Completed Trace Equivalence of Shuttle Run Motion Models

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Abstract **— As a special form of transferring the direction of running, shuttle run is a common way in university physical education. In order to optimize the trajectory of a shuttle run, the paper proposes completed trace equivalence of shuttle run motion models. Firstly, it combines Hybrid System theory and shuttle run motion to obtain a definition of shuttle run motion model. Secondly, it determines an equivalence algorithm using algebraic computation. Thirdly, it eliminates some branches of the model by completed trace equivalence to get simplified shuttle run motion model. Finally, a shuttle run motion example is used to prove the validity of the completed trace equivalence.**

Keywords - Shuttle run, completed trace equivalence, equivalence algorithm.

I. INTRODUCTION

Physical education teachers are the pioneers of physical practice reforms. They have important responsibility for promoting students' physique. When physical education sector makes continuous effort and attempt to improve students' physical health condition, physical education teachers should have the spirit of exploration and innovation. The university existing physical test project cannot really reflect students' physique condition. For example, 800 meters run and 1000 meters run being unable to accurately test students' heart function and lung function, but 20 meters shuttle run can test students' heart function and lung function. Such test projects are more popular with university students. Overseas mostly use 20 meters shuttle run to test children heart function and lung function.

Hybrid Systems [1-3] unify continual variable dynamic system and discrete event dynamic system and mutually exchanges information. Continuous subsystem dynamic characteristic evolves development over time. Discrete subsystem dynamic evolution is driven by event. Continuous subsystem and discrete subsystem interact with each other. Hybrid Automaton is the most commonly used model in the modeling of Hybrid System. In the Hybrid Automaton, each discrete location has a specific continuous dynamic process. It uses Hybrid Automaton to describe shuttle run motion and gets shuttle run motion model.

If a software program system has many completely same state transition labels, then it may be called function equivalence. It can remove redundant branches to simplify the software program system structure. For the system equivalence relation, its research source linear and branch time equivalence which is proposed by Glabbeek. Completed trace equivalence [4] plays an important role in the state space equivalence and has been applied in each kind of equivalence and approximate equivalence of complex systems.

Given the above analysis and demand, the paper establishes shuttle run motion which based on Hybrid System theory and algebraic method. It obtains equivalence algorithm by time analysis of discrete location continuous dynamic process. And then, completed trace equivalence theory is used to simplify shuttle run motion model. Finally, an example proves the validity of completed trace equivalence of shuttle run motion models.

II. SHUTTLE RUN MOTION MODEL

Before establish shuttle run motion model, it introduces Hybrid Automaton.

Definition 1(Hybrid Automaton[5,6]) A Hybrid Automaton is a tuple H=<Q, V, HX, Init, Lab, E, Inv, F, R >, *where*

(1) Q *is a finite set of discrete locations.*

(2) V *is a finite set of continuous variables.*

(3) HX *is a set of continuous variables values.*

(4) $Init \in Q \times HX$ *is a set of initial states.*

(5) Lab *is a finite set of discrete transition programs.*

(6) E *is a finite set of discrete transitions.*

(7) Inv *is a finite set of continuous variables invariants.*

(8) F is a finite set of programs which describe system continuous variables dynamic processes.

(9) R *is a finite set of discrete location transition conditions.*

Based on above definition of Hybrid System, the running process of Hybrid System is as follows [7-10]:

(1) The running process of Hybrid System begins with initial state $< q_0, X_0 > \in Init$. The transition process of Hybrid System contains continuous variables dynamic process and discrete event process.

(2) In the system discrete location *q*, if system continuous variables value *X* is in the value range of discrete location corresponding invariant *Inv*(*q*), $X \in Inv(q)$, continuous variables value *X* evolves with corresponding continuous variables dynamic process program. If system continuous variables value *X* is not in the value range of discrete location corresponding invariant *Inv*(*q*), $X \notin Inv(q)$, system occurs transition of discrete locations [11, 12].

(3) After transition of discrete locations, system continuous variable value *X* evolves with new program. The discrete location is unchanged until continuous variables value go beyond value range of invariant.

For the 20 meters shuttle run motion process, in each section 20 meters distance, a student's motion process is as follows. Firstly, he does uniform acceleration motion. Secondly, he does uniform motion. Finally, he does uniform deceleration motion. It gets shuttle run motion model by Hybrid Automaton [13-16].

