A New Evidence Combination Rule Based on Weight of Evidence

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***Abstract* - Pignistic probability distance can describe evidence distance accurately. The paper calculates the similarity of evidence based on Pignistic probability distance, then determines the relative weight of evidence. In the Dempster's combination rule, every piece of evidence has equal importance, which severely limits the application of evidence theory, thus we revise the combination rule by introducing the weight into it. The system simulation shows that the revised combination rule reaches better result which in accordance with basic human cognition.**

Keywords - Theory of evidence; Pignistic probability distance; weight of evidence; combination rule

I. INTRODUCTION

Theory of evidence has been widely used in many aspects as a kind of uncertain reasoning method, such as reasoning under uncertainty [1], data fusion [2; 3], and multiple attribute evaluation [4; 5]. In the evidence theory, evidence includes not only attributes and objective environment which the people analysis of proposition to get the basic credibility basis on, but also includes people's experience, knowledge, and the observation and study on the problem.

Dempster's Rule of Combination [6] is the core of Theory of Evidence, It reflects the combined effects between multiple evidences, but this rule considers the importance and reliability of every evidence the same. Thus, it limits the application range of Theory of Evidence. Because of the frame of discernment is generally complete for concrete evidence decision problem when using Theory of Evidence to make decisions. Researchers believe that the reason of paradox generated from combination of evidence is as follows: (1) The frame of discernment is incomplete [7]; (2) The importance and reliability of every evidence is the same in the combination process [8]. This paper considers that frame of discernment is generally complete for concrete evidence decision, so the way to solve the paradox problem is to revision the importance and reliability of evidence.

Many researchers revise combination according to different importance and reliability of evidence[9], such as, Shafer[6] proposed an idea of evidence of a discount based on the reliability of evidence, but he didn't give the specific algorithm of the rate of discount, Yang[10] developed this idea and gave an algorithm to determine the discount coefficient based on Neural Network Learning, and he applied this method to make expert group prediction in China's securities market. Jousselme [11] measured the different of evidence by calculating the distance of evidence, Liu [12] put forward Pignistic probability distance for measuring the distance of evidence and verified that Pignistic evidence distance is more accurate to represent evidence distance than that of Jousselme's. On the basis of Liu's research, this paper utilizes Pignistic evidence distance to measure the similarity of the evidence, we consider that one evidence has a high degree of reliability when it has larger similarity with others, and the evidence should be given a greater weight.

In this paper, section 2 reviews related concepts and formulas in evidence theory. In section 3, we calculate the weight of evidence by using Pignistic distance matrix. In section 4, we introduce the weight of evidence into the Dempster's Rule of Combination to revise combination rule. In section 5, it shows a concrete example to help the readers understanding the combination rule in detail. Section 6 analyzes weight of evidence impacts on the result of fusion by using Matlab simulation.

II. EVIDENCE THEORY

Definition 1 (frame of discernment) [6]. Assume: Θ= {θ1, θ2…，θn} is a finite set of identifiable elements, called the frame of discernment, Θ will acquire its meaning form what we know or think we know.

Definition 2 (basic probability assignment, BPA) [12].If Θ is a frame of discernment, then function M: 2θ→[0,1] is called basic probability assignment whenever:

$$\begin{cases} m(\emptyset) = 0 \\ \sum_{A\subset\Theta} m(A) = 1 \end{cases} \tag{1}$$

The quantity m(A) is called A's basic probability number, and it is understood to be the measure of the belief that is committed exactly to A. Any subset A of Θ such that m(A)>0 is called a focal element.

Definition 3 (belief function) [13].

$$Bel(A) = \sum_{B\subset A} m(B) \tag{2}$$

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a function Bel: 2θ→[0,1] is called a belief function over Θ if it is given by (2) for some basic probability assignment M: 2θ→[0,1].

Definition 4 (Dempster's rule of combination) [6]. Suppose Bel1 and Bel2 are belief functions over the same frame Θ, with basic probability assignments m1 and m2 and focal elements A1…Ak and B1…Bl, respectively

Suppose $\sum_{A_i \cap B_j} m_1(A_i) m_2(B_j) < 1$

Then the function M: 2θ→[0,1] defined by

$m(\emptyset) = 0$ And

$$\mathrm{m(C)} = (1-\mathrm{K})^{-1} \sum_{A_i \cap B_j = C} m_1(A_i) m_2(B_j) \quad (3)$$

$$\mathrm{K} = \sum_{\mathrm{A_i} \cap \mathrm{B_{j=\emptyset}}} \mathrm{m_1(A_i) m_1(B_j)}$$

For all non-empty A⊂θ is a basic probability assignment .The core of the belief function given by m is equal to the intersection of the cores of Bel1 and Bel2.

