# A Novel Adaptive Markov Matrix IMM Algorithm using Multi-Sensor Fusion

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***Abstract* — Because the Markov parameter of the initial Interacting Multiple Model, IMM, is a constant and the likelihood function of model's probability is the Gaussian approximation, which is a suboptimal result in the sense of Bayes, it may slow down model switching and reduce filtering precision. To solve the problem, we use multi-sensor fusion criterion so that state estimation of the interactive multiple models is reweighted, the mode probabilities are updated by the cross-covariance matrix of estimation errors, and the Baum's auxiliary function is maximized to estimate the transition probabilities of hidden Markov model, which is a time-varying Markov transition probabilities. The new algorithm has clearly improved the speed of estimation and filter precision compared to the original algorithm.**

***Keywords - scalar weight; Interacting Multiple Model (IMM); hidden Markov model; multi-sensor fusion***

## I. INTRODUCTION

Because of the complexity of the target motion and the interference of the environmental factors, It is difficult to trace the unknown maneuvering. the IMM algorithm that is put forward by Blom in [1] is proved an effective way by practice. for maneuvering target tracking, the known multiple models make up a target motion aggregate, the filter of each model works in parallel, as well as they compute the state estimation which will be fused, finally state estimation of the tracking system is output . the SCT-IMM algorithm in [1] can track any flight phase of maneuvering targets, however, can't solve model switching lagging and filter smoothing. the hidden markov model in [5] makes the markov matrix of the IMM algorithm adaptive, which can reduce switching time of the motion model.

In the IMM algorithm, the probability value of each model represents the proximity between the state estimated value and the actual value. In fact, there is a continuous uncertainty state and a discrete uncertain state in a mixture system, therefore, the likelihood function of model's probability is the Gaussian approximation, which is a suboptimal result in the sense of Bayes, at the same time, Gaussian approximation of the likelihood function increase the error of model probability. In [4], SIMM that is different with previous method is proposed in the criterion of the multi-sensor optimal information fusion. which can solve model probability calculation under the circumstances of assuming filter estimation error not related.

In this paper, we present a improved algorithm about model probability of IMM in the sense of linear minimum variance calculation based on multi-sensor fusion optimal theory. The hidden Marko model's parameter estimation is introduced, so that it can improve the speed of estimation and the filter precision, this paper is organized as follows. the hidden Marko state transition probability model is given in Section II, the probability of multi-sensor optimal information fusion is illustrated Section III. the improved IMM interactive multiple model algorithm and simulation discussion are given in Section IV and Section V respectively. Finally, some conclusions are drawn in Section VI.

## II. HIDDEN MARKO STATE TRANSITION PROBABILITY MODEL

The struct of HMM is usually described by the following parameters:

1) $\pi$ : the struct of HMM is usually described by the following parameters:

$$\pi = \{\pi_1, \pi_2, \cdots, \pi_N\} \quad (1)$$

where $N$ is the number of the model state.

2) $A$ : state transition probability matrix.

$$A = \{a_{ij}\} \quad a_{ij} = P(q_{i+1} = S_j \mid q_i = S_i). \quad (2)$$

3) $B$ : the observed sequence probability distribution

$$B = \{b_{ij}\}, b_{jk} = P(o_t = V_k \mid q_t = S_j), 1 \le j \le N, 1 \le k \le M. \quad (3)$$

The above is to define the composition of the HMM.

The operated mode of the target tracking system at time $k$ can be described as a discrete one and a half order Marko process where there are $S$ states, let $M_j(k)$ denote the effective model $j$ at time $k$, so the Marko transition probability is defined as a process model:

$$A = (a_{ij}) \Leftrightarrow P\{M_j(k) \mid M_i(k-1)\}, i, j \in S. \quad (4)$$

$$0 < a_{ij} < 1 \text{ 且 } \sum_{j=1}^{s} a_{ij} = 1.$$

Let the initial state probability distribution of markov model set $\mu = \{\mu_1, \ \mu_2, \cdots, \ \mu_s\} \Leftrightarrow \pi$, due to the transition probability matrix is time-varying, if there is a measuring sequence $\mathbf{Z}^k = \{z_1, \cdots, z_k\}$ at time $k$, the characteristics probability density of the target motion measuring is $f_i(z_k) = p(z_k \mid M_i(k))$ $i \in s$, namely $B = \{b_i(z_k)\}$, clearly it has some HMM features. This idea motives us to develop a

