V_i g \tag{2}$$

Where V_i is the volume of string unit.



Fig, (2). Force Analysis of the ith Unit

B. The Insufficient of Considering the Fluid Force on String Unit by Equations (1, 2)

Through the analysis of static force of fluid on the pipe string units, the deficiency of the existing models are: (1) hydraulic pressure environment to pipe string, the existing models consider only fluid force in the Z direction by buoyancy ; (2) as the actual string unit on the both ends up and down out of contact with fluid , then the fluid force to the unit structure in the Z direction is not equal to the buoyancy shown as Eq.(2) ; (3) Due to both ends up and down of the string unit structure are not exposed in fluids but only the side of string in practice. So the fluid force of string unit in addition to acts in the Z direction, also additional forces acts in X and Y direction. The paper aims at these deficiencies to carry out the following work.

III. THEORETICAL ANALYSIS OF FLUID FORCES ON PIPE STRING STRUCTURE UNIT

A. Analysis of Fluid Forces on Submerged Pipe String Structure Unit i.

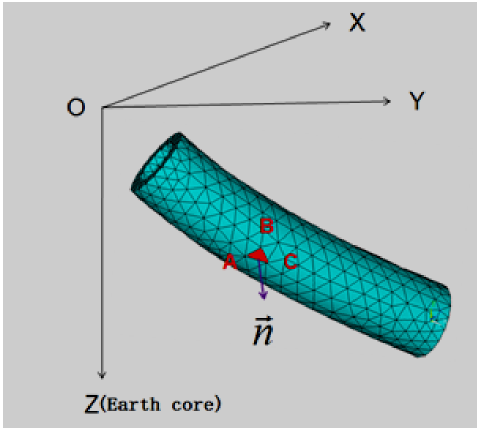
Show as Fig.(3), the area ABC in the string unit is infinitesimal. The normal vector of the area is:

$$\vec{n} = \cos\partial \vec{i} + \cos\beta \vec{j} + \cos\chi \vec{k} \tag{3}$$

Where ∂, β, χ are the angle between the $\vec{n}$ and axis X,Y,Z. The fluid force on the surface of the area ABC becomes:

$$F_i = \sigma \Delta A_i \vec{n} \tag{4}$$

Where σ is the surface pressure of area ABC.



Fig,(3). Schematic Diagram of the String Unit

A1. Fluid Force of the Submerged String Unit in the X, Y, Z Direction

1) The fluid force on the ith string in the X direction becomes:

$$F_{iX} = F_i \cos\partial \vec{n} = \sigma \Delta A_i \cos\partial \vec{n} \tag{5}$$

Then, the fluid force acting on the tube in the X direction becomes:

$$F_X = \sum \sigma \Delta A_i \cos\partial = \oiint_{\Sigma} \sigma \cos\partial dA \tag{6}$$

Where Σ is the closed area of the ith string.

For an arbitrary unit, assuming a vertical prism of height is z and wellhead pressure is P_0, then the surface pressure of area ABC is:

$$\sigma = P_0 + \rho h z \tag{7}$$

Thus, the surface integral is transformed into a volume integral with the help of the Gauss divergence theorem, the fluid force becomes:

$$\begin{aligned} F_X &= \sum \sigma \Delta A_i \cos\partial = \oiint_{\Sigma} \sigma \cos\partial dA \\ &= \iiint_{\Omega} \frac{\partial \sigma}{\partial x} dxdydz = \iiint_{\Omega} 0 dxdydz = 0 \end{aligned} \tag{8}$$

2) The fluid force of on the ith string structure in the Y direction becomes:

$$F_{iY} = F_i \cos\beta \vec{j} = \sigma \Delta A_i \cos\beta \vec{j} \tag{9}$$

So the fluid force in the Y direction of the entire string becomes:

$$F_Y = \sum \sigma \Delta A_i \cos\beta = \oiint_{\Sigma} \sigma\cos\beta dA = \iiint_{\Omega} \frac{\partial \sigma}{\partial y} dxdydz = \iiint_{\Omega} 0 dxdydz = 0 \tag{10}$$

3) Similarly, the fluid force on the ith string in the Z direction becomes:

$$F_{iZ} = F_i \cos\chi \vec{k} = \sigma \Delta A_i \cos\chi \vec{k} \tag{11}$$

So fluid forces produced in the Z direction of the entire string becomes:

$$F_Z = \sum \sigma \Delta A_i \cos\chi = \oiint_{\Sigma} \sigma\cos\chi dA = \iiint_{\Omega} \frac{\partial \sigma}{\partial z} \mathrm{dv} = \iiint_{\Omega} -\rho \mathrm{g} \mathrm{dv} = -\rho g V \tag{12}$$

Where V is the volume of the ith string.

From Eqs. (8),(10) and (12),the fluid force acting on the string in the X ,Y direction are zero but that in the Z direction is equal to the buoyancy by the principle of Archimedes.

