A Study on the Critical Friction Angle in Tennis Table

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***Abstract* — In this paper we: i) introduce the concept of critical friction angle, ii) view the process of table tennis as a tracking problem, iii) confirm the feasibility of the proposed racquet and table rebound model, iv) consider statistical and numerical calculation methods of model building, v) assess the model using fuzzy mathematics theory.**

Keywords – critical friction angle; physical rebound; Numerical calculation; Fuzzy mathematics; biomechanics

I. INTRODUCTION

Table tennis as a sport in our country, its development and the performance affects the image of our country sports. In the research of table tennis, there are a lot of excellent works, but most are for the development of table tennis athletes and referees, coaches, performances, for table tennis racquet and table's research also not many. So this article, focusing on the physical recovery of table tennis and table tennis are studied, and from the perspective of critical friction Angle, in-depth, eventually come to the conclusion.

Critical friction Angle is one of the theoretical basis of table tennis movement is analyzed, and the introduction of the critical friction Angle to solve the friction between table tennis racket and the table. To this, many scholars have made a feasibility study for the critical Angle of friction problem. Yan Feng in view of the friction Angle proposed the five application model, by introducing the concept of friction Angle, introduced the five kinds of mode, and the friction Angle of concrete analysis in the study on the application problems in several patterns [1-5]; Xu Chris hayes from the suction Angle of internal friction Angle, Angle of internal friction of the premise, through the analysis of suction determine conditions, determine the way, as well as the methods to determine, and affirmed the importance of the friction Angle are pointed out [6-8].

In addition, the physical recovery is also an important concept in the process of table tennis, it is a combination of movement and mechanics theory, is an important point of table tennis in-depth analysis. Here, a lot of scholars have made the feasibility for physical recovery theory and its application problem of research. First, Ren Yanqing by studying the rotation of the table tennis in the process, and the table tennis and table tennis racket collision phenomenon, as a springboard, build the rotation model, and table tennis ball and racket rebound paper model to build rebound as the goal, the basis of the basic law and trajectories of table tennis, from the Angle of mathematical theory, analyzes the rally rebound phenomenon and the building of the model [9,10].

Whether it is a critical friction Angle or physical rebound phenomenon of the research is to table tennis technology to contribute. Therefore, scholars also made research for table tennis technology. Jian-jun tang, for example, through the construction of table tennis tactical system, analyzes the technical movement of tactical formation, and the formation of technology solved the use of table tennis tactical model further. Albala c for another example, table tennis technology system construction, the theory elaboration, through examples of specific examples and analysis, expounds on the scientific, make readers have a scientific understanding of table tennis technology system.

Is studied in this paper, based on the critical Angle of friction, table tennis rackets and table the rebound in the physical model of the build process. Through the theoretical analysis of critical friction Angle, the basic concepts, such as physical rebound trajectory and characteristics of table tennis, as well as quantitative analysis from the Angle of mathematics and mechanics, constructed based on the critical friction Angle of table tennis racquet and table the rebound in the physical model, a theoretical contribution to the research of table tennis.

II. MODEL ESTABLISHMENT

A. Research Base

In table tennis technical research process, it tends to get involved in mechanical problems. As force effects, table tennis movement process parabola problems, and table tennis racket as well as table tennis table physical rebound problem is one of inevitable problems in these problems. Firstly, introduce physical rebound significances. No doubt as force effects are mutual, physical rebound refers to exert opponent acted physical assault rebounding to opponents. In table tennis movement process, table tennis

racket due to suffer table tennis impacts and suffer its acted force, and table tennis table also suffers certain force effects because of table tennis falling and impacting, in the process, physical rebound phenomenon is very common.

Secondly, introduce critical friction angle concept. When object lies in sliding critical state, static friction force FS arrives at maximum value $F_{\max}$, now F_R and F_N included angle is also the largest, now ϕ_m is called friction angle. When object lies in sliding and static critical state now ϕ_m is called critical friction angle.