HMM transition probability estimates, based on a maximize Baum auxiliary function, we defined a before sequence variable as follows:

$$\alpha_i(k) = f(Z^k, M_i(k) \mid \lambda).$$

a after sequence variable:

$$\beta_i(t) = f(Z_{t+1}{}^k \mid M_i(t), \lambda).$$

$\eta_t(i)$ is a probability of target mode $i$ at time $t$ , $\xi_t(i,j)$ is a probability that the target is in mode $i$ at time $t$ or in mode $j$ at time

$$\eta_t(i) = f(M_i(t) \mid Z^k, \lambda),$$

$$\xi_t(i,j) = f(M_t(i), M_{t+1}(j) \mid Z^k, \lambda). \tag{5}$$

HMM transition probability matrix is estimated at time $t$ as $A_t = \{a_{ij}(t)\}$

$$\alpha_i(t) = f(Z^k, M_i(t) \mid \lambda) = \sum_{j=1}^{s} \alpha_j(t-1) \cdot a_{ij}(t-1) \cdot b_i(z_k),$$

$$i, j \in S. \tag{6}$$

$$\beta_i(t) = \sum_{j=1}^{s} a_{ij}(t-1) \cdot b_i(z_k) \cdot \beta_j(t+1), \tag{7}$$

$$\eta_t(i) = f(M_i(t) \mid Z^k, \lambda) = \frac{\alpha_i(t)\beta_i(t)}{\sum_{j=1}^{s} \alpha_i(t)\beta_i(t)}, \tag{8}$$

$$\begin{aligned}\xi_t(i,j) &= f(M_t(i), M_{t+1}(j) \mid Z^k, \lambda) \\ &= \frac{\alpha_i(t) \cdot a_{ij}(t-1) \cdot b_j(z_{t+1}) \cdot \beta_i(t+1)}{\sum_{i=1}^{s}\sum_{j=1}^{s} \alpha_i(t) \cdot a_{ij}(t-1) \cdot b_j(z_{t+1}) \cdot \beta_i(t+1)} \\ &= \frac{\alpha_i(t) \cdot a_{ij}(t-1) \cdot b_j(z_{t+1}) \cdot \beta_i(t+1)}{\sum_{i=1}^{s} \alpha_i(t) \cdot \beta_i(t)}.\end{aligned} \tag{9}$$

There are clearly established the following relation:

$$\eta_t(i) = \sum_{j=1}^{s} \xi_t(i,j),$$

The sequence of Target motion mode at time $k$ can be expressed as follows:

$$\mathbf{M(k)} = \{M_1, M_2, \cdots, M_k\},$$

According to the Baum-Welch function, so

$$Q(\lambda, \bar{\lambda}) = \sum_{\mathbf{M(k)}} P(\mathbf{M(k)} \mid Z^k, \lambda) \ln P(\mathbf{M(k)}, Z^k \mid \bar{\lambda}),$$

We maximize the Baum-Welch auxiliary function repeatedly, so that a local optimal value can be made by the estimated parameters eventually. So compared the above steps, the HMM model transition probability is estimated：

$$\bar{a}_{ij} = \frac{\sum_{t=1}^{k-1} \xi_t(i,j)}{\sum_{t=1}^{k-1} \eta_t(i)}. \tag{10}$$

Considering the real time factor, we need do a compromise dispose, the transition probability matrix is output only for one iteration, the iteration results is for the current estimation of target tracking transfer probability matrix.