A2. The Fluid Moment Acting on the Centroid of the i-th String in The X, Y, Z Direction

In Fig.(3), Assuming the point (x_0, y_0, z_0) is the shape center coordinate of the ith string and the centroid coordinates of the small area ABC is (x, y, z) .the radius vector of the infinitesimal area ABC to the centroid can also be written as:

$$\mathrm{r} = (x - x_0)\vec{i} + (y - y_0)\vec{j} + (z - z_0)\vec{k} \tag{13}$$

The moment of the fluid force about the shape center coordinate can be expressed as:

$$\begin{aligned}
M_i = r \times F_i &= \begin{vmatrix} i & j & k \\ x - x_0 & y - y_0 & z - z_0 \\ F_{ix} & F_{iy} & F_{iz} \end{vmatrix} \\
&= \begin{vmatrix} i & j & k \\ x - x_0 & y - y_0 & z - z_0 \\ \sigma\Delta A_i \cos\partial & \sigma\Delta A_i \cos\beta & \sigma\Delta A_i \cos\chi \end{vmatrix} \\
&= \left((y - y_0)\sigma\Delta A_i \cos\chi - (z - z_0)\sigma\Delta A_i \cos\beta\right)\vec{i} \\
&+ \left((z - z_0)\sigma\Delta A_i \cos\partial - (x - x_0)\sigma\Delta A_i \cos\chi\right)\vec{j} \\
&+ \left((x - x_0)\sigma\Delta A_i \cos\beta - (y - y_0)\sigma\Delta A_i \cos\partial\right)\vec{k}
\end{aligned} \tag{14}$$

Then, the moment is defined as:

$$\begin{aligned}
M_i &= M_x \vec{i} + M_y \vec{j} + M_Z \vec{k} = \sum M_i \\
&= \oiint_{\Sigma} \left((y - y_0)\sigma \cos\chi - (z - z_0)\sigma \cos\beta\right) dA \vec{i} \\
&+ \oiint_{\Sigma} \left((z - z_0)\sigma \cos\partial - (x - x_0)\sigma \cos\chi\right) dA \vec{j} \\
&+ \oiint_{\Sigma} \left((x - x_0)\sigma \cos\beta - (y - y_0)\sigma \cos\partial\right) dA \vec{k}
\end{aligned} \tag{15}$$

By the Gauss divergence theorem, the Eq.(15) is:

$$\begin{aligned}
M_i &= M_x \vec{i} + M_y \vec{j} + M_Z \vec{k} \\
&= \iiint_{\Omega} \left(\frac{\partial((y - y_0)(p_0 + \rho g z))}{\partial z} - \frac{\partial((z - z_0)(p_0 + \rho g z))}{\partial y} \right) dv \vec{i} \\
&+ \iiint_{\Omega} \left(\frac{\partial((z - z_0)(p_0 + \rho g z))}{\partial x} - \frac{\partial((x - x_0)(p_0 + \rho g z))}{\partial z} \right) dv \vec{j} \\
&+ \iiint_{\Omega} \left(\frac{\partial((x - x_0)(p_0 + \rho g z))}{\partial y} - \frac{\partial((y - y_0)(p_0 + \rho g z))}{\partial x} \right) dv \vec{k} \\
&= \iiint_{\Omega} (y - y_0)\rho g dv \vec{i} + \iiint_{\Omega} (x - x_0)\rho g dv \vec{j} + o \vec{k} \\
&= \left(\rho g \iiint_{\Omega} y dv - \rho g y_0 V \right)\vec{i} + \left(\rho g \iiint_{\Omega} x dv - \rho g x_0 V \right)\vec{j} + o \vec{k}
\end{aligned} \tag{16}$$

The shape center coordinate (x_0, y_0, z_0) can be defined as:

$$\begin{cases} x_0 = \dfrac{\iiint_{\Omega} x dv}{V} \\ y_0 = \dfrac{\iiint_{\Omega} y dv}{V} \\ z_0 = \dfrac{\iiint_{\Omega} z dv}{V} \end{cases} \tag{17}$$

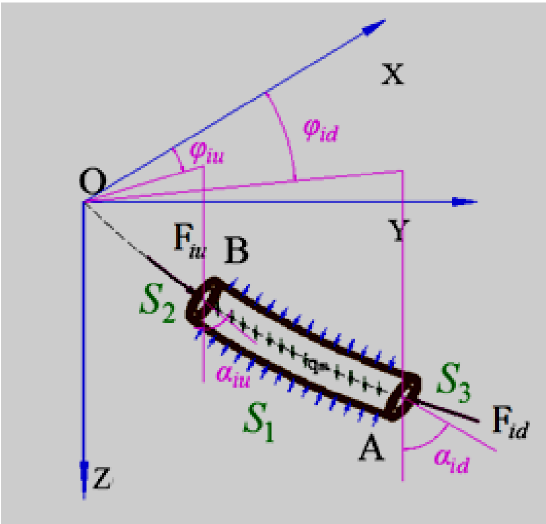
Putting the Formula (17) into Formula (16), the moment can be written as:

$$M = M_x \vec{i} + M_y \vec{j} + M_z \vec{k} \tag{18}$$

As it is shown in Eq. (18) that the moment of the fluid force in the X, Y, Z direction on the centroid of immersion unit are all zero.