Definition 2(Shuttle Run Motion Model) A shuttle run motion model is a tuple H=<Q, V, HX, Init, Lab, E, Inv, F, R >, *where*

(1) Q *is a finite set of discrete locations.*

(2) V *is a finite set of continuous variables.*

(3) HX *is a set of continuous variables values.*

(4) $Init \in Q \times HX$ *is a set of initial states.*

(5) Lab *is a finite set of discrete transition programs. Each discrete transition program is an algebraic program.*

(6) E *is a finite set of discrete transitions.*

(7) Inv *is a finite set of continuous variables invariants.*

(8) F is a finite set of algebraic programs which describe system continuous variables dynamic processes.

(9) R *is a finite set of discrete location transition conditions.*

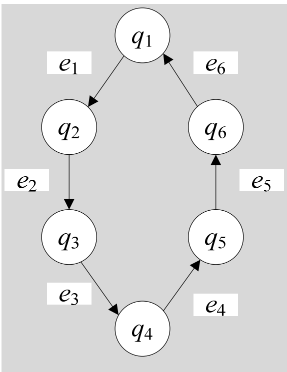

Fig. (1). A shuttle run motion model

In the Figure 1, $Q=\{q_1,q_2,q_3,q_4,q_5,q_6\}$ is a finite set of discrete locations. $V=\{s,v\}$ is a finite set of continuous variables. *s* represents displacement. *v* represents speed. The initial state $<q_1,X_0>\in Init$. $X=(s,v)^{\mathrm{T}}$, $X_0=(s_0,v_0)^{\mathrm{T}}=(0,0,0)^{\mathrm{T}}$. $Lab=\{lab\}$, the discrete transition program *lab* is $X'=X$. In the discrete location q_1, the algebraic program f_1 is $\begin{cases} s'=s+t^2 \\ v'=v+2t \end{cases}$, the invariant Inv_1 is $\{(0\le s\le 4)\wedge(0\le v\le 4)\}$. In the discrete location q_2, the algebraic program f_2 is $\begin{cases} s'=s+vt \\ v'=v \end{cases}$, the invariant Inv_2 is $\{(4\le s\le 16)\wedge(v=4)\}$. In the discrete location q_3, the algebraic program f_3 is $\begin{cases} s'=s+4t-t^2 \\ v'=v-2t \end{cases}$, the invariant Inv_3 is $\{(16\le s\le 20)\wedge(0\le v\le 4)\}$. In the discrete location q_4, the algebraic program f_4 is $\begin{cases} s'=s-t^2 \\ v'=v+2t \end{cases}$, the invariant Inv_4 is $\{(16\le s\le 20)\wedge(0\le v\le 4)\}$. In the discrete location q_5, the algebraic program f_5 is $\begin{cases} s'=s-vt \\ v'=v \end{cases}$, the invariant Inv_5 is $\{(4\le s\le 16)\wedge(v=4)\}$. In the discrete location q_6, the algebraic program f_6 is $\begin{cases} s'=s-4t+t^2 \\ v'=v-2t \end{cases}$, the invariant Inv_6 is $\{(0\le s\le 4)\wedge(0\le v\le 4)\}$.

III. EQUIVALENCE ALGORITHM

For the 20 meters shuttle run motion process, in each same direction section 20 meters distance, if two branches spend the same time, then these two branches are equivalence. It is the equivalence algorithm

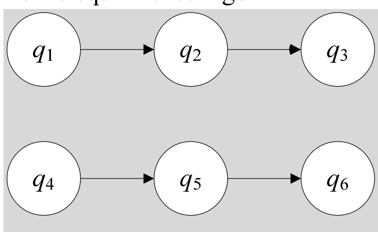

Fig. (2). Two branches

In the Figure 2, initial state value is $X_0=(s_0,v_0)^{\mathrm{T}}=(0,0)^{\mathrm{T}}$, in the discrete location q_1, the algebraic program f_1 is $\begin{cases} s'=s+t^2 \\ v'=v+2t \end{cases}$, the invariant Inv_1 is $\{(0\le s\le 4)\wedge(0\le v\le 4)\}$. In the discrete location q_2, the algebraic program f_2 is $\begin{cases} s'=s+vt \\ v'=v \end{cases}$, the invariant Inv_2 is $\{(4\le s\le 16)\wedge(v=4)\}$. In the discrete location q_3, the algebraic program f_3 is $\begin{cases} s'=s+4t-t^2 \\ v'=v-2t \end{cases}$, the invariant Inv_3 is $\{(16\le s\le 20)\wedge(0\le v\le 4)\}$. In the discrete location q_4, the algebraic program f_4 is $\begin{cases} s'=s+\frac{5}{6}t^2 \\ v'=v+\frac{5}{3}t \end{cases}$, the invariant Inv_4 is $\left\{(0\le s\le \frac{15}{2})\wedge(0\le v\le 5)\right\}$. In the discrete location q_5, the algebraic program f_5 is $\begin{cases} s'=s+vt \\ v'=v \end{cases}$, the invariant Inv_5 is

$\left\{(\frac{15}{2} \le s \le \frac{25}{2}) \wedge (v=5)\right\}$. In the discrete location q_6, the algebraic program f_6 is $\begin{cases} s' = s + 5t - \frac{5}{6}t^2 \\ v' = v - \frac{5}{3}t \end{cases}$, the invariant Inv_6 is $\left\{(\frac{25}{2} \le s \le 20) \wedge (0 \le v \le 5)\right\}$. Because two branches have the same time, they are equivalence.