## III. Model Probability Based on Multi -Sensor Optimal Information Fusion

The optimal information fusion estimation problem of the interactive multiple model is aimed to get a optimal value $\hat{x}(k)$ that the scalar weight multiplies by every filter's optimal value $\hat{x}_i(k)$ and consume.

$$\hat{x}(k) = \sum_{i=1}^{s} \varepsilon_i(k) \cdot \hat{x}_i(k),$$

Where $\varepsilon_i(k)$ is a optimal scalar weight, its value is obtained under the minimum variance. the index function $J = tr(\mathbf{P}_0^x(k))$ is minimal, $tr(\mathbf{P}_0^x(k)) = \min(tr(P^x(k)))$ , $P^x(k)$ is the error variance matrix of system state including weight filter method. the optimal scalar weight $\varepsilon_i(k)$ can be calculated by the following[4]:

$$\hat{x}(k) = \sum_{i=1}^{s} \varepsilon_i(k) \cdot \hat{x}_i(k) = \alpha_{s-1} x_{s-1} + \beta_{s-1} x_s. \tag{11}$$

$$\sum_{i=1}^{s} \varepsilon_i(k) = 1, \quad \alpha_{s-1} + \beta_{s-1} = 1.$$

Where

$$\alpha_{s-1} = \frac{tr(P_s{}^x)}{tr(P_{s-1}{}^x) + tr(P_s{}^x)}, \ \beta_{s-1} = \frac{tr(P_{s\text{-}1}{}^x)}{tr(P_{s-1}{}^x) + tr(P_s{}^x)},$$

$$tr(P^s) = \frac{tr(P^1)tr(P^2)\cdots \to}{tr(P^1)tr(P^2)\cdots tr(P^{s-1}) + tr(P^1)tr(P^2)\cdots \to}$$

$$\frac{tr(P^s)}{tr(P^{s-2})tr(p^s) + \cdots + tr(P^2)tr(P^3)\cdots tr(P^s)}.$$

$$s = 2, 3 \cdots, N$$

$$\begin{aligned}P^s = {} & (\alpha_{s-1})^2[(\alpha_{s-2})^2[(\alpha_{s-3})^2 \cdots [(\alpha_2)^2[ \\ & (\alpha_1)^2 P^1 + (\beta_1)^2 P^2] + (\beta_2)^2 P^3] + \cdots (\beta_{s-3})^2 P^{s-2}] \\ & + (\beta_{s-2})^2 P^{s-1}] + (\beta_{s-1})^2 P^s,\end{aligned}$$

$$\begin{cases} \varepsilon_1 = \alpha_{N-1}\alpha_{N-2} \cdots \alpha_2 \alpha_1 \\ \varepsilon_2 = \alpha_{N-1}\alpha_{N-2} \cdots \alpha_2 \beta_1 \\ \varepsilon_s = \alpha_{N-1}\alpha_{N-2} \cdots \alpha_s \beta_{s-1} \end{cases} \tag{12}$$

The interactive multiple model of scalar weight mentioned above has a condition of an related estimate error, as well the method can also apply to other situations, which filter estimate error is not related. the feature of the fusion should be satisfied: unbiasedness and optimality. the model probability of the interactive multiple model algorithm is improved, its theoretical analysis can be referenced in [4].

## IV. Improved IMM Interactive Multiple Model Algorithm

There are two aspects of measurement update and time update in the conventional IMM algorithm, they are a circular process, which can be divided into four parts: initializing the model condition,model filtering,updating probability model and combining the output. because the Marko transition probability is set with a very strong subjectivity and Gaussian approximation of likelihood function makes the model probability to get a local optimal value. the structure diagram of the improved IMM algorithm is as shown Fig.1.

*A. Input interaction*

$\hat{p}_{ij}$ is the transition probability from model $i$ to model $j$ .which is get by the following section , $\mu_j(k-1)$ is the probability of model $j$ at time $k-1$ , the prediction probability of model $j$ can be given below,

$$\bar{c}_j = \sum_{i=1}^{s} \hat{p}_{ij}(k-1) \cdot \mu_i(k-1),$$

$$\mu_{ij}(k-1) = \bar{c}_j^{\,-1} \cdot \hat{p}_{ij}(k-1) \cdot \mu_i(k-1).$$

Where $\mu_{ij}(k-1)$ is a mixed probability of every model. according to $\hat{x}_j(k-1)$ that is a filter estimated value and $P_j(k-1)$ that is a variance value at time $k-1$, the mixedd state estimation and covariance of model $j$ can be given below:

