An Image Segmentation Method Based on Partial Differential Equation Models

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***Abstract* — In this paper, we propose an image segmentation method based on a partial differential equation model. Partial Differential Equations and Fuzzy Image Segmentation algorithms have become important fields in image processing research. In an image processing system, the segmentation process is one of the most important steps. More precisely, image segmentation is defined as the process of assigning a label to every pixel in an image such that pixels with the same label share certain visual characteristics. Several general-purpose algorithms and techniques have been developed for image segmentation. In order to effectively solve an image segmentation problem for a specific problem domain, these techniques often have to be combined with knowledge domains, as there is no general solution to image segmentation problems. Our results show that the performance of image segmentation can be improved by using partial differential equation models.**

Keywords - Image segmentation method; Partial differential equation; fuzzy image segmentation.

I. INTRODUCTION

Image segmentation is one of fundamental and important tasks in image analysis and computer vision. Given an image, the segmentation goal is to separate the image domain into dissimilar regions, each of which has a consistent trait (intensity, color or texture, etc.) throughout that is different from other regions in the image. Once a decision is made on the desired trait, various methods are available to reach the segmentation goal. This paper will focus on variation level set methods and partial differential equation (PDE) methods for image segmentation.

The basic idea behind these methods is explained as follows. The segmentation problem is formulated in terms of minimizing (or at least finding critical points of) an energy functional that takes a level set function. The level set functions evolve according to an evolution partial differential equation (PDE), which is derived from the minimization of the energy functional by calculating the L~2(ordinary) gradient of energy functional and using continuous gradient descent method. A signed level set method to solve the Mumford-Shah model. The Mumford-Shah model for image segmentation is a powerful and robust region-based technique; however, the numerical method for solving the Mumford-Shah model is difficult to implement. Peng [1] solved a particular case of the Mumford-Shah model using the curve evolution and level set method for image segmentation, where the binary case of two regions was considered. As a result, a number of generalizations have been developed to improve both its applicability and efficiency. However, the Zhang's [2] method based on the traditional level set method has slightly some intrinsic limitations. The Dirac function has to be involved in the associated gradient descent equation when minimizing with respect to level set function.

The reinitializing procedure is quite complicated and expensive, and is fraught with its own problems, such as when and how to reinitialize. The numerical approximation of the evolution equation has to utilize a complex semi-implicit scheme. In Li's [3] paper, he presents a signed level set method to solve the two-phase piecewise constant case of the Mumford-Shah model for image segmentation, pursuing the mechanism of the traditional level set method. The proposed method avoids some intrinsic limitations of solving methods in the traditional level set framework, and allows for more robustness to the locations and sizes of initial contour and more computational efficiency. Numerical results demonstrated that the proposed method is fast enough for near real-time bimodal segmentation applications while still retaining enough accuracy.

Variational level set methods for image segmentation based on both L~2andSobolev gradients Variational level set methods for image segmentation involve minimizing energy functional over a space of level set functions using continuous gradient descent method in Ji's paper [4]. The functional includes the internal energy (curve length, usually) for regularization and the external energy that aligns the curves with object boundaries. Current practice is in general to minimize the energy functional by calculating the L~2gradient of the total energy. However, the gradient is particularly effective for minimizing the curve length functional by gradient descent method in that it produces the solution in a single iteration. In this paper, we thus propose to use the gradient for the internal energy, while still using L~2gradient for the external energy. The test results show that the "L~2plus" gradient scheme has much more computational efficiency than the methods only based on L~2gradient.

Implicit active contour model with local and global intensity fitting energy intensity in homogeneities often occur in real-world images and may cause considerable difficulties in image segmentation in Xu's paper [5]. To handle intensity inhomogeneity efficiently, some localized region-based models have been proposed recently. For example, De [6] et al. recently proposed a region-scalable fitting (RSF) active contour model. Very recently, Wang et al. [7] proposed a novel active contour model driven by local image fitting energy, which also can handle intensity inhomogeneity efficiently. However, these models easily get stuck in local minimums for most of contour initializations. This makes it need user intervention to define the initial contours professionally. In this study, Peng [8] propose a new active contour model, which integrates a local intensity fitting (LIF) energy with an auxiliary global intensity fitting (GIF) energy.

