Sliding Mode Backstepping Control for the Flexible Manipulator

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***Abstract*—As the dynamics model of flexible manipulator are very complex, traditional mode based control strategies are not suitable for solving the control problem. A new sliding mode control algorithm was proposed based on back-stepping method. The bounds of uncertainties applied in the sliding mode control are reduced by applying back-stepping method. This provides a robust control system with a less error. The back-stepping method is used to provide uniform ultimate boundedness stability purpose. The control performance of the proposed control algorithms were verified in terms of tracking capability, vibration suppression performance as well as time response specifications..**

Keywords- flexible manipulator; trajectory tracking; sliding mode control; back-stepping control

I. INTRODUCTION

In the recent years flexible robot manipulators not only have been used in manufacturing but also used in vast area such as medical area and working in International Space Station. Control methodologies and the mechanical design of robot manipulators have started in the last two decades and the most of researchers work in these methodologies [1].

Flexible manipulators have the advantages of light weight, small inertia, low energy consumption with respect to the rigid manipulators. It is difficult to establish accurate model for flexible manipulator system because it is a very complex nonlinear systems. Therefore its control faces many problems and the traditional control strategy is difficult to work, or has poor control effect.

Sliding mode controller (SMC) is one of the influential nonlinear controllers in certain and uncertain systems which are used to present a methodical solution for friction [2], disturbance, and load changing, and it has been attracting considerable interesting [3–5]. Xu et al. [6] presented a robust neural network-based sliding mode trajectory tracking control approach of omnidirectional wheeled mobile manipulators. Due to contain a discontinuous function, the sliding-mode control strategy can cause chattering phenomena and affect the performance of the controller.

To reduce the chattering, several works based on high order sliding mode techniques have been published to improve the performance [7] . Back-stepping control is a new type recursive and systematic design methodology for the feedback control of uncertain nonlinear system, particularly for the system with matched uncertainties [8]. The most appealing point of it is to use the virtual control variable to make the original high order system simple, thus the final outputs can be derived systematically through suitable Lyapunov functions [9]

Based on the systematic construction of Lyapunov functions, sliding mode control based on back-stepping technology [10] is a nonlinear recursive design methodology for tracking and regulation strategies, and it offers a choice to accommodate the unknown parameters and nonlinear effects. The key idea of back-stepping technique is to select recursively some appropriate functions of state variables as fictitious control inputs for lower dimension subsystems of the overall system [11].However, the back-stepping technique is only used to control strict feedback systems without considering the uncertain factors in practice [12], and its application is severely restricted. To obtain satisfying performances of control algorithms, many researchers have devoted to the sliding mode control based on back-stepping design methodology and applied it in various fields, such as chaos synchronization [12,13], quadrotor [14,15], motor drive [16], tracking control [17], aeroelastic system [18], etc.

This paper combined the sliding mode control and back-stepping control together on the basis of above knowledge. We mainly investigate back-stepping principle controller for manipulator based on sliding model observer. A sliding model controller, which adopted a saturation function, is designed to reduce the chattering problem and to estimate the rotor velocity and rotor position angle, while it has good tracking performance for the frequency and phase. Meanwhile, the practical stability of the controller-observer scheme is studied via Lyapunov theorem. The Lyapunov based theory of guaranteed stability of uncertain system is then used for avoiding chattering and thus, providing a continuous control law.

II. PROBLEM FORMULATION

First, based on Newton's laws of motion and Lagrangian method [9], the manipulator's dynamic model can be expressed as

$$\begin{cases} I\ddot{q}_1 + Mgl\sin q_1 + K(q_1 - q_2) = 0 \\ J\ddot{q}_2 + K(q_2 - q_1) = u - dt \end{cases} \quad (1)$$

where I_{load} is the inertia of the load-side, and J_{motor} is the inertia of the motor rotor, k is the coefficient of elasticity of the link, M is the mass of the load. g is the gravity acceleration, l is the distance from the mass center of the load to the link, u is the driven torque of the motor, q_1 is the angular displacement of the load, and q_2 is the angular displacement of the motor rotor. If K is big, the flexible arm has big elastic stiffness and less flexibility while q_1 is close to q_2. If K is small, the flexible arm has small elastic stiffness and much flexibility whiles the arm easy to bend. d_t denote the disturbance. By denoting $x_1=q_1$, $x_3=q_2$, system (1) could be defined as state equation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{1}{I}(Mgl\sin x_1 + K(x_1 - x_3)) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{J}(u - K(x_3 - x_1) - dt) \end{cases} \quad (2)$$

Control process should be described as that the joint angle x_1 follows the tracking instruction x_{1d}. Because the controlled object is mismatched system, the traditional control method can't be used to design the stable controller.

