Progressive Collapse Mechanism Analysis of RC Frame Structure Based on Energy Arithmetic

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***Abstract* — The paper proposes a collapse mechanism analysis method on the basis of existing researches. Specific to the circumstance of theoretical resistance absence for progressive collapse caused by component failure of structure resulted from accidental load, the paper proposes computational methods for collapse resistance of structural collapse at beam mechanism phase, catenary mechanism phase, and collapse phase respectively based on energy balance. In addition, it deduced the transforming relationship between structural internal resistance and energy consumption at the transition period from beam mechanism phase to catenary mechanism phase, as well as explored the relationship with nonlinear dynamic collapse demand The accuracy of the derivation has been verified through computation examples.**

***Keywords* - Progressive collapse; energy arithmetic; beam mechanism; catenary mechanism; energy balance.**

I.INTRODUCTION

In recent years, reports on architectural structure collapses as a result of accidents have been increasing. Especially, the damage scope and economic losses of progressive collapse are the most dramatic which makes the researchers focus on in-depth investigation and analysis on structural failure mechanism while pay attention to accidents. Progressive collapse refers to component failure of structure resulted from accidental load which further causes bearing parts being subject to unconstrained gravity load at the upper structure as well as damages resulted from chain reaction. The researches on progressive collapse have experienced three big booms. The first one was Ronan Point apartment bombing occurred in Britain in 1968. After that accident, Britain, Canada and other countries incorporated progressive collapse resistance into codes for structural design for the first time; the second one was Alfred P. Murrah building collapse happened in the United States in 1995. Under the influence of the accident, General Affairs Department and Department of Commerce of the United States issued specific codes on progressive collapse resistance respectively (GSA2003, DoD2005); the third one was the world trade center crash in New York in 2001, which attracted great attention from scientific research fields of all countries. The accident played a great role in promoting researches on progressive collapse [1-2].

At present, the researches on continuous collapse mechanism are mainly as alternative path method and tie force method (GSA2003, DoD2007). The structural collapse resistance solutions can be divided to linear static method, nonlinear static method, linear dynamic method and nonlinear dynamic method. The computation results of nonlinear static method are more precise since it can more comprehensively the mechanics behaviors of structure during progressive collapse; however, it is only applicable to special working conditions [3-5] since it analysis process is complex and requires higher capacity of analyst as well as computer program process. Comparatively speaking, linear static method and nonlinear static method are simpler but can hardly describe the collapse behavior of the structure at elastic-plastic stage correctly. Therefore, the contribution of beam mechanism and catenary mechanism on structural collapse resistance which makes the structural analysis tends to be conservative.

In recent years, relevant scholars proposed to analyze structure collapse resistance mechanism with energy algorithm. For example, Dusenberry[6] proposed two energy methods for structural collapse mechanism, determined the relationship between structure unbalanced load action and structural energy dissipation and introduced the concept of kinetic energy which were conducive to evaluate the progressive collapse resistance capacity of the structure; however, the method is only applicable to narrow scope. Besides, it needs solution to structural energy dissipation and kinetic energy of the structure. Thus the process is complicated and can hardly be calculated. Lzzuddin[7-8] et. al. put forward the concept of structural instability node displacement. The energy balance equation was established based on SDOF; however, it can hardly be put into application since the unit energy consumption calculation of structural collapse is complicated. The relationship between structure linear static resistance demands and nonlinear dynamic demands was established by Liyi [9-10] based on energy equation to solve correction coefficient which simplified the computational process; however, the nonlinear dynamic resistance may be varied from the actual power demand.

Based on the aforementioned researches, the relationship between unbalanced gravity action and internal energy dissipation was established based on energy balance algorithm, resistance demand relationship under beam mechanism and catenary mechanism was analyzed and the relationship between linear static demand and nonlinear dynamic resistance of structural members was further explored in the paper.