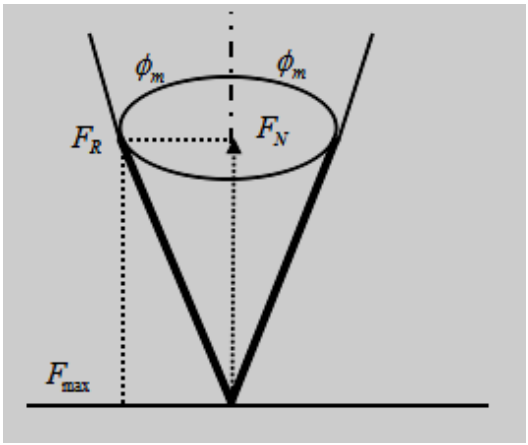

Figure 1. The Critical Angle of Friction

Refer to Figure 1:

Friction angle tangent is equal to static friction coefficient, that: $\tan\varphi_m = F_{\max} / F_N = f \bullet F_R / F_N = f$;

By friction angle, it can define static friction coefficient;

When movement initiative direction that is trend direction changing, $F_{\max}$ and supporting plane whole counter-force F_R direction will also change;

When whole counter-force F_R active line continuous changes in space, it will appear friction cone that is special cone ;

For object balance range, $\varphi \le \varphi_m (F \le F_{\max})$, it can use friction angle to express as Figure 2.

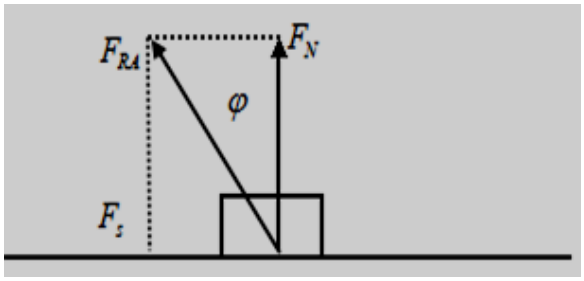

Figure 2. The critical friction Angle diagram

Thereupon, for table tennis racket and table physical rebounding process, friction angle, especially for critical friction angle application is very important.

For table tennis racket and table tennis table physical rebounding research under critical friction angle, it has very important significances, it can summarize as following Table 1:

TABLE 1. RESEARCH SIGNIFICANCES

	Research Significance
1	Research table tennis racket and table physical rebounding is helpful for analyzing table tennis movement problem of movement trajectory;
2	For appeared physical rebounding phenomenon, reasonable arrange table tennis racket and table positions:
3	Research on it table tennis movement process physical rebounding has profound significances in implementing more regular table tennis movement and normative motions;

By above research significances summarizing, it is clear that table tennis racket and table tennis table physical rebound model construction in critical friction angle is a feasible research.

B. Table Tennis Movement Trajectory using Critical Friction Angle

In table tennis and table physical rebounding process, table tennis movement trajectory shows parabolic shapes, in the process, it can be used for critical friction angle solving. Below Figure is table tennis movement trajectory simple graph, analyze the simple graph; its analysis process is shown in Figure 3.

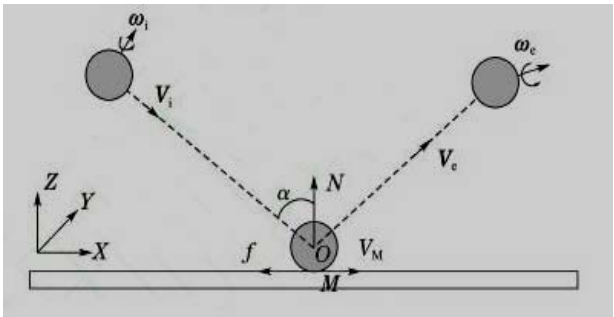

Figure 3. Table tennis movement

At first, establish three-dimensional coordinate system that are respectively x, y, z three coordinates axes, from which define table as x, y axis, and longer side is x axis, shorter side is y axis, take direction that is vertical to table tennis table plane is z axis positive direction, according to table tennis flight trajectory, it can get following scale Table 2:

TABLE 2. VARIABLES TABLE

	Variable
Incidence before collision	Table tennis incidence speed: V
	Angular speed: ω_i
After collision	Table tennis releasing speed is V_e
	Angular speed is ω_e

Among them, Speed and angular speed components in x, y, z axis are respectively using suffix x, y, z to express.