B. Analysis the Fluid Force and Moment of the Actual String Unit

The above analysis shows that Eq. (3) to Eq.(18) can be applied for a condition that the action area is closed region. Due to the section A and section B out of contact with fluid shown as Fig. (4), the fluid force acting on the string unit actually equal to the fluid force acting on the side of the string unit reaction. Therefore, the paper uses the equivalent method to handle the fluid force of actual string unit. This method is as follows.



Fig,(4) .Mechanical Analysis Schematic

In Fig, (4), S_1 is the outer side area of string unit; S_2, S_3 are the area of the section A and section B; F_{ic} is the fluid force acting on the side of the string unit; F_{iu} and F_{id} are the fluid force acting on the section A and section B .Then, the real fluid force acting on the string unit actually equal to F_{ic} and F_{ic} can be expressed as:

$$\vec{F}_{ic} = \iint_{\Sigma s_1} \sigma \cos\partial dA\,\vec{i} + \iint_{\Sigma s_1} \sigma\cos\beta dA\,\vec{j} + \iint_{\Sigma s_1} \sigma\cos\chi dA\,\vec{k} \tag{19}$$

The F_{iu} and F_{id} can also be expressed as:

$$\vec{F}_{iu} = \iint_{\Sigma s_2} \sigma \cos\partial dA\,\vec{i} + \iint_{\Sigma s_2} \sigma\cos\beta dA\,\vec{j} + \iint_{\Sigma s_2} \sigma\cos\chi dA\,\vec{k} \tag{20}$$

$$\vec{F}_{id} = \iint_{\Sigma s_3} \sigma \cos\partial dA\,\vec{i} + \iint_{\Sigma s_3} \sigma\cos\beta dA\,\vec{j} + \iint_{\Sigma s_3} \sigma\cos\chi dA\,\vec{k} \tag{21}$$

By equivalent thought, the real fluid force also can be written as：

$$\begin{aligned}\vec{F}_{ic} &= \left(\vec{F}_{ic} + \vec{F}_{iu} + \vec{F}_{id}\right) - \left(\vec{F}_{iu} + \vec{F}_{id}\right) \\ &= \iint_{\Sigma s_1+s_2+s_3} \sigma\cos\partial dA\,\vec{i} + \iint_{\Sigma ss_1+s_2+s_3} \sigma\cos\beta dA\,\vec{j} \\ &+ \iint_{\Sigma s_1+s_2+s_3} \sigma\cos\chi dA\,\vec{k} - \left(\vec{F}_{iu} + \vec{F}_{id}\right)\end{aligned} \tag{22}$$

Where the integral surface $s_1+s_2+s_3$ of the $\vec{F}_{ic} + \vec{F}_{iu} + \vec{F}_{id}$ form a closed are Ω .

Substituting Eqs (8), (10) and (12) into Eq. (22), then Eq. (22) becomes:

$$\vec{F}_{ic} = -\left(\vec{F}_{iu} + \vec{F}_{id}\right) - \rho g V\,\vec{k} \tag{23}$$

In the same way, M_{ic} is the moment of the fluid force F_{ic} about the shape center; M_{iu} is the moment of the fluid force F_{iu} about the shape center; M_{id} is the moment of the fluid force F_{id} about the shape center. The real moment acting on the string unit actually equal to M_{ic} .The moment M_{ic} is:

$$\begin{aligned}M_{ic} &= \vec{r}\times\vec{F}_{ic} \\ &= \iint_{\Sigma s_1}\left((y-y_0)\sigma\cos\chi - (z-z_0)\sigma\cos\beta\right)dA\,\vec{i} \\ &+ \iint_{\Sigma s_1}\left((z-z_0)\sigma\cos\partial - (x-x_0)\sigma\cos\chi\right)dA\,\vec{j} \\ &+ \iint_{\Sigma s_1}\left((x-x_0)\sigma\cos\beta - (y-y_0)\sigma\cos\partial\right)dA\,\vec{k}\end{aligned} \tag{24}$$

Where $\vec{r}$ is the radius vector of the side of string unit to the centroid.