II. SIMPLIFIED ALGORITHM

A. Energy Arithmetic

The schematic diagrams of structural unconstrained collapsed caused by local component failure resulted from accidental load of frame structure are as shown in Figure 2, 3 and 4. The resistance reaction of structure under the effect of beam mechanism after column failure is as shown in Figure 3. And the resistance of structure under the effect of catenary mechanism after sub-component failure is as shown in Figure 4. During the process of structural collapse analysis, invalid upper column component is equivalent to the downward gravity load and will form external force with downward vertical displacement of structural collapse. According to the principle of energy balance, it will reach equilibrium condition when the external force is equivalent to energy dissipation of structural collapse resistance deformation. The following relationship will be established:

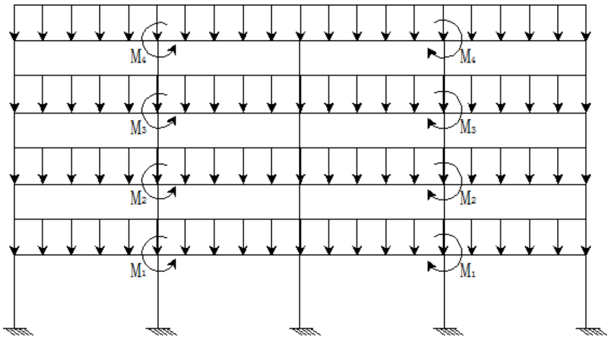


Figure 1. Initial phase of the structure

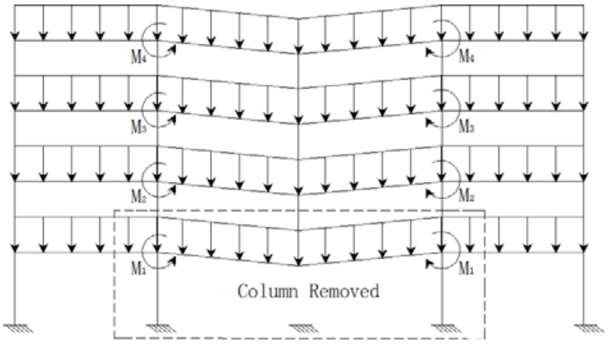
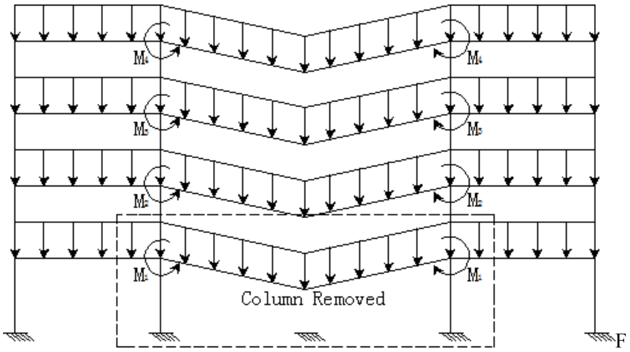


Figure 2. Central column failure phase



igure 3-a. Beam mechanism phase

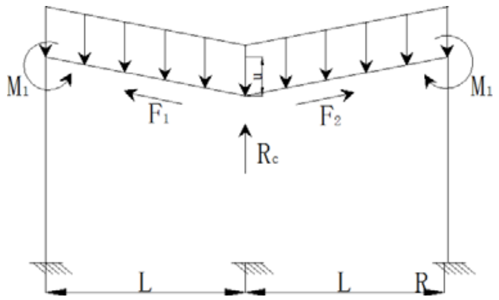


Figure 3-b. Schematic diagram of mechanism phase resistance

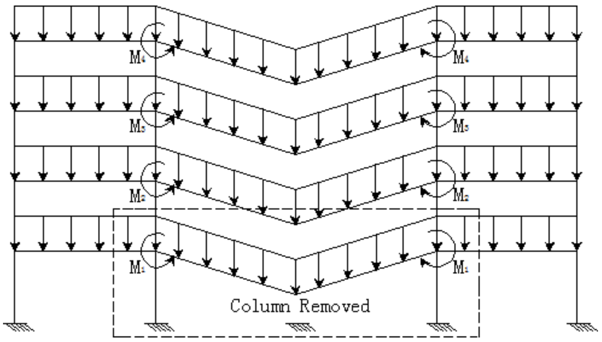


Figure 4-a. Catenary phase

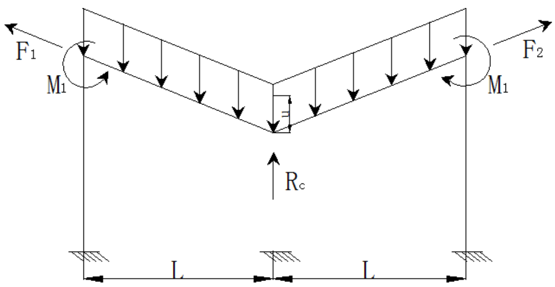


Figure 4-b Schematic diagram of resistance at catenary phase

$$\text{WE=UI} \tag{1}$$

WE is external force action.