Definition 5 (Pignistic probability function) [14; 15]. Let m be a BPA on Θ. Its associated pignistic probability function BetPm: 2θ→[0,1] is defined as

$$BetP_m(\alpha) = \sum_{A \in \Theta, \alpha \in A} \frac{m(A)}{|A|} \quad (4)$$

where $|A|$ is the cardinality of subset A.

Definition 6 (pignistic evidence distance). [12] Let m1 and m2 be two BPAs on frame Θ and let BetPm1 and BetPm2 be the results of two pignistic transformations from them respectively. Then

$$difBetP_{m2}^{m1} = \max_{A \in \Theta}(|BetP_{m1}(A) - BetP_{m2}(A)|) \quad (5)$$

is called pignistic evidence distance of the two BPAs, denoted difBetP12 hereafter.

Definition 7 (pignistic similarity of evidence).

$$simBetP_{m2}^{m1} = 1 - difBetP_{m2}^{m1} \quad (6)$$

is called pignistic similarity of evidence of the two BPAs, denoted simBetP12 hereafter.

III. WEIGHT OF EVIDENCE

Pignistic probability metrics is evidence distance which represents the largest distance in all subset from the construction of frame, Pignistic evidence distance represent evidence distance clearer than Jousselme's distance of evidence. Based on this, this section choses pignistic evidence distance to calculate evidence of similarity to get weight of evidence.

Firstly, calculate pignistic distance by the formula 5 and construct pignistic distance matrix.

$$DifBetP = \begin{vmatrix} 0 & difBetP_{12} & difBetP_{1n} \\ difBetP_{21} & \ddots & difBetP_{2n} \\ difBetP_{n1} & difBetP_{n2} & 0 \end{vmatrix} \quad (7)$$

Secondly, on the basis of definition 8, get a pignistic evidence similarity matrix.

$$SimBetP = 1 - DifBetP = \begin{vmatrix} 1 & simBetP_{12} & simBetP_{1n} \\ simBetP_{21} & \ddots & simBetP_{2n} \\ simBetP_{n1} & simBetP_{n2} & 1 \end{vmatrix} \quad (8)$$

Thirdly, get the reliability degree of evidence i.

$$SimBetP(m_i) = \sum_{j=1}^{n} simBetP_{ij} \quad (9)$$

Denoted SimBetPi hereafter.

Finally, by normalizing reliability of evidence, we can get the weight of evidence ωi.

$$\omega_i = \frac{SimBetPi}{\sum_{i=1}^{n} simBetP_i} \quad (10)$$

IV. A NEW COMBINATION RULE

In the Dempster's combination rule, all evidences have equal importance and reliability, which is often inconsistent with actual decision problems and will limit the application range of D-S theory. In this section, we revise the combination rule on the basis of section 3. The weight of evidence is defined as the importance of the evidence against other evidence, so the weight of evidence is effective only in the process of integration. In this section, we revise the Dempster's combination rule by introducing the weight into it. The revised combination rule in this text as follows:

$$\mathrm{m(C)} = \begin{cases} K^{-1} \sum_{A_i \cap B_j = C} (m_1(A_i))^{\omega 1} (m_2(B_j))^{\omega 2} & C \neq \emptyset \\ 0 & C = \emptyset \end{cases} \quad (11)$$

$\mathrm{K} = \sum_{A_i \cap B_j \neq \emptyset} (m_1(A_i))^{\omega 1} (m_2(B_j))^{\omega 2}$, ω1 is the weight of m1, ω2 is the weight of m2, the revised combination rule do not change the commutativity and associativity of Dempster's Rule of Combination, at the same time , section 5 will show how the result of synthesis will change when the weight of evidence changed.

V. A CALCULATION EXAMPLE

In order to help readers understand the aforementioned calculation method, in this section, an example is given to demonstrate the calculation process.

Let the three BPAs m1, m2 and m3 defined on Θ= {a, b, c}. Let

m1(a)=0.6, m1(b)=0.2, m1 (c)=0.2;
m2(a)=0.5, m2(b)=0.4, m2(c)=0.1;
m3(a)=0.2, m3 (b)=0.5, m3(c)=0.3

Step 1. Calculate Pignistic distance matrix of evidence.

$$DifBetP = \begin{vmatrix} 0 & 0.2 & 0.4 \\ 0.2 & 0 & 0.3 \\ 0.4 & 0.3 & 0 \end{vmatrix}$$

Step 2. Calculate Pignistic similarity matrix of evidence.

$$SimBetP = \begin{vmatrix} 1 & 0.8 & 0.6 \\ 0.8 & 1 & 0.7 \\ 0.6 & 0.7 & 1 \end{vmatrix}$$

Step 3. Compute the weight of evidence.
ω1=0.33, ω2=0.35, ω3=0.32
Step 4. Compute the result of combination
m(a)=0.432, m(b)=0.373, m(c)=0.195.