The M_{iu} and M_{id} can also be expressed as:

$$\begin{aligned}M_{iu} &= \vec{r}\times\vec{F}_{iu} \\ &= \iint_{\Sigma s_2}\left((y-y_0)\sigma\cos\chi - (z-z_0)\sigma\cos\beta\right)dA\,\vec{i} \\ &+ \iint_{\Sigma s_2}\left((z-z_0)\sigma\cos\partial - (x-x_0)\sigma\cos\chi\right)dA\,\vec{j} \\ &+ \iint_{\Sigma s_2}\left((x-x_0)\sigma\cos\beta - (y-y_0)\sigma\cos\partial\right)dA\,\vec{k}\end{aligned} \tag{25}$$

$$\begin{aligned} M_{id} &= \vec{r} \times \vec{F}_{id} \\ &= \iint_{\Sigma s_3} ((y-y_0)\sigma\cos\chi - (z-z_0)\sigma\cos\beta) dA\,\vec{i} \\ &+ \iint_{\Sigma s_3} ((z-z_0)\sigma\cos\partial - (x-x_0)\sigma\cos\chi) dA\,\vec{j} \\ &+ \iint_{\Sigma s_3} ((x-x_0)\sigma\cos\beta - (y-y_0)\sigma\cos\partial) dA\,\vec{k} \end{aligned} \quad (26)$$

Where $\vec{r}_{iu}$ and $\vec{r}_{id}$ are the radius vector of upper and lower end face of pipe string unit to the centroid.

$$\begin{aligned} M_{ic} &= M_{ic} + M_{iu} + M_{id} - (M_{iu} + M_{id}) \\ &= \oiint_{\Sigma s_2+s_2+s_3} ((y-y_0)\sigma\cos\chi - (z-z_0)\sigma\cos\beta) dA\,\vec{r} \\ &+ \oiint_{\Sigma s_1+s_2+s_3} ((z-z_0)\sigma\cos\partial - (x-x_0)\sigma\cos\chi) dA\,\vec{j} \\ &+ \oiint_{\Sigma s_1+s_2+s_3} ((x-x_0)\sigma\cos\beta - (y-y_0)\sigma\cos\partial) dA\,\vec{k} \end{aligned} \quad (27)$$

Because S_1+S_2+S_3 form a closed area .According to the Eq. (18), the Eq. (27) can be written as:

$$M_{ic} = 0 - (M_{iu} + M_{id}) = -(M_{iu} + M_{id}) \quad (28)$$

IV. MODEL TO CALCULATE THE FORCES AND MOMENT OF PIPE STRING UNIT

A. Model to Calculate the Forces Fic

As shown in Fig,(5), the vertical height △H of the section A is very small , it can assume that the fluid pressure σ_1 acting on the section A is a constant, $\sigma_1 = p_0 + \rho g H_{iu}$. In the same way, the fluid pressure σ_2 acting on the section B is a constant, $\sigma_2 = p_0 + \rho g H_{id}$; $\vec{n}_1$ and $\vec{n}_2$ are the normal vector of the section A and section B:

$$\begin{cases} \vec{n}_1 = \cos\alpha_1\,\vec{i} + \cos\beta_1\,\vec{j} + \cos\chi_1\,\vec{k} \\ \vec{n}_2 = \cos\alpha_2\,\vec{i} + \cos\beta_2\,\vec{j} + \cos\chi_2\,\vec{k} \end{cases} \quad (29)$$

In Fig,(5), it's easy to get the equation:

$$\begin{cases} \cos\alpha_1 = \sin\alpha_{iu}\cos\varphi_{iu} \\ \cos\beta_1 = \sin\alpha_{iu}\sin\varphi_{iu} \\ \cos\chi_1 = \cos\alpha_{iu} \end{cases}$$

$$\begin{cases} \cos\alpha_2 = \sin\alpha_{id}\cos\varphi_{id} \\ \cos\beta_2 = \sin\alpha_{id}\sin\varphi_{id} \\ \cos\chi_2 = \cos\alpha_{id} \end{cases}$$

Where α is the deviation angle; φ is the azimuth angle.

Then the Eq. (20) and Eq.(21) become:

$$\begin{aligned} \vec{F}_{iu} &= \vec{n}\,\sigma A \\ &= \left(\sin\alpha_{iu}\cos\varphi_{iu}\,\vec{i} + \sin\alpha_{iu}\sin_{iu}\,\vec{j} + \cos\alpha_{iu}\,\vec{k}\right)\sigma_1 A \end{aligned} \quad (30)$$

$$\begin{aligned} \vec{F}_{id} &= \vec{n}\,\sigma A \\ &= \left(\sin\alpha_{id}\cos\varphi_{id}\,\vec{i} + \sin\alpha_{id}\sin_{id}\,\vec{j} + \cos\alpha_{id}\,\vec{k}\right)\sigma_1 A \end{aligned} \quad (31)$$

Substituting Eq. (30) and Eq. (31) into Eq.(23), it can obtain the fluid force acting on the strings unit:

$$\begin{aligned} \vec{F}_{ic} &= (\sin\alpha_{id}\cos\varphi_{id}\sigma_2 A - \sin\alpha_{iu}\cos\varphi_{iu}\sigma_1 A)\vec{i} \\ &+ (\sin\alpha_{id}\sin\varphi_{id}\sigma_2 A - \sin\alpha_{iu}\sin\varphi_{iu}\sigma_1 A)\vec{j} \\ &+ (\cos\alpha_{id}\sigma_2 A - \cos\alpha_{iu}\sigma_1 A - \rho g V)\vec{i} \end{aligned} \quad (32)$$