In the process, set table tennis ball center O to contact point M vector is $\vec{OM} = (0,0,-r)^T$, M point movement speed is :

$$V_M = V + \vec{OM} \times \omega = \begin{bmatrix} v_x - r\omega_y \\ v_y + r\omega_x \\ 0 \end{bmatrix}$$

M point speed in table tangential direction is: $v_{xy} = \sqrt{(v_x - r\omega_y)^2 + (v_y + r\omega_x)^2}$

After table tennis racket and table occurring physical rebounding, table tennis angular speed and speed meet:

$$\begin{cases} V_e = \begin{bmatrix} V_{ix} \\ V_{iy} \\ -eV_{iz} \end{bmatrix} \\ \omega_e = \omega_i \end{cases} \qquad \begin{cases} V_{ex} = k_{vx1}V_{ix} + k_{vx2}V_{iy} + b_{vx} \\ V_{ey} = k_{vy1}V_{ix} + k_{vy2}V_{iy} + b_{vy} \\ \omega_{ex} = k_{wx1}V_{ix} + k_{wx2}V_{iy} + b_{wx} \\ \omega_{ey} = k_{wy1}V_{ix} + k_{wy2}V_{iy} + b_{wy} \end{cases}$$

On this basis, construct critical friction angle theory-based table tennis racket physical rebounding model and table tennis table physical rebounding model.

C. *Critical Friction Angle Theory-Based Table Tennis Racquet Physical Rebounding Model*

For moving table tennis, in establishing physical rebounding model process.

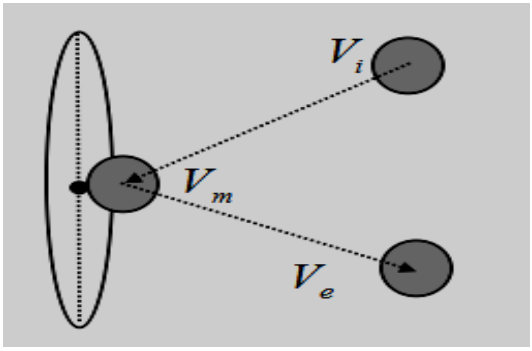

Figure 4. Table tennis racquet rebound physical model

As Figure 4, then it can establish critical friction angle theory-based table tennis racket physical rebounding model. Among them, in three-dimensional coordinate system, established speed expression method and racket coordinate system established speed expression method relation is:

$$V = \dot{T} \bullet {}^eQ + T \bullet {}^eV$$

Where:

V : Speed in three-dimensional coordinate system;

T : Three-dimensional coordinate system to racket coordinate system transformation matrix;

$\dot{T}$: T derivative; ${}^eQ$: Racket coordinate system's homogeneous coordinate;

${}^eV$: Racket coordinate system's speed;

By calculating, it can get:

$$ {}^eV_i = T^{-1}(V_i - V_{ph}) ;$$

$$^{e}V_e = T^{-1}(V_e - V_{ph})$$

Input above two formulas into (1) and (2), it can get regarding critical friction angle theory-based table tennis racket physical rebounding model.

D. *Critical Friction Angle Theory Based on Table Physical Rebounding Model*

Table tennis movement in table can be understood as parabolic movement, is defined according to its movement trajectory. In general, it can be described as shown in Figure 5.

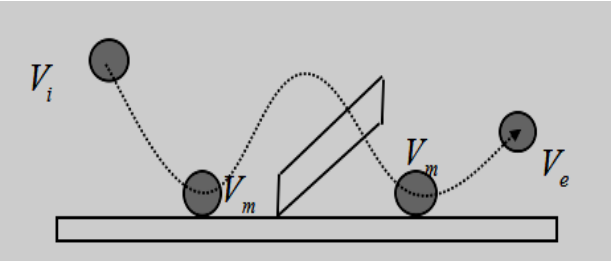

Figure 5. Table tennis physical model of the rebound.