III. CONTROLLER DESIGN

In the method of back-stepping sliding mode controller design, the position error is defined by:

$$z_1 = x_1 - x_{1d} \quad (3)$$

Then

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - \dot{x}_{1d} = x_2 - \dot{x}_{1d} \\ \ddot{z}_1 &= -\frac{1}{I}(Mgl\sin x_1 + K(x_1 - x_3)) - \ddot{x}_{1d} \end{aligned} \quad (4)$$

There are 4 steps in the design method of back-stepping control law:

The first Lyapunov function is defined by

$$V_1 = \frac{1}{2}z_1^2 \quad (5)$$

Then

$$\dot{V}_1 = z_1(x_2 - \dot{x}_{1d})$$

$$x_2 = -c_1 z_1 + \dot{x}_{1d} + z_2$$

Where c_1 is positive constant, z_2 is virtual control section, $z_2 = x_2 + c_1 z_1 - \dot{x}_{1d}$. Then

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2 \quad (6)$$

Obviously when z_2=0, $\dot{V}_1 \le 0$.

The second Lyapunov function is defined by

$$V_2 = V_1 + \frac{1}{2}z_2^2 \quad (7)$$

Then the derivative of z_2 becomes

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 + c_1\dot{z}_1 - \ddot{x}_{1d} \\ &= -\frac{1}{I}(Mgl\sin x_1 + K(x_1 - x_3)) + c_1(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d} \\ \ddot{z}_2 &= -\frac{1}{I}(Mglx_2\cos x_1 + K(x_2 - x_4)) \\ &\quad + c_1\left(\left(-\frac{1}{I}(Mgl\sin x_1 + K(x_1 - x_3))\right) - \ddot{x}_{1d}\right) - \dddot{x}_{1d} \end{aligned} \quad (8)$$

Then

$$\begin{aligned} \dot{V}_2 &= -c_1 z_1^2 + z_1 z_2 + z_2\dot{z}_2 \\ &= -c_1 z_1^2 + z_1 z_2 + \\ &\quad z_2\left[-\frac{1}{I}(Mgl\sin x_1 + K(x_1 - x_3)) + c_1(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d}\right] \\ &= -c_1 z_1^2 + z_1 z_2 + \\ &\quad z_2\left[-\frac{1}{I}(Mgl\sin x_1 + Kx_1) + c_1(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d}\right] + z_2\frac{K}{I}x_3 \end{aligned} \quad (9)$$

Select

$$x_3 = -\frac{I}{K}\begin{bmatrix} -\dfrac{1}{I}(Mgl\sin x_1 + Kx_1) + \\ c_1(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d} + z_1 + c_2 z_2 \end{bmatrix} + z_3 \quad (10)$$

Where c_2>0, z_3 is virtual control part,

$$z_3 = x_3 + \frac{I}{K}\begin{bmatrix} -\dfrac{1}{I}(Mgl\sin x_1 + Kx_1) + \\ c_1(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d} + z_1 + c_2 z_2 \end{bmatrix} \quad (11)$$

then

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 + \frac{K}{I}z_2 z_3 \quad (12)$$

Obviously, when z_3 is zero, $\dot{V}_2 \le 0$. A new virtual control part should be introduced to let z_3 is zero.

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The third Lyapunov function is defined by

$$V_3 = V_2 + \frac{1}{2}z_3^2 \tag{13}$$

Then the derivative of z_3 becomes

$$\dot{z}_3 = x_4 + \frac{I}{K}\left[\begin{array}{c} -\frac{1}{I}(Mgl\cos x_1 \cdot x_2 + Kx_2) + \\ c_1(\dot{x}_2 - \ddot{x}_{1d}) - \dddot{x}_{1d} + \dot{z}_1 + c_2\dot{z}_2 \end{array}\right] \tag{14}$$

Define S

$$\begin{aligned} S = &-\frac{1}{I}(Mgl\cos x_1 \cdot x_2 + Kx_2) + \\ &c_1(\dot{x}_2 - \ddot{x}_{1d}) - \dddot{x}_{1d} + \dot{z}_1 + c_2\dot{z}_2 \end{aligned} \tag{15}$$

Then equation (14) turns into

$$\dot{z}_3 = x_4 + \frac{I}{K}S \tag{16}$$

Take equation (16) into equation (13)

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 + \frac{K}{I}z_2 z_3 + z_3\left(x_4 + \frac{I}{K}S\right) \tag{17}$$

select

$$x_4 = -\frac{I}{K}S - c_3 z_3 - \frac{K}{I}z_2 + z_4$$

Where $c_3>0$, z_4 is virtual control part,

$$z_4 = -x_4 + \frac{I}{K}S + c_3 z_3 + \frac{K}{I}z_2$$

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 + z_3 z_4$$

Obviously, when z_4 is zero, $\dot{V}_3 \le 0$. A new virtual control part should be introduced to let z_4 is zero. In the method of sliding mode control, the sliding surface s is defined as z_4. Then the fourth *Lyapunov function* is defined by

$$V_4 = V_3 + \frac{1}{2}z_4^2 \tag{18}$$

then

$$\dot{V}_4 = \dot{V}_3 + z_4\dot{z}_4 \tag{19}$$

Then the derivative of S becomes

$$\begin{aligned} \dot{S} = &-\frac{1}{I}(-Mgl\sin x_1 \cdot x_2^2 + K\dot{x}_2) + \\ &c_1(\ddot{x}_2 - \dddot{x}_{1d}) - \ddddot{x}_{1d} + \ddot{z}_1 + c_2\ddot{z}_2 \end{aligned} \tag{20}$$