II. Methodology and Framework of Image Segmentation Method

The LIF energy is responsible for attracting the contour toward object boundaries and is dominant near object boundaries, while the GIF energy incorporates global image information to improve the robustness to initialization of the contours. The proposed model can efficiently handle intensity inhomogeneity, while allowing for more flexible initialization and maintaining the sub-pixel accuracy. The implicit active contours have proved to be an efficient framework for image segmentation. This implicit model is derived from motion by mean curvature and uses the image gradient to stop the evolution process. a new formulation of implicit active contours based on mean curvature motion is useful for solving this problem. The basic process for image segmentation is shown in the following figure 1.

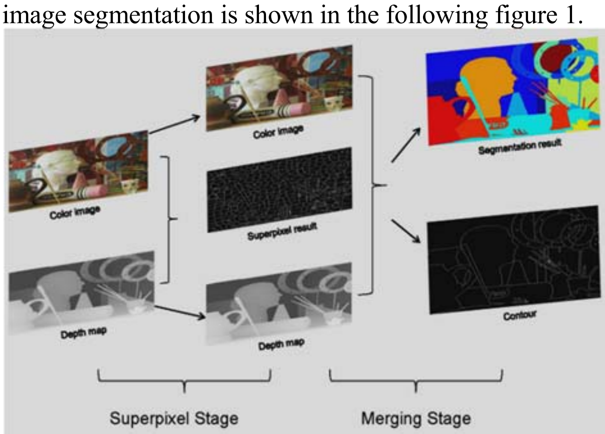

Figure 1. The basic process for image segmentation

Image segmentation, extracting the objects of interest from images, is a most fundamental and important problem in image processing. It has always been a hot but a formidable task in an image project. Recently, segmentation methods based on partial differential equation (PDE) have been widely paid attention by many researchers, due to their variable form, flexible structure and excellent performance. The basic idea is to deform a curve, surface or image according to a PDE with initial and boundary conditions, and obtain the desired segmentation results as the solution of the equation. The evolution PDE can be designed directly or indirectly according to image characteristic and user demand.

However, the RSF model easily gets stuck in local a minimum which makes it sensitive to the contours initialization. Besides, the RSF model is also sensitive to high noise. To these issues, we proposed an improvement on the RSF model. First, the Gaussian kernel for the RSF energy is replaced with a ‘‘mollifying’’ kernel with compact support. Second, the RSF energy is redefined as a weighted energy integral, where the weight is local entropy deriving from a grey level distribution of image. The total energy functional is then incorporated into a variation level set formulation with two extra internal energy terms. The new RSF model not only handles better intensity inhomogeneity, but also allows for more flexible initialization and more robustness to noise compared to the original RSF model.

Studying on the initialization problem of level set function, we proposed an adaptive level set evolution equation starting with a constant function. For the segmentation technique based on partial differential equations, segmentation can be regarded as a process of seeking the numerical solution of a partial differential equation with initial condition. Because the segmentation results typically depend on the selection of initial contours, most of the existing methods need user intervention to define the initial contours professionally. This means that they may be fraught with the problems of how and where to define the initial contours.

Up to now, it is still a great challenge to find an efficient way to tackle the contour initialization problem. Combining the TV (Total Variation) regularization, we proposed an adaptive level set evolution equation starting with a constant function. The formulation is composed by an adaptive driving force and a TV-based regularizing force. The adaptive driving force makes the level set function to have the opposite sign along the edges at convergence and the regularizing force is used to smooth the level set function. Due to the adaptive driving force, the level set function can be initialized to a constant function, which completely eliminates the need of initial contours. This implies that the new formulation is robust to initialization or even free of manual initialization. In addition, the evolution PDE can be solved numerically via a simple explicit finite difference scheme with a significantly larger time step. The proposed model is fast enough for near real-time segmentation applications while still retaining enough accuracy; in general, only a few iterations are needed to obtain segmentation results accurately.