The force F_{iX}, F_{iY}, F_{iZ} component of the fluid force in X, Y, Z directions are:

$$\begin{cases} F_{iX} = \sin\alpha_{id}\cos\varphi_{id}\sigma_2 A - \sin\alpha_{iu}\cos\varphi_{iu}\sigma_1 A \\ F_{iY} = \sin\alpha_{id}\sin\varphi_{id}\sigma_2 A - \sin\alpha_{iu}\sin\varphi_{iu}\sigma_1 A \\ F_{iZ} = \cos\alpha_{id}\sigma_2 A - \cos\alpha_{iu}\sigma_1 A - \rho g V \end{cases} \quad (33)$$

Defined by the structure of buoyancy factor can be concluded that the unit structure i buoyancy coefficient is:

$$k_{fi} = \frac{G - F_{iz}}{G} \quad (34)$$

Through the above analysis can be concluded that there is obviously difference between the equations deduced in the paper with that of the existing one.

B. Model to Calculate the Moment

In Fig.5, the center coordinates of the section A is (x_1, y_1, z_1),the center coordinates of the section B is(x_2, y_2, z_2).The shape center coordinates of the string unit is (x_0, y_0, z_0) , the radius vector $\vec{r}_{iu}$, $\vec{r}_{id}$ of the section A and the section B to the centroid are:

$$\begin{cases} \vec{r}_{iu} = (x_1 - x_0)\vec{i} + (y_1 - y_0)\vec{j} + (z_1 - z_0)\vec{k} \\ \vec{r}_{id} = (x_2 - x_0)\vec{i} + (y_2 - y_0)\vec{j} + (z_2 - z_0)\vec{k} \end{cases} \quad (35)$$

Then, the moments Miu, Mid of the force Fiu, Fid acting on the centroid of the string units can be described as:

$$\begin{aligned} M_{iu} &= \vec{r}_{iu} \times F_{iu} \\ &= [(y_1 - y_0)\cos\alpha_{iu} - (z_1 - z_0)\sin\alpha_{iu}\sin\varphi_{iu}]\sigma_1 A\vec{i} \\ &+ [(z_1 - z_0)\cos\alpha_{iu}\cos\varphi_{iu} - (x_1 - x_0)\cos\alpha_{iu}]\sigma_1 A\vec{j} \\ &+ [(x_1 - x_0)\sin\alpha_{iu}\sin\varphi_{iu} - (y_1 - y_0)\sin\alpha_{iu}\sin\varphi_{iu}]\sigma_1 A\vec{k} \end{aligned} \tag{36}$$

$$\begin{aligned} M_{id} &= \vec{r}_{id} \times F_{id} \\ &= [(y_2 - y_0)\cos\alpha_{id} - (z_2 - z_0)\sin\alpha_{id}\sin\varphi_{id}]\sigma_2 A\vec{i} \\ &+ [(z_2 - z_0)\cos\alpha_{id}\cos\varphi_{id} - (x_2 - x_0)\cos\alpha_{id}]\sigma_2 A\vec{j} \\ &+ [(x_2 - x_0)\sin\alpha_{id}\sin\varphi_{id} - (y_2 - y_0)\sin\alpha_{id}\sin\varphi_{id}]\sigma_2 A\vec{k} \end{aligned} \tag{37}$$

Substituting Eq. (36) and Eq. (37) into Eq.(28) to obtain the moment acting on the centroid of the string units:

$$\begin{aligned} M_{ic} &= -(M_{iu} + M_{id}) \\ &= \{[(z_2 - z_0)\sin\alpha_{id}\sin\varphi_{id} - (y_2 - y_0)\cos\alpha_{id}]\sigma_2 A - [(y_1 - y_0)\cos\alpha_{iu} - (z_1 - z_0)\cos\alpha_{iu}\sin\varphi_{iu}]\sigma_1 A\}\vec{i} \\ &+ \{[(x_2 - x_0)\sin\alpha_{id}\cos\varphi_{id} - (z_2 - z_0)\cos\alpha_{id}]\sigma_2 A - [(z_1 - z_0)\cos\alpha_{iu}\cos\varphi_{iu} - (x_1 - x_0)\cos\alpha_{iu}]\sigma_1 A\}\vec{j} \\ &+ \{[(y_2 - y_0)\sin\alpha_{id}\cos\varphi_{id} - (x_2 - x_0)\sin\alpha_{id}\sin\varphi_{id}]\sigma_2 A - [(x_1 - x_0)\cos\alpha_{iu}\sin\varphi_{iu} - (y_1 - y_0)\cos\alpha_{iu}\cos\varphi_{iu}]\sigma_1 A\}\vec{k} \end{aligned} \tag{38}$$