$$WE=\sum_{i=1}^{n} M_i g\delta_i(x,y)u \tag{2}$$

Whereas in the formula, i refers to structure node, M_i refers to the node quality at the point of i

g refers to acceleration of gravity

(x,y) refers to coordinates of the node in the structure

δ_i (x,y) refers to vertical offset of the node in the coordinate system

u refers to failure coefficient:

$$u=\begin{cases}1, \ \textit{Intact}\ \text{structure}\\ 0，\textit{Steucture}\ \text{damaged}\end{cases}$$

In case of member failure, namely u=0, u $\delta_i(x,y)$ =0

Therefore, if there are j nodes damaged under external force, the node gravity will be exempted from external force. The structural external force is as listed below:

$$\text{WE=}\sum_{i=1}^{n-j} M_i g\ \ \delta_i(x,y) \tag{3}$$

In addition, based on structural dynamics theory, the dissipation of kinetic energy of the structure is as:

$$UI=\sum\int F_i du=\sum\int M_i \ddot{u}_i du_i=\sum\int\frac{1}{2}(\dot{B}_i{}^2-\dot{A}_i{}^2) \tag{4}$$

Whereas in the formula, $\dot{u}_i$ is the accelerated speed. $\ddot{u}$ is the accelerated speed.

However, the kinetic energy algorithm of the structure shall calculate the velocity and acceleration while calculating the structure collapsed node. The calculation is to complicated and can hardly be controlled. Izzuddin [7] proposed that in case of initial failure due to accidental load of a single column on frame structure, the node displacement of instable components can be presented through instable displacement. The energy balance equation can be established based on SDOF theoretical model. Displacement based method was adopted in the paper to describe node instability displacement. Improvement will be made specific to kinetic energy formula as below:

$$UI=\sum_{i=1}^{n}\int_C F_i du_i=\sum_{i=1}^{n}\int R_i(\delta)dU \tag{5}$$

Whereas in the formula, R_i refers to structural collapse resistance.

δ U refers to coordinate deformation and displacement deformation.

In case of structural energy balance,

$$\text{WE=UI，}\quad \sum_{i=1}^{n} M_i g\delta_i(x,y)u=\sum_{i=1}^{n}\int R(\delta)dU \tag{6}$$

Since the node calculation is too complicated, it can be transformed to concentrated force formula as shown below:

$$WE=\sum_{i=1}^{n} M_i g\delta(x,y)u=Mg\delta u \tag{7}$$

$$UI=\sum_{i=1}^{n}\int R_i(\delta)dU=\int R(\delta)dU \tag{8}$$

B. Structural resistance balance relationship under beam mechanism and catenary mechanism.

When the structure is at the position of Figure 1, the relationship between structure external action and kinetic energy at initial stage of collapse is as shown below:

$$WE=Mg\delta u \tag{9}$$

$$UI=\int R(\delta)dU \tag{10}$$

When the collapse is as shown in Figure 2, structural collapse between external force and dissipation of kinetic energy under beam mechanism is as derived below,

$$WE=Mg\delta_{ls,1}u \tag{11}$$

$\delta_{l,1}$ is vertical offset of under beam mechanism

$$UI=\sum_{i=1}^{n}\int R(\delta)dU \tag{12}$$

As shown in Figure 3-1, the resistance of component under beam mechanism R is

$$R=R_{e1}+R_{e2}=\frac{M_1}{L_1}+\frac{M_2}{L_2} \tag{13}$$

Whereas in the formula, R_{e1} , R_{e2} are the resistance at left and right side of the beam. M_1 and M_2 are bending moment at left and right side of the beam.