III. The Basic Algorithm

If the blurry edge, the strong noise and the intensity inhomogeneity appear in an image, a traditional active contour model fails to segment contours, especially, for a magnetic resonance image and an ultrasound image in medicine. Because of these reasons, we propose an active contour based on local linear fitting energies for a difference image. The active contour model is solved by minimizing its energy functional. The optimum local linear fitting parameters in the model are obtained. Contours for some images with intensity inhomogeneity are successfully extracted. Experimental results show that the method has the capacity of extracting weak edge and objects with intensity inhomogeneity. Because only average intensity information is considered in the local binary fitting model, the model can successfully segment some magnetic resonance images in medicine. However, the local binary fitting model fails to segment an ultrasound image with a lot of noise that affects the distribution of intensity. For extending the application field of the local binary fitting model, we propose an active contour based on local intensities and local gradient fitting energy.

By utilizing the level set method to solve, we successfully segment the weak edge in magnetic resonance images and contours with noise in ultrasound images. Experimental results show that the proposed method has the capacity of anti-noise. The segmentation accuracy is higher than that of the local binary fitting, local and global intensity fitting models. An ultrasound image has serious noise, and its target edge is very weak. For solving these problems, we propose an active contour based on local intensity and local Bhattacharyya distance energy for image segmentation. Through using the level set method, the weak edge successfully extracted. The proposed model weakens the influence of noise. Forward equations were generated as the equation (1):

$$\boldsymbol{q} = \arg\min_{\boldsymbol{q}} \sum_{i=1}^{N} \left\| \vec{r}_i(\boldsymbol{q}) - \vec{r}_{i,meas} \right\|^2 \tag{1}$$

A full body marker set consisting of N = 47 markers was defined to provide redundancy and robustness against occasional marker dropout which is inevitable in real-time image capture. After solving (1), the estimated body pose is processed that outputs the smoothed pose $\boldsymbol{q}$ as well as the generalized $\dot{\boldsymbol{q}}$ and generalized accelerations $\ddot{\boldsymbol{q}}$.

In the inverse dynamics processing step, a vector $\boldsymbol{c}$ can be expressed as:

$$\boldsymbol{c} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{B}(\boldsymbol{q})\boldsymbol{c}_{ext} \tag{2}$$

Where $\boldsymbol{M}$ is a square matrix, and $\boldsymbol{n}$ are terms related to Coriolis and centrifugal effects then we have:

$$l(\boldsymbol{q}) = \sum_{i=1}^{N_{terms}} n_i \prod_{j=1}^{N_{DOF}} q_i^{E_{ij}} \tag{3}$$

The k is computed analytically by partial differentiation:

$$d_k = \frac{\partial l(\boldsymbol{q})}{dq_k} = -\sum_{i=1}^{N_{terms}} n_i E_{ik} \prod_{j=1}^{N_{DOF}} q_i^{E_{ij}-\delta_{kj}} \tag{4}$$

So we can get the equation (5):

$$v = -\frac{dl(\boldsymbol{q})}{dt} = -\sum_k \frac{\partial l(\boldsymbol{q})}{\partial q_k}\frac{dq_k}{dt} = \boldsymbol{d}^T\dot{\boldsymbol{q}} \tag{5}$$

The final processing step performed static optimization is formulated as a quadratic programming problem:

$$\boldsymbol{F} = \arg\min_{\boldsymbol{F}} \sum_{i=1}^{N_m} V_i \left(\frac{F_i}{F_{\max,i}}\right)^2, \quad \text{subject to} \begin{cases} \boldsymbol{D}(\boldsymbol{q})\boldsymbol{F} = \boldsymbol{n} \\ F_i \geq 0 \end{cases} \tag{6}$$

$$AUC = 1 + \frac{\sum_{i=1}^{n} f_i}{f_0} \tag{7}$$

The formula generates labels for each file block.

$$for(j=0; j \leq n-1; j++); \quad \{W_j = r*(j+1); T_i = [h(W_j)*m_j]^c \bmod N\}; \tag{8}$$

So we can get the following equation (8):

$$ {}_aI_b^{(\alpha)} f(t) = \frac{1}{\Gamma(1+\alpha)}\int_a^b f(t)(dt)^{\alpha} = \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{j=N-1} f(t_j)(\Delta t_j)^{\alpha} \tag{9}$$