Substituting Eq. (36) and Eq. (37) into Eq.(28) to obtain the moment acting on the centroid of the string units:

$$\begin{cases} M_{iX} = [(z_2 - z_0)\sin\alpha_{id}\sin\varphi_{id} - (y_2 - y_0)\cos\alpha_{id}]\sigma_2 A - [(y_1 - y_0)\cos\alpha_{iu} - (z_1 - z_0)\cos\alpha_{iu}\sin\varphi_{iu}]\sigma_1 A \\ M_{iY} = [(x_2 - x_0)\sin\alpha_{id}\cos\varphi_{id} - (z_2 - z_0)\cos\alpha_{id}]\sigma_2 A - [(z_1 - z_0)\cos\alpha_{iu}\cos\varphi_{iu} - (x_1 - x_0)\cos\alpha_{iu}]\sigma_1 A \\ M_{iZ} = [(y_2 - y_0)\sin\alpha_{id}\cos\varphi_{id} - (x_2 - x_0)\sin\alpha_{id}\sin\varphi_{id}]\sigma_2 A - [(x_1 - x_0)\cos\alpha_{iu}\sin\varphi_{iu} - (y_1 - y_0)\cos\alpha_{iu}\cos\varphi_{iu}]\sigma_1 A \end{cases} \tag{39}$$

V. EXAMPLES

To verify the correctness of the above theoretical analysis, two special string unite model are analyzed. Assuming that string unit is located in the YZ plane, the material of pipe column is steel and its density is 7850 Kg/m3; The fluid is water and its density is 1000 Kg/m3 ; the gravitational acceleration g is 9.8N/Kg ; the wellhead pressure P0 is 3 kPa.

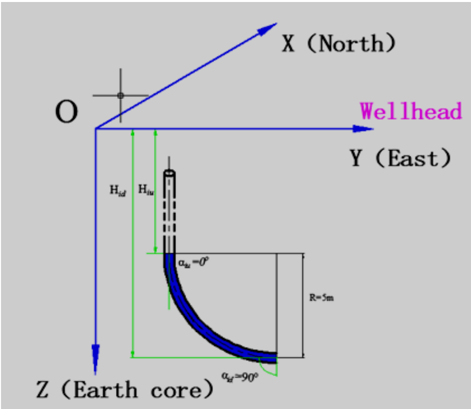
A. Example 1

As shown in Fig,(6), the special string unite model is a 1/4 round string unit which the radius D of the cross section is 1 m ; the curvature radius r of 1/4 round is 5 m and the length l_i is $\frac{5}{2}\pi$ .Substituting the above parameters into Eq.(33) and Eq.(39) ,the fluid force and moment are deduced as follow:

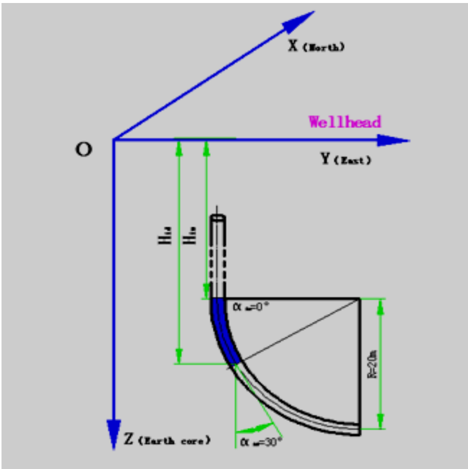
$$\begin{cases} F_{iX} = 0 \\ F_{iY} = p_0 A + \rho_m A g H_{id} = 3000 \times 0.7854 + 1000 \times 0.7584 \times 9.8 \times (5 + H_{iu}) \\ F_{iZ} = p_0 A + \rho_m A g (l_i + H_{iu}) = 3000 \times 0.7854 + 1000 \times 0.7584 \times 9.8 \times \left(\frac{5\pi}{2} + H_{iu}\right) \\ k_{fi} = \frac{G - F_{iZ}}{G} = 1 - \frac{\rho_m}{\rho_s}\left(1 + \frac{H_{iu}}{l_i}\right) - \frac{p_0}{\rho_s g l_i} = 1 - \frac{1000}{7850}\left(1 + \frac{2H_{iu}}{5\pi}\right) - \frac{3000 \times 2}{7850 \times 9.8 \times 5\pi} \end{cases} \tag{40}$$

$$\begin{cases} M_{iX} = 0.5 \times 1000 \times 9.8 \times 0.7854 \times 9.8 \times (5 + H_{iu}) \\ M_{iY} = 0 \\ M_{iZ} = 0 \end{cases} \tag{41}$$

When the vertical height Hiu changes from 300m to 3000m, the results of the fluid force and moment are shown in table1.



Fig,(6). Model 1 of Quadrant-circle Unit structure



Fig,(7). Model 2 of string Unit structure.

Table 1. The theories calculation results of model 1.