$$UI=\sum_{i=1}^{n}\int R(\delta)dU=\sum_{i=1}^{n}\int(\frac{M_1}{L_1}+\frac{M_2}{L_2})dU=(\frac{M_1}{L_1}+\frac{M_2}{L_2})\delta_{ls,1} \tag{14}$$

When the structural collapse is as shown in Figure 3, the structure will enter catenary mechanism phase after being affected by beam mechanism. The paper will consider resistance mechanism of component collapse separately under catenary mechanism. In the next section, further discuss the impact of rebar catenary mechanism on collapse resistance under the influence of beam mechanism.

In case of catenary mechanism, external imbalance and Internal collapse resistance of concrete structure at structural collapse are derived as below:

$$WE=Mgu\delta_{ls,2} \tag{15}$$

$$UI = \int R(\delta)dU \tag{16}$$

$$R_1 = \frac{M_1}{L_1} + \frac{M_2}{L_2} \tag{17}$$

$$R_2 = R_{2,a} + R_{2,b} = \frac{\delta_{ls,2}}{\sqrt{L_a^2 + \delta_{ls,2}^2}} \square F_a + \frac{\delta_{ls,2}}{\sqrt{L_b^2 + \delta_{ls,2}^2}} \square F_b$$

Whereas in the formula, $\delta_{ls,2}$ is the vertical offset under catenary mechanism. And R_2 is the resistance under catenary mechanism.

Due to the impact of load balance on structure,

$$F_a = F_b, \quad \frac{\delta_{ls,2}}{\sqrt{L_a^2 + \delta_{ls,2}}} = \frac{\delta_{ls,2}}{\sqrt{L_b^2 + \delta_{ls,2}}} \tag{18}$$

$$R_2 = 2\frac{F_a \square \delta_{ls,2}}{\sqrt{L_a^2 + \delta_{ls,2}}} = 2\frac{F_b \bullet \delta_{ls,2}}{\sqrt{L_b^2 + \delta_{ls,2}}} \tag{19}$$

$$F_a = F_b = E \square \varepsilon \square A \tag{20}$$

δ_2 is vertical offset of under catenary mechanism

F_a, F_b are axial tensile force at the left and right side of the beam

Under the assumption that on plastic hinge at beam support before structure yield failure, the main deformation is as rotational deformation. The deformation ε is to be calculated as below.

$$\varepsilon = (\frac{\sqrt{L_a^2 + \delta_{ls,2}^2} - L_a}{L_a}) \tag{21}$$

$$F_a = F_b = E_a A_a (\frac{\sqrt{L_a^2 + \delta_{ls,2}^2} - L_a}{L_a}) \tag{22}$$

$$R_{ls,2} = \delta_{ls,2}\frac{F_a}{\sqrt{L_a^2 + \delta_{ls,2}}} = E_a A_a (\frac{\sqrt{L_a^2 + \delta^2{}_{ls,2}} - L_a}{L_a}) \frac{\delta_{ls,2}}{\sqrt{L_a^2 + \delta^2{}_{ls,2}}} \tag{23}$$

$$UI = \sum_{i=1}^{n} E_a A_a (\frac{\sqrt{L_a^2 + \delta^2{}_{ls,2}} - L_a}{L_a}) \frac{\delta^2{}_{ls,2}}{\sqrt{L_a^2 + \delta^2{}_{ls,2}}} \tag{24}$$

When the structure is completely collapsed, external power and structure dissipation of kinetic energy will reach to equilibrium state. It is as shown in the following relational expression.

WE=UI

$$WE = Mg\delta_{\max}u \tag{25}$$

$$UI = R_{\max}\delta_{\max} \tag{26}$$

Whereas in the formula, $\delta_{\max}$ refers to the maximum vertical displacement

And $R_{\max}$ is the largest internal resistance.