Equation (8) can be converted into the following form:

$$f(y,\omega) = f^0(y,\omega) + \int_s S(y-y',\omega) \, \mathrm{L}^1 F(\mathrm{y}',\omega)\mathrm{d}y' + \rho_1\omega^2 \int_s \frac{\mathbf{g}(y-y',\omega)}{\mathrm{J}f(\mathrm{y}',\omega)\mathrm{d}y'} \tag{10}$$

Image segmentation is a vital image processing technique in computer vision and image analysis. So far, there are many ways for image segmentation. Among them the active contour model based on variation method and level set method is one of important image segmentation methods. It has embodied the superiority of partial differential equation in image segmentation. It utilizes the idea of dynamic evolution. Researching active contour model is important for image segmentation. Image segmentation technology research based on partial differential equations can promote multidisciplinary cross fusion. Moreover, the flexible numerical computational method has better stability during the discretization of evaluative partial differential equation, and it can meet the demand in high quality image restoration and accurate image segmentation, and so on. In which, S is cylinder cross section, $y = (x_1, x_2)$, and

$$\mathbf{g}(y-y',\omega) = \frac{1}{(2\pi)^2}\int_0^{\infty} \bar{k} d\bar{k} \int_0^{2\pi} \mathbf{g}(\bar{k},\omega)\exp(-i\mathrm{k}\cdot(\mathrm{y}-\mathrm{y}'))d\phi \qquad \bar{k} = (k_1, k_2) \tag{11}$$

Suppose $k_3 = 0$, $\mathbf{g}(\bar{k},\omega)$ can be obtained from Equation (8).

Owing to introducing a shrinkage velocity term in the geodesic active contour model, the constant velocity is set in advance. If it is chosen too large, the over-segmented result may be obtained. If it is very small, the model fails to segment correctly deep concave boundaries. In addition, while there are multiple object boundaries, the method still fails to extract all boundaries. Because of these problems, a geodesic active contour model including gradient error control is used in this model. An error function term about gradient norm is introduced into the geodesic active contour model. For such kind of material, general form of equation (10) is expressed as following equation (12-14):

$$G_{ik}(\bar{k},\omega)=\frac{1}{\rho_0\omega^2}[\frac{\beta^2}{\bar{k}^2-\beta^2}\theta_{ik} \\ +\bar{k}_i\bar{k}_k\left(\frac{1}{\bar{k}^2-\alpha^2}-\frac{1}{\bar{k}^2-\beta^2}\right)+m_im_k\frac{\beta_\perp^2}{\bar{k}^2-\beta_\perp^2}] \tag{12}$$

$$g_{ik}(\bar{k},\omega)=-\frac{1}{\eta_{11}^0}\frac{1}{\bar{k}^2}+\frac{1}{\rho_0\omega^2}\left(\frac{e_{15}^0}{\eta_{11}^0}\right)^2\frac{\beta_\perp^2}{\bar{k}^2-\beta_\perp^2} \tag{13}$$

$$\gamma_i(\bar{k}_i,\omega)=\frac{1}{\rho_0\omega^2}\left(\frac{e_{15}^0}{\eta_{11}^0}\right)^2\frac{\beta_\perp^2}{\bar{k}^2-\beta_\perp^2}m_i \tag{14}$$

In which,

$$\alpha^2=\frac{\rho_0\omega^2}{C_{11}^0},\quad \alpha^2=\frac{\rho_0\omega^2}{C_{66}^0}\ \beta_\perp^2=\frac{\rho_0\omega^2}{C_{44}'},$$

$$C_{44}'=C_{44}^0+\frac{(e_{15}^0)^2}{\eta_{11}^0} \tag{15}$$

A shrinkage velocity term in the geodesic active contour model is introduced. The model can segment deep concave contours. But shrinkage velocity is specified before segmentation. If it is set differently, the segmented results are different. Additionally, the geodesic active contour model fails to segment contour whose edge is weak and blurry. Because of these problems, we propose a local adaptive parameter setting method for parameters automatize setting. It integrates local spatial points distance and local intensity information into the geodesic active contour model, the original geodesic active contour model is improved. Automatic setup parameters in the method can be achieved. It shows that the model enhances the segmentation accuracy, and realizes the segmentation for blurry boundaries.