IV. COMPLETED TRACE EQUIVALENCE OF SHUTTLE RUN MOTION MODELS

Definition 3(Completed Trace Equivalence) If there exists a process q, $p \xrightarrow{\sigma} q$ and $I(q) = \varnothing$, then $\sigma \in Act$ is a completed trace of a process q. Let $CT(p)$ is the set of process p completed trace. If $T(p)= T(q)$ and $CT(p)= CT(q)$, then two processes p and q are completed trace equivalence. It is written as $p =_{CT} q$. In the completed trace semantic, two processes are equal if and only if completed trace equivalence of two processes.

In the completed trace equivalence semantic, a process can be described as a process diagram of a completed trace equivalence class.

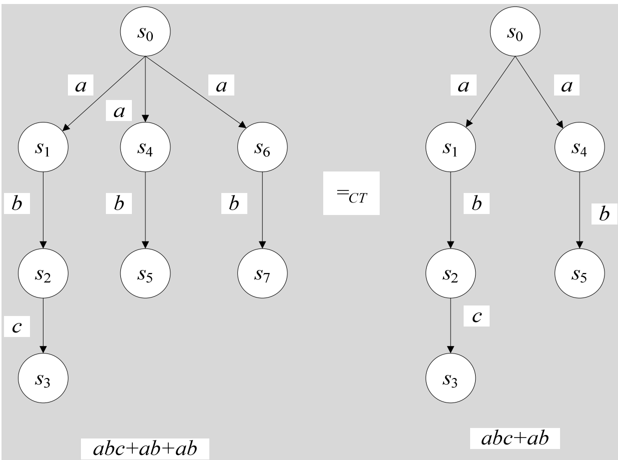

Fig. (3). A completed trace equivalence example

Based on completed trace equivalence theory, it gets completed trace equivalence of shuttle run motion models. Completed trace equivalence of shuttle run motion models can reduce discrete locations and optimize shuttle run motion model.

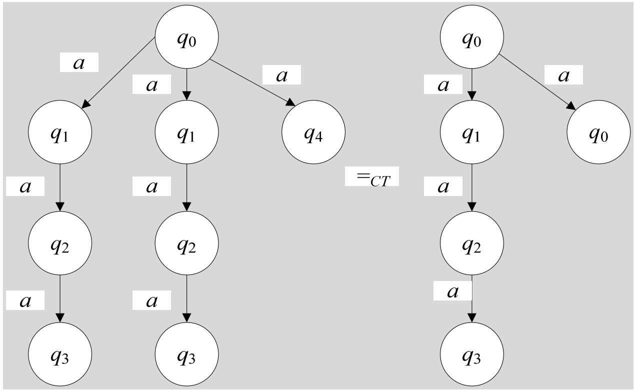

Fig. (4). A completed trace equivalence of shuttle run motion models example.