C. Impact of beam mechanism on catenary mechanism under structural collapse destruction

The concrete construction will enter the process of collapse after removal of the component. First of all, the bending mechanism at beam end will resist the unbalanced action on the upper structure. With the completion of beam mechanism phase caused by concrete structure damage, it will enter concrete tensile resistance on the unbalanced gravity subsequently, namely catenary mechanism phase. But since the schedule and collapse resistance transforming from beam mechanism to catenary mechanism are too complicated, it can hardly be distinguished by adopting common calculation method. The kinetic energy algorithm proposed by Dusenberry [6] was put forward in the paper to analyze the changes of structural energy consumption at each phases of structural collapse, and further consider the influence mechanism of beam mechanism on subsequent catenary mechanism resistance requirements.

Kinetic theory algorithm: it is believed that structural kinetic energy will be generated under gravity unbalanced gravity of structure being subject to accidental load and when the external force can not be absorbed by structural internal energy dissipation. When the value is equal to or less than zero, it indicates that there are sufficient deformation energy dissipation can neutralize the negative effects on the structure in case of external unbalanced gravity.

$$KE = WE - UI \tag{27}$$

Whereas in the formula, KE refers to structural kinetic energy.

When the concrete construction is under beam mechanism phase, the structural kinetic energy is as shown in the formula below:

$$WE_1 = Mg\delta_{ls,1}u \tag{28}$$

$$UI_1 = (\frac{M_1}{L_1} + \frac{M_2}{L_2})\delta_{ls,1} \tag{29}$$

$$KE_1 = \delta_{ls,1}[Mgu - (\frac{M_1}{L_1} + \frac{M_2}{L_2})] \tag{30}$$

When the structural collapse is as shown in the Figure 4, the concrete construction will enter from beam mechanism to catenary mechanism, namely the transition phase from concrete beam resistance to rebar resistance. It is assumed that the resistance at structural collapse is the sum of the two, the structural internal energy dissipation at the transition phase is as shown in the calculation formula as below:

$$WE_{1-2} = Mgu(\delta_{ls,2} - \delta_{ls,1}) \tag{31}$$

$$UI_{1-2} = (R_c + R_s)(\delta_{ls,2} - \delta_{ls,1})$$
$$= [\frac{M_1}{L_1} + \frac{M_2}{L_2} + 2E_a A_a (\frac{\sqrt{L^2 + {\delta^2}_{ls,2}} - L^2}{L^2}) \tag{32}$$
$$\frac{\delta_{ls,2}}{\sqrt{L^2 + {\delta^2}_{ls,2}}}](\delta_{ls,2} - \delta_1)$$

Combining the resistance effect of structural collapse at initial phase, beam mechanism phase, transition phase and catenary mechanism phase, directly consider the energy consumption relation under catenary mechanism being affected by beam mechanism, the following equation may be obtained:

$$WE = Mgu\delta_{ls,2} \tag{33}$$

$$UI = R_c\delta_{ls,2} + R_s(\delta_{ls,2} - \delta_{ls,2})$$
$$= (\frac{M_1}{L_1} + \frac{M_2}{L_2})\delta_{ls,2} + [2E_a A_a (\frac{\sqrt{L^2 + {\delta^2}_{ls,2}} - L^2}{L^2}) \tag{34}$$
$$\frac{\delta_{ls,2}}{\sqrt{L^2 + {\delta^2}_{ls,2}}}](\delta_{ls,2} - \delta_{ls,1})$$

D. Non-linear dynamic analysis

The relationship between structural nonlinear static resistance and nonlinear dynamic resistance is as shown in Figure 5. Among existing research results, both GSA2003 and DoD2005 proposed to introduce dynamic amplification coefficient in static nonlinear method to consider dynamic loading during structural collapse process in order to avoid dynamic reanalysis and structural inertia force effects. The defined value of the coefficient as specified in U.S. GSP standard is 2.0-2.5. But most of experiments indicate that the value range is too conservative and that it is prone to cause excessive structure design. The paper mainly discussed the reaction mechanism of structural damage under dynamic action to represent dynamic expansion coefficient λ as well as perform non-linear dynamic analysis of structural collapse.