$$P(C\ |\ X)=\prod\nolimits_{k=1}^{n}P(x_k\ |\ C_i) \tag{16}$$

$$P(C_k\ |\ X)=P(X\ |\ C_k)P(C_k) \\ =P(C_k)\prod_{i=1}^{n}P(X_i\ |\ C_k) \tag{17}$$

IV. RESULTS AND DISCUSSION

In the framework of level set method, we developed a nonlinear diffusion equation directly for image segmentation nonlinear diffusion equations have received a lot of attention in the area of image analysis and computer vision. However, segmentation is integrated into smoothing process which generates a piecewise constant approximation to the geometrical description of image. So, segmentation results rely closely on the performance of smoothing. Besides, a smoothing algorithm with good performance in preserving image features (edges) usually needs to design intricate diffusion term, which may introduce complex computation that may lead to the inefficiency of the whole process. The pre-disposal process for an image is shown in the figure 2.

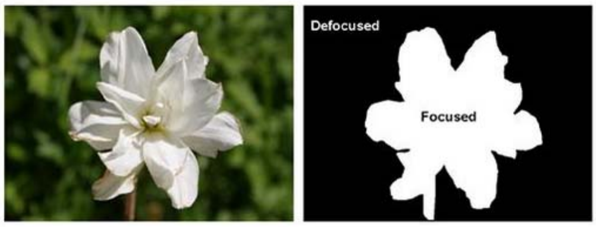

Figure 2. The pre-disposal process

Based on the mechanism of nonlinear diffusion, we develop a nonlinear diffusion equation (with the initial and boundary conditions directly for image segmentation. The zero contour line of level set function starting with a zero function can be smoothly generated, and quickly come to a steady state which separates object from its background. This work constitutes a framework for further investigations on nonlinear diffusion equations directly for segmentation. The segmentation results on the synthetic image which is corrupted by Gaussian noise are shown in Fig. 3. Figure 4 shows a case study result of the whole process.

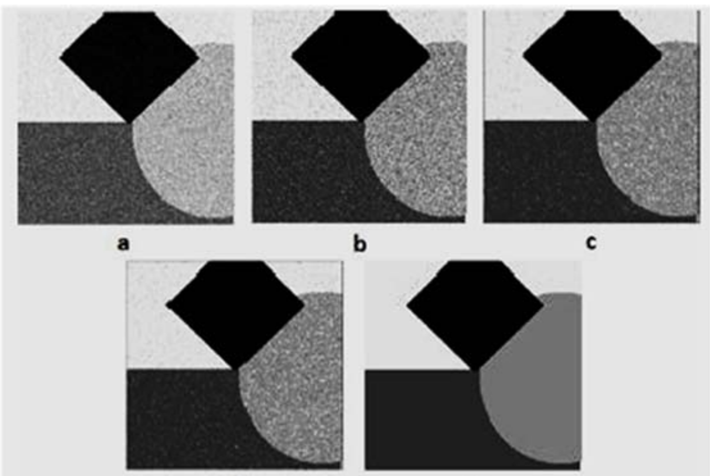

Figure 3. Segmentation of the synthetic image corrupted by Gaussian noise (0,0.01) (a) noisy image; (b) FCM_S1; (c) FCM_S2; (d) GIFP_FCM; (e) RFCM_SSI.