VI. SIMULATION OF NEW COMBINATION RULE

This part applies simulation methods to test the impact of the change of weight on the result of combination. Simply, this paper assumes that there are two sets of evidence, and the frame of discernment consists of two elements A and B, Basic belief assignment function is randomly generated by Matlab, testing the change of the result when weight ω is in the range [0, 1].

Simulation 1:
m1(A)=0.2423, m1(B)=0.7577;
m2(A)=0.3213, m2(B)=0.6787.

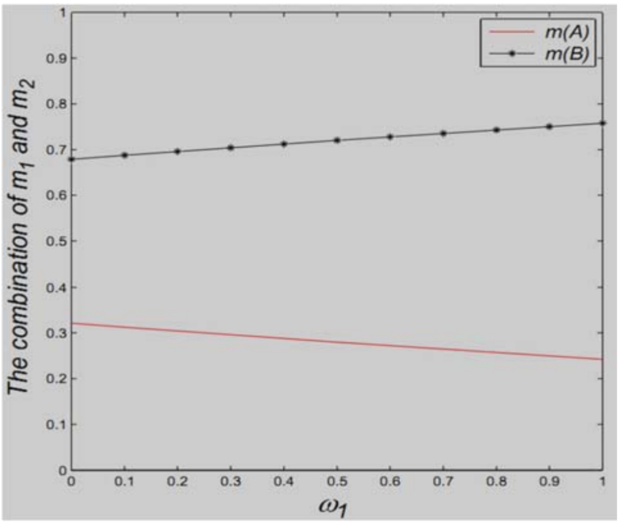

Fig. (1). The Result of Simulation 1

Simulation 2:
m1(A)=0.8288, m1(B)=0.1712;
m2(B)=0.6555, m2(A)=0.3445

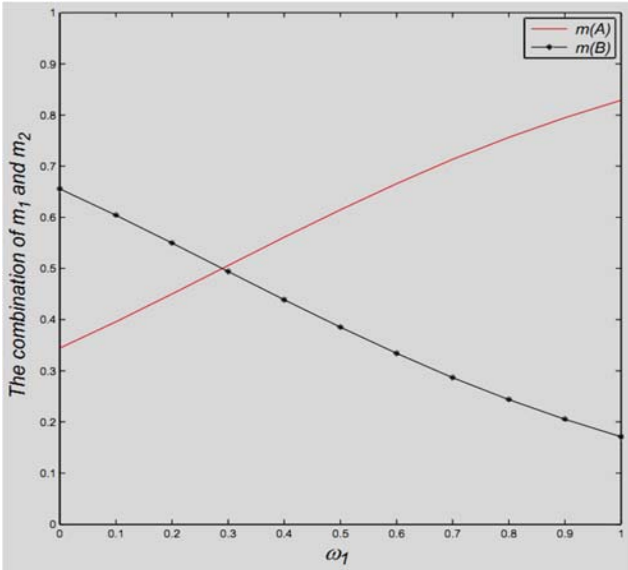

Fig. (2). The Result of Simulation 2.

(1) As can be seen from Figure (1), the result of combination changes with weight change from 0 to 1, the result of combination biases to the evidence of greater weight, the bigger "force of evidence" with big weight scilicet, it obviously showed in Figure (2), in particular, when weight ω1=0.28, the result of combination mutates, from m(A)<m(B) mutates to m(A)>m(B), this shows that the change of weight will produce different result of combination.

(2) When ω1=0, the result of combination is same as m2, it shows that when weight of m1 is 0, m1 will not have any impact on the results in the combination process. Similarly, when ω1=1, the result of combination is the same as m1.

It can be seen in simulation that the impact of weight on the result of combination is accord with human cognition completely.

VII. CONCLUSIONS

Calculate pignistic evidence distance to get the degree of similarity between the evidence, and give greater confidence to the similar evidence, these reflect the principle "Majority rules" in group decision. Introducing weight of evidence into Dempster's Rule of Combination reflects that weight of evidence has an effect in the process of combination, and this will also simplify the calculation procedure. Given less weight, the evidence with larger conflict plays a weak role in the process of combination. In this way, we can effectively deal with the conflict problem in the process of evidence combination.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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