Whereas in the formula, in the non-linear dynamic analysis

$$G_{nd} = R_{nd} = \lambda R_{ns} \qquad \frac{R_{nd}}{R_{ns}} = \frac{\delta_{nd}}{\delta_{ns}} = \lambda \tag{35}$$

$$\delta_{nd} = \lambda\delta_{ls} \tag{36}$$

Substituted it to the aforementioned equation, the nonlinear dynamic resistance complies with the following relationship at beam mechanism phase.

$$WE = Mgu\delta_{nd} = \lambda Mgu\delta_{ns} \tag{37}$$

$$UI = R_{nd} \cdot \delta_{nd} = \lambda R_{ls} \cdot \lambda\delta_{ls} = \lambda^2 (\frac{M_1}{L_1} + \frac{M_2}{L_2})\delta_{ls} \tag{38}$$

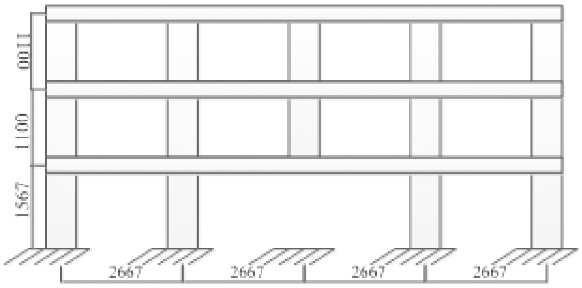
Combining the aforementioned relational expression, the structural dynamic energy dissipation under catenary mechanism is as shown below:

$$WE = \lambda Mgu\delta_2 \tag{39}$$

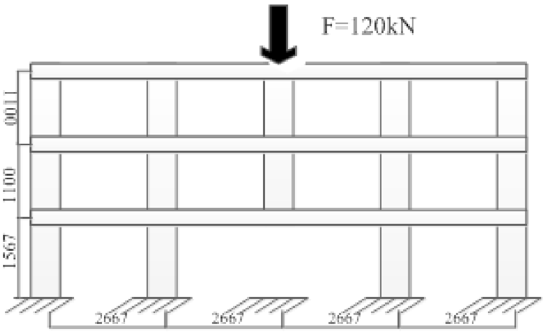
$$UI = R_{nd,c}\delta_{nd,2} + R_{nd,s}(\delta_{nd,2} - \delta_{nd,1}) = \lambda^2 R_c\delta_2 + \lambda^2 R_s(\delta_2 - \delta_1)$$
$$= \lambda^2 \{(\frac{M_1}{L_1} + \frac{M_2}{L_2})\delta_2 \tag{40}$$
$$+[2E_a A_a (\frac{\sqrt{L^2 + {\delta^2}_2} - L^2}{L^2}) \frac{\delta_2}{\sqrt{L^2 + {\delta^2}_2}}](\delta_2 - \delta_1)\}$$

III. VERIFICATION THROUGH NUMERICAL EXAMPLES

By setting a four-span three-layer plane reinforced concrete specimen, the literature adopted quasi-static calculation method to load the loading process of progressive collapse of simulated framework structure. The beams reinforcement was HRB400, concrete grade as C30, and beam and column sections were 200*200mm and 100mm*200mm respectively. The paper set it as an example and verified the theoretical derivation specific to progressive collapse of the frame structure through calculation. The frame structure schematic diagram is as shown in Figure 5.



a.Model schematic diagram of plane frame structure



b. Schematic diagram of structure static loading

Figure 5. Schematic diagram of plane frame

Due to load application on the center column in the structure based on column failure sample on simulation flat structure of progressive collapse of the simulated structure, it entered elastic phase, beam mechanism phase, catenary phase and failure phase successively. The section mainly performed computational analysis specific to beam mechanism phase and catenary mechanism phase of structural collapse resistance.

First of all, the bending moment at beam end at central column collapsed was determined and the collapse resistance under beam mechanism was calculated. The vertical displacement of experimental data can be obtained under the beam mechanism and further calculated the collapse resistance under beam mechanism; structural energy equilibrium conversion was carried out through calculation of the collapse resistance under catenary mechanism thus to obtain the internal force changing process while transforming from beam mechanism to catenary phase at structural collapse.