V. EXPERIMENTS

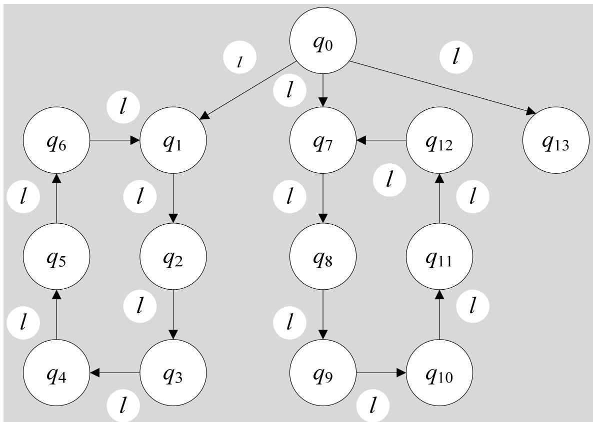

Fig. (5). A shuttle run motion model

In the Figure 5, $V = \{s, v\}$ is a finite set of continuous variables, initial state value is $X_0 = (s_0, v_0)^{\mathrm{T}} = (0,0)^{\mathrm{T}}$, discrete transition program l is $X' = X$. In the discrete location q_0, it has direct discrete transition. In the discrete location q_1 and q_{13}, the algebraic program f_1 and f_{13} are $\begin{cases} s' = s + t^2 \\ v' = v + 2t \end{cases}$, the invariant Inv_1 and Inv_{13} are $\{(0 \le s \le 4) \wedge (0 \le v \le 4)\}$. In the discrete location q_2, the algebraic program f_2 is $\begin{cases} s' = s + vt \\ v' = v \end{cases}$, the invariant Inv_2 is $\{(4 \le s \le 16) \wedge (v = 4)\}$. In the discrete location q_3, the algebraic program f_3 is $\begin{cases} s' = s + 4t - t^2 \\ v' = v - 2t \end{cases}$, the invariant Inv_3 is $\{(16 \le s \le 20) \wedge (0 \le v \le 4)\}$. In the discrete location q_4, the algebraic program f_4 is $\begin{cases} s' = s - t^2 \\ v' = v + 2t \end{cases}$, the invariant Inv_4 is $\{(16 \le s \le 20) \wedge (0 \le v \le 4)\}$. In the discrete location q_5, the algebraic program f_5 is $\begin{cases} s' = s\text{-}vt \\ v' = v \end{cases}$, the invariant Inv_5 is $\{(4 \le s \le 16) \wedge (v = 4)\}$. In the discrete location q_6, the algebraic program f_6 is $\begin{cases} s' = s - 4t + t^2 \\ v' = v - 2t \end{cases}$, the invariant Inv_6 is $\{(0 \le s \le 4) \wedge (0 \le v \le 4)\}$. In the discrete location q_7, the algebraic program f_7 is $\begin{cases} s' = s + \frac{5}{6}t^2 \\ v' = v + \frac{5}{3}t \end{cases}$, the invariant Inv_7 is $\left\{(0 \le s \le \frac{15}{2}) \wedge (0 \le v \le 5)\right\}$. In the discrete location q_8, the algebraic program f_8 is $\begin{cases} s' = s + vt \\ v' = v \end{cases}$, the invariant Inv_8 is $\left\{(\frac{15}{2} \le s \le \frac{25}{2}) \wedge (v = 5)\right\}$. In the discrete location q_9, the algebraic program f_9 is $\begin{cases} s' = s + 5t - \frac{5}{6}t^2 \\ v' = v - \frac{5}{3}t \end{cases}$, the invariant Inv_9 is $\left\{(\frac{25}{2} \le s \le 20) \wedge (0 \le v \le 5)\right\}$. In the discrete location q_{10}, the algebraic program f_{10} is $\begin{cases} s' = s\text{-}\frac{5}{6}t^2 \\ v' = v + \frac{5}{3}t \end{cases}$, the invariant Inv_{10} is $\left\{(\frac{25}{2} \le s \le 20) \wedge (0 \le v \le 5)\right\}$. In the discrete location q_{11}, the algebraic program f_{11} is $\begin{cases} s' = s\text{-}vt \\ v' = v \end{cases}$, the invariant Inv_{11} is $\left\{(\frac{15}{2} \le s \le \frac{25}{2}) \wedge (v = 5)\right\}$. In the discrete location q_{12}, the algebraic program f_{12} is $\begin{cases} s' = s\text{-}5t + \frac{5}{6}t^2 \\ v' = v - \frac{5}{3}t \end{cases}$, the invariant Inv_{12} is $\left\{(0 \le s \le \frac{15}{2}) \wedge (0 \le v \le 5)\right\}$. $q_1 \to q_2 \to q_3$ and $q_7 \to q_8 \to q_9$ are equivalence, $q_4 \to q_5 \to q_6$ and $q_{10} \to q_{11} \to q_{12}$ are equivalence. It gets completed trace equivalence model. Completed trace equivalence of shuttle run motion models can reduce discrete locations and optimize shuttle run motion model.

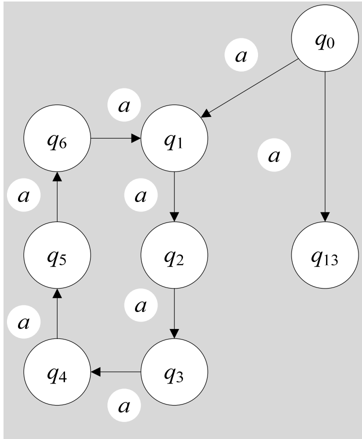

Fig. (6). Completed trace equivalence model.

VI. CONCLUSION

In this paper, completed trace equivalence of shuttle run motion models is proposed. It can eliminate some redundant states of shuttle run motion model. In future work, we will study approximate equivalence of shuttle run motion models.

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