The vertical depth of water （m）	x(N)	Y(N)	Z(N)	kft
300	0	2349917	2371883	-3.99327
500	0	3889301	3911267	-7.23719
800	0	6198377	6220343	-12.1031
1000	0	7737761	7759727	-15.347
1300	0	10046837	10068803	-20.2129
1500	0	11586221	11608178	-23.4568
1800	0	13895297	13917263	-28.3227
2000	0	15434681	15456647	-31.5666
2300	0	17743757	17765723	-36.4325
2500	0	19283141	19305107	-39.6764
2800	0	21592217	21614183	-44.5423
3000	0	23131601	23153567	-47.7862

B. Example 2

As shown in Fig,(7), the special string unite model is a circular cylinder unit which the central angle is 30^0 ;the radius D of circular cross-sections is 1 m ;the curvature radius r is 20 m ;the length is $l_i = \frac{10}{3}\pi$m and $\alpha_{iu} = 0, \alpha_{id} = \alpha_{iu} + kl_i = 30^0$.
Substituting the above parameters into Eq.(33) and Eq.(39) ,the fluid force and moment calculation modes are deduced as follow :

$$\begin{cases} F_{iX} = 0 \\ F_{iY} = 0.5p_0A + 0.5\rho_m Ag(H_{iu} + 0.5R) = 0.5\times3000\times0.7854 + 0.5\times1000\times0.7584\times9.8\times(10 + H_{iu}) \\ F_{iZ} = 0.13397p_0A + \rho_m Ag\left(l_i + 0.13397H_{iu} - \frac{\sqrt{3}}{4}R\right) = 315.67 + 1000\times0.7584\times9.8\times\left(\frac{10\pi}{3} + 0.13397H_{iu} - 8.66\right) \\ k_{fi} = \frac{G - F_{iZ}}{G} = 1 - \frac{\rho_m}{\rho_s}\left(1 + \frac{0{,}133H_{iu} - 8.66}{l_i}\right) - \frac{p_0}{\rho_s g l_i} = 1 - \frac{1000}{7850}\left(1 + \frac{0.402H_{iu} - 25.98}{10\pi}\right) - \frac{3000\times3}{7850\times9.8\times10\pi} \end{cases} \tag{42}$$

$$\begin{cases} M_{iX} = 0.9014068\times1000\times9.8\times0.7854\times(H_{id} - H_{iu}) \\ M_{iY} = 0 \\ M_{iZ} = 0 \end{cases} \tag{43}$$

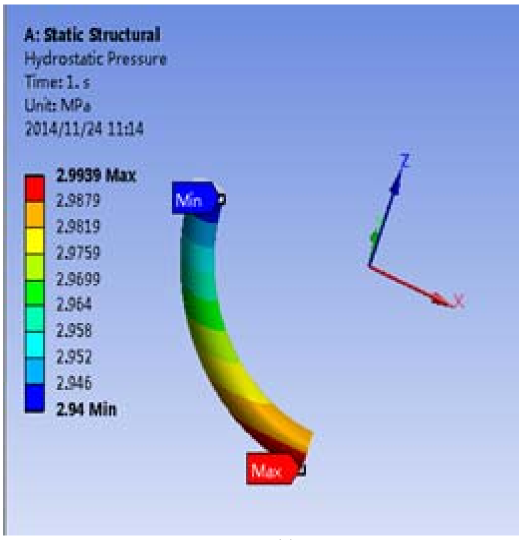
When the vertical height Hiu changes from 300m to 3000m, the results of the fluid force and moment are shown in table 2

TABLE 2 .THE THEORIES CALCULATION RESULTS OF MODEL 2.

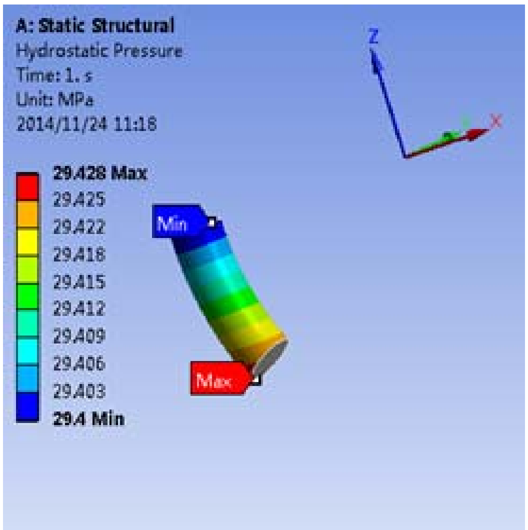
The vertical depth of water （m）	X(N)	Y(N)	Z(N)	kft
300	0	1194200	323460	0.464653499
500	0	1963892	529699	0.123315074
800	0	3118430	839056	-0.388690081
1000	0	3888122	1045295	-0.730028507
1300	0	5042660	1354652	-1.242033662
1500	0	5812352	1560890	-1.583370432
1800	0	696690	1870248	-2.095377242
2000	0	7736582	2076487	-2.436715667
2300	0	8891120	2385844	-2.948720823
2500	0	9660812	2592082	-3.290057593
2800	0	10815350	2901440	-3.802064403
3000	0	11585042	3107678	-4.143401173

VI. APPLICATIONS

To verify the theoretical analysis proposed in this paper , two 3D models of above special string unites by using the finite element code are built to simulate this complex mechanical problem as shown in Fig.,(8) . Both ends of the pipe string unit are restricted and applied a hydrostatic pressure as and an initial pressure as P=3KPa in the lateral surface of string unit. the material of pipe column is steel and its density   is 7850 Kg/m3; The fluid is water and its density   is 1000 Kg/m3 ; the gravitational acceleration g is 9.8N/Kg ; the wellhead pressure P0 is 3 kPa.Through the simulation of ANSYS calculate the constraint force   and   of upper and lower end surface separately. Then based on force balance to calculate the fluid forces as   on the side of the pipe column unit. Compared the results of ANSYS calculation with that of theoretical calculation vertical depth from 300 m to 3000 shown in tables 3, 4.