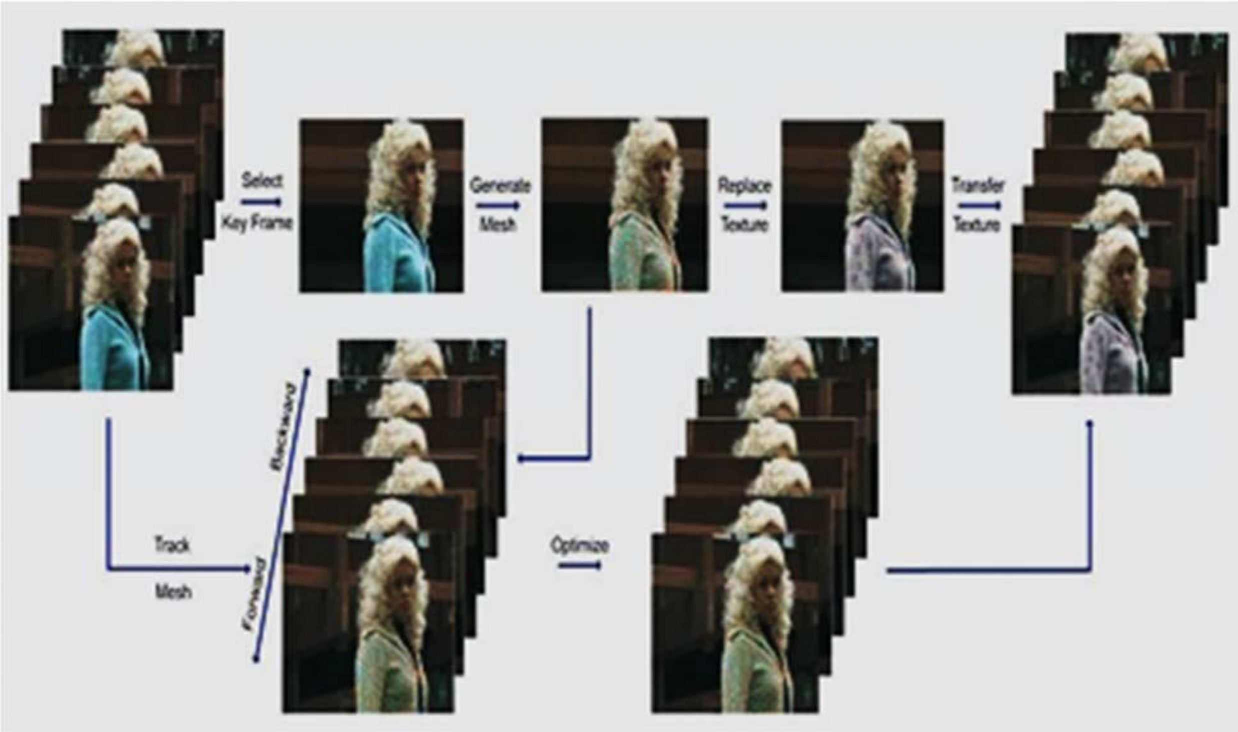

Figure 4. A case study result of the whole process

The diffusion term in our equation is response for the smoothness of the level set function during the evolution. The source term is used for identifying object and its background with "source" and "sink". The level set function can be initialized to any bounded function, e.g., a zero function, which completely eliminates the need of initial contours. This implies that our model is robust to initialization or even free of manual initialization. The proposed model has four main advantages: First, it doesn't use image gradient to stop the evolution process. Second, it allows robustness to initialization or even is free of manual initialization since the level set function can be initialized to a binary function that contains both positive and negative values. Third, the zero-level line of level set function starting with such binary function finally comes to a unique steady state, thus it allows setting a termination criterion on the algorithm by determining the binary length of zero-level line at each of iterations. Fourth, the evolution PDE is easily resolved numerically by the use of the semi-implicit additive operator splitting (AOS) scheme introduced to nonlinear diffusion filtering, which remains numerically stable for a large time step and so less iteration numbers are needed to converge to the steady state solution. The proposed algorithm has been successfully applied to both synthetic and real images with homogeneous intensity regions.

V. CONCLUSIONS

In this paper, the author studies on the image segmentation method based on partial differential equation. Currently, Partial Differential Equation and Fuzzy Image Segmentation algorithms have become important fields in image processing research. Given an image, the segmentation goal is to separate the image domain into dissimilar regions, each of which has a consistent trait (intensity, color or texture, etc.) throughout that is different from other regions in the image. Experimental results show that the proposed method has the capacity of anti-noise. The segmentation accuracy is higher than that of the local binary fitting, local and global intensity fitting models. An ultrasound image has serious noise, and its target edge is very weak. For solving these problems, we propose an active contour based on local intensity and local Bhattacharyya distance energy for image segmentation.

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