Based on the calculation, the bending moment at beam end of central column and displacement value can be obtained. The resistance under beam mechanism can be calculated. Set the collapse resistance of first floor beam as an example. The following formula can be obtained according to equation 14:

$$M_1 = 2.1kN\cdot m$$

$$UI = (\frac{2.1}{2667} + \frac{2.1}{2667}) * 27 = 4.25\times 10^4 N\cdot mm^2$$

According to formula 24, the collapse resistance under catenary mechanism is

$$UI = 26.14 * 10^5 * \frac{\sqrt{2667^2 + 127^2} - 127}{2667}$$

$$* \frac{127^2}{\sqrt{2667^2 + 127^2}} = 15\times 10^6 N\cdot mm^2$$

According to equation 28-34, the energy dissipation from structure beam mechanism to catenary mechanism can be obtained.

Due to the dead-weight of structure of 7.5KN, 120kN of exterior load was applied on upper structure central column.

$$WE_1 = 127.5 * 10^3 * 27 = 34.4\times 10^5 N\cdot mm$$

$$KE_1 = WE_1 - UI_1 = 33.97\times 10^5 N\cdot mm$$

$$WE_{1-2} = 127.5 * 10^3 * 100 = 12.75 * 10^6 N\cdot mm$$

$$UI_{1-2} = (0.015\times 10^4 + 11.8\times 10^4)\times(127-27)$$

$$= 11.815\times 10^6$$

$$KE_{1-2} = 0.935 N\cdot mm$$

It turned out that the calculation results can describe the internal collapse resistance action of plane frame structures specimen with few discrepancy compared to test results.

IV. CONCLUSION

Based on the principle of energy balance, definite force mechanism analysis was performed and computational methods for collapse resistance at each phase of structural progressive collapse were established in the paper specific to progressive collapse of frame structure due to accidental load failure which have provided basis for collapsed design. The main conclusions are as follows:

1) Based on the principle of energy balance, it analyzed collapse resistance of beam mechanism and catenary mechanism at structural collapse process, introduced the method of structural energy dissipation, and further analyzed dissipating energy demand at transition period from beam mechanism to catenary mechanism.

2) The equation relationship between structural nonlinear static demands and nonlinear dynamic demands under beam mechanism and catenary mechanism by adopting energy balance and enlargement coefficient.

3) The accuracy of the derivation has been verified through computation examples.

REFERENCES

[1] Lu Xinzheng, Liyi, Ye Lieping. Research on Progressive Collapse Resistance of Concrete Structure and Design Method , CA:Beijing,China,2011

[2] Liyi. Research on Progressive Collapse Resistance of RC Frame Structure. Ph.d Thesis, Tsinghua University, Beijing, On China,2011

[3] GSA 2003 Progressive collapse analysis and design guidelines for new federal office buildings and major modernization project. Washington(DC):United States General Services Administration,2003

[4] DoD 2005 . Design of structures to resist progressive collapse. Washington, (DC): Department of Defense.,2005.

[5] ASCE. ASCE-7, Minimum design loads for building and other structures. Reston(VA):Structural Engineering Institute, American Society of Civil Engineers.

[6] O. Dusenberry, P.E.,F.ASCE, O.Hamburger . Practical means for energy-based analyses of disproportionate collapse potential[J].Journal of Performance of Constructed Facilities, 20:336-348, 2006.

[7] Izzuddin B. A., A.G.Vlassis, A.Y.Elghazouli et al. Progress collapse of multi-storey buildings due to sudden column loss——Part I: Simplified assessment framework[J].EngineeringStructure,30(5):1308-1318, ,2008.

[8] A.G.Vlassis, Izzuddin B. A., A.Y.Elghazouli.et al. Progress collapse of multi-storey buildings due to sudden column loss——Part II: application[J]. Engineering Structure, 30:1424-1438,2008

[9] Liyi, Ye Lieping, Lu Xinzheng Demand Analysis of Progressive Collapse Resistance of RC Frame Structure Based on Energy Method I: Beam Mechanism[J]. Journal of Building Structures, 32:1-8, 2011.

[10] Liyi, Lu Xinzheng, Ye Lieping Demand Analysis of Progressive Collapse Resistance of RC Frame Structure Based on Energy Method II: Catenary Mechanism[J]. Journal of Building Structures,32:9-16, 2011.