Then the derivative of z_4 becomes

$$\dot{z}_4 = \frac{1}{J}(u - dt) - \frac{K}{J}(x_3 - x_1) + \frac{I}{K}\dot{S} + c_3\dot{z}_3 + \frac{K}{I}\dot{z}_2 \tag{21}$$

Take equation (21) into equation (19)

$$\dot{V}_4 = \dot{V}_3 + z_4\left(\begin{array}{c} \frac{1}{J}(u - dt) - \frac{K}{J}(x_3 - x_1) + \\ \frac{I}{K}\dot{S} + c_3\dot{z}_3 + \frac{K}{I}\dot{z}_2 \end{array}\right) \tag{22}$$

The switching control law is used to suppress disturbances. In order to make $\dot{V}_4 \le 0$, the control law is defined as

$$u = -\eta\,\mathrm{sgn}(z_4) - J\left(\begin{array}{c} -\frac{K}{J}(x_3 - x_1) + \frac{I}{K}\dot{S} + \\ c_3\dot{z}_3 + \frac{K}{I}\dot{z}_2 + z_3 + c_4 z_4 \end{array}\right) \tag{23}$$

Where $c_4>0$, $\eta \ge D$. Take equation (23) into equation (22)

$$\begin{aligned} \dot{V}_4 = &\frac{1}{J}(-\eta|z_4| - dt \cdot z_4) - c_1 z_1^2 - c_2 z_2^2 - \\ &c_3 z_3^2 + z_3 z_4 + z_4(-z_3 - c_4 z_4) \\ &\le -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 - c_4 z_4^2 \end{aligned} \tag{24}$$

IV. SIMULATION RESULTS

In system (1) I=J=1.0, Mgl=5.0, K=1200. The joint angle command can be defined as x_{1d}=0.5sin($6\pi t$), disturbance dt=200000sin($3\pi t$), then D=200000, using control law system (23) select η=D+0.10, control parameter c_1= c_2= c_3= c_4=50. The simulation results are given in Fig. (1-2). The results show that the controller can be used to ensure the subject tracking error converges to zero.

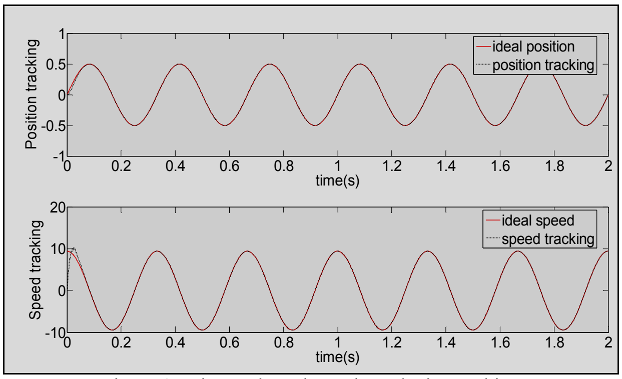

Figure 1. Joint angle and angular velocity tracking

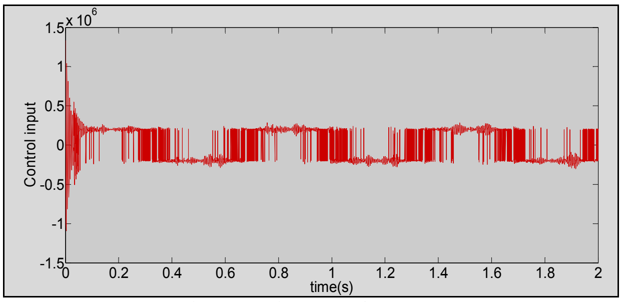

Figure 2. Joint Control Moment

V. CONCLUSION

A novel approach was developed for trajectory tracking control of electrically-driven robotic manipulators in task space. The simulation results confirmed that the proposed control law can provide a desired tracking performance for a robotic manipulator with uncertain dynamics and uncertainties in actuator models. In proposed control, we use feedback linearization technique for reduce of known nonlinear terms. For compensation of remained terms, a sliding mode control is proposed by using of back-stepping method. Analytical mathematic are shown that the closed loop system is global asymptotical stable. For remove of chattering control, we modify proposed control. By modified control, the closed loop system is uniform ultimate boundedness stable. The simulation results illustrate that the designed controller performs well in presence of uncertainties and the tracking errors converge to zero rapidly.

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