(a)



(b)

Figs. 8 (a) ,(b)String Unit is Located in the Well Depth of 1800 m When Applying the Hydrostatic Pressure

TABLE 3.COMPARISON OF THE RESULTS OF THEORETICAL WITH THAT OF ANSYS OF STRING UNIT 1

The vertical depth of water Hiu（m）	Results of ANSYS calculation (N)			Results of theories calculation (N)			Deviation （%）	
	$X(10^{-3})$	$Y(10^{6})$	$Z(10^{6})$	X	Y	Z	Y	Z
300	5.1	2.34855	2.37012	0	2349917	2371883	0.058172267	0.07432913
500	6.4	3.88742	3.90899	0	3889301	3911267	0.048363446	0.05821643
800	11.1	6.19578	6.21735	0	6198377	6220343	0.041898065	0.048116318
1000	11.8	7.73465	7.75622	0	7737761	7759727	0.040205429	0.045194889
1300	18.2	10.0429	10.06457	0	10046837	10068803	0.039186462	0.042040747
1500	20.3	11.5815	11.60344	0	11586221	11608178	0.040056201	0.040816052
1800	23.4	13.8897	13.9116	0	13895297	13917263	0.039991948	0.040690472
2000	42	15.4295	15.45037	0	15434681	15456647	0.033502474	0.04061036
2300	48	17.7377	17.7585	0	17743757	17765723	0.03413595	0.040656944
2500	52	19.2764	19.2983	0	19283141	19305107	0.034957998	0.0352601
2800	58	21.5846	21.6065	0	21592217	21614183	0.0352766	0.035546104
3000	62	23.1234	23.1452	0	23131601	23153567	0.035453664	0.036136981

TABLE 4.COMPARISON OF THE RESULTS OF THEORETICAL WITH THAT OF ANSYS OF STRING UNIT 2

	Results of ANSYS calculation（N）			Results of theories calculation (N)			Deviation （%）	
The vertical depth of water Hiu（m）	X	$Y(10^{6})$	$Z(10^{6})$	X	Y	Z	Y	Z
300	-0.0026	1.20006	0.3236	0	1194200	323460	-0.490705075	0.043282013
500	-0.0043	1.9925	0.5367	0	1963892	529699	-1.456699248	1.321694019
800	-0.0069	3.18126	0.8478	0	3118430	839056	-2.014795907	1.042123529
1000	-0.0088	3.97364	1.0569	0	3888122	1045295	-2.199468021	1.110212906
1300	-0.0112	5.1624	1.3588	0	5042660	1354652	-2.374540421	0.306204103
1500	-0.0127	5.9549	1.5723	0	5812352	1560890	-2.452501156	0.730993215
1800	-0.0155	7.1435	1.8823	0	6966891	1870248	-2.534975788	0.644406517
2000	-0.0175	7.936	2.07889	0	7736582	2076487	-2.577598221	0.115724298
2300	-0.0201	9.1247	2.4002	0	8891120	2385844	-2.627115594	0.601715787
2500	-0.0218	9.9172	2.6103	0	9660812	2592082	-2.653897002	0.702832704
2800	-0.0242	11.1058	2.9203	0	1081535	2901440	-2.685534911	0.650022058
3000	-0.0264	11.8983	3.1225	0	1158504	3107678	-2.703986744	0.47694774

As shown in tables 3, 4, the deviation between theoretical analyses with numerical solution is very tiny and the maximum of the deviation is 2.7%.Obviously, this deviation of results is tolerant for petroleum engineering and those calculation models are proved to be correct. Taking the results of quadrant-circle unit structure as an example, the fluid force acting on string unit in the Y, Z direction is more than 600 KN when the vertical depth of water over 800 m, far more than its own gravity 474.8321 KN. As shown in Eq. (2) the existing model considering fluid force of string unit as the buoyancy, while ignoring the fluid forces of the string units in X, Y direction. Substituting relevant parameters into Eq.(2),the buoyancy force of pipe string units in the Z direction is 60.4189KN.So the result obtained from existing model is bound to create a great error in mechanical calculation of string.

VII. CONCLUSION

The paper presents a generalized model to calculate fluid force and moment in the X, Y and Z direction of arbitrary units (line unit or curve unit) of the rod string under static fluid and derive the calculation formula of special string unit.

Calculation models are proved to be correct by compared the results ANSYS finite element calculated with the results derived formula calculation.

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