For static table tennis table, in table tennis parabolic movement trajectory process, force status can be decomposed into two cases in tangential direction and normal direction. In tangential direction, speed gradually reduces, but angular speed remains unchanged, and then before and after physical rebounding, their relations can be:

$$\begin{cases} V_e = \begin{bmatrix} V_{ix} \\ V_{iy} \\ -eV_{iz} \end{bmatrix} \\ \omega_e = \omega_i \end{cases}$$

And in its tangential direction, the two relations can be expressed as:

$$\begin{cases} V_{ex} = k_{vx1}V_{ix} + k_{vx2}V_{iy} + b_{vx} \\ V_{ey} = k_{vy1}V_{ix} + k_{vy2}V_{iy} + b_{vy} \\ \omega_{ex} = k_{wx1}V_{ix} + k_{wx2}V_{iy} + b_{wx} \\ \omega_{ey} = k_{wy1}V_{ix} + k_{wy2}V_{iy} + b_{wy} \end{cases}$$

By experiment, measured individual parameters multiple groups of numerical values, input into above formulas and then can get critical friction angle theory-based table tennis table physical rebounding model.

III. Model Evaluation

For table tennis racket and table physical rebounding model construction in critical friction angle, to further evaluate two models' reliability, it can make quantitative evaluation on the two evaluation index through mathematics fuzzy comprehensive evaluation method and then define above analysis reliability.

A. *Define Evaluation Indicator Set*

According to lots of literature information and above analysis process, it can get two kinds of table tennis movement physical rebounding model in critical friction angle theory that is table tennis racket physical rebounding model, table tennis table physical rebounding model, table tennis racket and table relative sliding rebounding model. According to: $U = \{u_1, u_2, \cdots, u_m\}, m = 1,2,3$

Evaluation indicator set is ={table tennis racket physical rebounding model, table tennis table physical rebounding model, table tennis racket and table relative sliding rebounding model }.

B. *Define Evaluation Grade Set*

For systematic evaluation grade, mainly determination method is expert evaluation method. In table tennis movement physical rebounding model evaluation, its evaluation grade set is as following, according to: $V = \{v_1, v_2, \cdots, v_n\}, n = 1,2,3,4$

Table tennis movement physical rebounding model evaluation grade set is={very good, good, general, bad }.

C. *Define Each Evaluation Indicator Weight*

Weight mainly expression method is:

$$w = \{\mu_1, \mu_2, \cdots, \mu_m\}, m = 1,2,3$$

Where: $$\sum_{m=1}^{6} \mu_m = 1$$

Define evaluation grade indicator weights methods are mainly analytic hierarchy process and normalization method, from which normalization formula is:

$$w_i = \frac{\frac{C_i}{\overline{S_i}}}{\sum_{i=1}^{n} \frac{C_i}{\overline{S_i}}}, (i = 1,2,\cdots,m)$$

Where:

w_i is evaluation parameter i monitoring value;

$\overline{S_i}$ is evaluation parameter i standard arithmetic mean value of m grade,

the weight set is: $w = \{w_1, w_2, \cdots, w_m\}$.

Here, apply normalization method to calculate weight, result is:

$$w = \{0.345 \quad 0.350 \quad 0.305\}.$$

D. Define Evaluation Matrix

Comprehensive evaluation matrix R evaluation method is mainly experts' evaluation method, analytic hierarchy process, membership function method. Here use membership function method, define fuzzy relation matrix R, from which: $R = (R_1, \quad R_2, \quad R_3,)^T$.

Evaluation grade on the 1 grade:

$$\mu_{i1(u_i)} = \begin{cases} 0 & u_i \geq v_{i2} \\ -\dfrac{u_i - v_{i2}}{v_{i2} - v_{i1}} & v_{i1} < u_i < v_{i2} \\ 1 & u_i \leq v_{i1} \end{cases} \quad (1)$$

Evaluation grade on the j grade:

$$\mu_{ij(u_i)} = \begin{cases} 0 & u_i \leq v_{ij-1} \text{ or } u_i \geq v_{ij+1} \\ \dfrac{u_i - v_{ij-1}}{v_{ij} - v_{ij-1}} & v_{ij-1} < u_i < v_{ij} \\ -\dfrac{u_i - v_{ij+1}}{v_{ij+1} - v_{ij}} & v_{ij} \leq u_i < v_{ij+1} \end{cases} \quad (2)$$

Evaluation grade on the n grade:

$$\mu_{in(u_i)} = \begin{cases} 0 & u_i \leq v_{in-1} \\ \dfrac{u_i - v_{in-1}}{v_{in} - v_{in-1}} & v_{in-1} < u_i < v_{in} \\ 1 & u_i \geq v_{in} \end{cases} \quad (3)$$

Input data into above each parameter's each grade standard membership function formula, it can solve each evaluation parameter membership to each evaluation grade, and then construct fuzzy relation matrix R, that is:

$$R = \begin{pmatrix} 0.1 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.25 \\ 0.5 & 0.4 & 0.5 \\ 0.1 & 0.1 & 0.05 \end{pmatrix}$$

E. Carry on Comprehensive Evaluation

Known:

$$W = (\mu_j)_{1\times m}, \quad R = (r_{ji})_{m\times n},$$

by:

$$S = w \circ R = (\mu_1, \mu_2, \cdots; \mu_m) \circ \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix} = (s_1, s_2, \cdots; s_n)$$

It can get fuzzy evaluation set S, from which "$\circ$" is fuzzy composition operator. Here we take fuzzy operator as:

$M(\cdot, \oplus)$

operator, that:

$$s_k = \min\left(1, \sum_{j=1}^{m} \mu_j r_{jk}\right), k = 1, 2, \cdots, n$$

Input above computation result into above formula and can get: $S = (0.3431 \quad 0.3452 \quad 0.3117)$

By above evaluation, it is clear that in critical friction angle theory, table tennis racquet and table physical rebounding model comprehensive evaluation are both above 0.34, and table tennis racquet and table tennis table physical rebounding model < 0.34, so it is clear that above provided two models construction has certain reliability.

IV. CONCLUSION

We formulated table tennis racquet and table rebound physical models and introduced the concept of critical friction angle, combining movement and mechanics. It is a preliminary feasibility study that lays the foundation for the next step in research.

REFERENCES

[1] Gao Ying, The Study of Loop Track under Dynamic Mathematical Model. Journal of Hebei Institute of Physical Education, 2013, 24(4):79-82.

[2] Liang Tiehuai, Based on the statistics of the long jump athletes three-dimensional force analysis of jumping. Information Technology Journal, 2013, 12(15): 3345-3348. DOI: 10.3923/itj.2013.3345.3348.

[3] Pu Sheng, Biomechanical analysis of the influencing factors on fitness running leg's stomp effect. Information Technology Journal, 2013, 12(19): 5085-5090. DOI: 10.3923/itj.2013.5085.5090.

[4] Sun Zai, Yu Guang-xin, Guo Mei, Zhu Li-li, Yang Jun, He Zheng-bing, Aerodynamic Principles of Table Tennis Loop and Numerical Analysis of Its Flying Route. China Sport Science, 2008, 28(4):69-71.

[5] Tang Dingyu, Ma Gang, Guo Jun, Applications of Monte Carlo algorithm in research on the basketball hit rate of ideal hollow shooting based on Matlab Simulation. Information Technology Journal, 2013, 12(15): 3315-3319. DOI: 10.3923/itj.2013.3315.3319.

[6] Xing Jinming, Jiang Yong, Yuan Wenxue, Liu Liqing, The application of grey comprehensive evaluation model in the sports industry research. International Journal of Applied Mathematics and Statistics, 2013, 48(18): 461-468.

[7] Yang Hua, Guan Zhi-ming, Simulation of Ping-pong Trajectory Based on ODE. Computer Simulation, 2011, 28(9):230-233.

[8] Zhang Cui-cui, Lin Lin, Hu Hong-quan, Sociological Analysis on Overseas Corps of Table Tennis in China. Bulletin of Sport Science & Technology, 2010, 18(3):123-124.

[9] Zhang Qiu-fen, Su Jing, Analysis on Medal Distribution and Medallist's Playing Type of Table Tennis in the Olympic Games. China Sport Science and Technology, 2005, 41(5):90-92.

[10] Zhong Yu-jing, Wang Da-zhong, Wang Juan, Philosophy in Table Tennis Development. Journal of Beijing Sport University, 2008, 31(4):456-459.