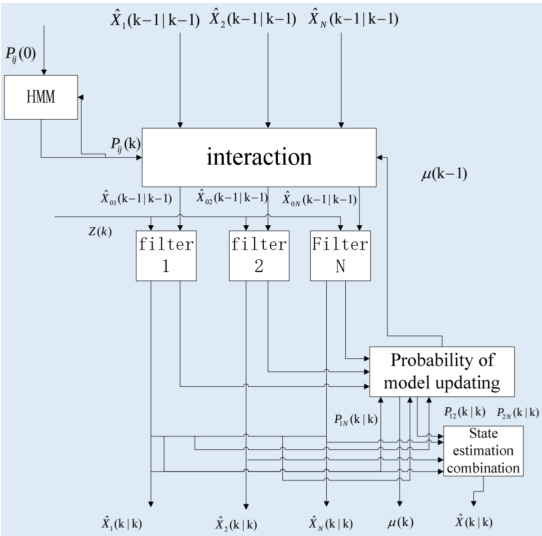


Figure. 1. Structure diagram of the improved IMM algorithm.

$$\hat{x}_{0j}(k-1 \mid k-1) = \sum_{i=1}^{s} \mu_{ij}(k-1)\hat{x} \ ,$$

$$P_{0j}(k-1 \mid k-1) = \sum_{i=1}^{s} \mu_{ij}(k-1 \mid k-1)\{P_i(k-1) + (\hat{x}_i(k-1) - \hat{x}_{oj}(k-1))(\hat{x}_i(k-1) - \hat{x}_{oj}(k-1))^T\}.$$

*B. The model $j$ matched filtering*

Each filter works in parallel mode, Kalman filter is used for filtering,. the state estimation $\hat{x}_i(k)$ and self-variance $P_i(k)$ can be obtained by Kalman formula.

*C. transition probability matrix at time $k$*

The transition probability matrix can be solved with Eq.(10), In order to simplify the calculation, we only do one iteration result as transition probability matrix of the current moment.

*D. model prabability updating*

According to the optimal information fusion theory, every model probability is obtained with Eq.(12).

*E. output combination*

Using Eq.(11), the value of each filter, get the final state estimation.

## V. The Simulation Calculation And The Result Analysis

In this paper, Target are tracked in a two-dimensional coordinate plane, in total, target motion time is 70 s, target maneuver time and its acceleration as shown in table 1.

TABLE I. Motion Target Maneuve

| Maneuver Moment /s | X Acceleration /ms$^{-2}$ | Y Acceleration /ms$^{-2}$ |
|---|---|---|
| 1 | 4 | -2 |
| 41 | 1 | 5 |
| 56 | 16 | 15 |

The state $\mathbf{x}_0 = [x, \dot{x}, y, \dot{y}]$ consists of position, velocity, their initial values are $[2000m, 0m/s, 10000m, -15m/s]$, we assume that the x direction and y direction are independently observed, the measurement noise covariance matrix $R$ =100, the sampling period $T$ =1S, Monte Carlo simulation is 50 times.

As for the IMM estimator, two motion models, i.e., Constant Velocity model(CV), the Constant Acceleration model(CA), are employed here.

1) CA model

The dynamic transition matrix is given below.

$$\Phi_1 = \begin{bmatrix} \Phi_{ca} & 0 \\ 0 & \Phi_{ca} \end{bmatrix}, \ \Phi_{ca} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}.$$

Process noise covariance is given below.

$$G_1 = \begin{bmatrix} G_{ca} & 0 \\ 0 & G_{ca} \end{bmatrix}, \ G_{ca} = m\begin{bmatrix} \frac{T^4}{8} & \frac{T^3}{6} & \frac{T^2}{4} \\ \frac{T^3}{3} & \frac{T^2}{2} & T \\ T^2 & T & 1 \end{bmatrix}.$$

Where the process noise covariance coefficient $m$ of CA is 10.

2) CV model

The dynamic transition matrix is given below.

$$\Phi_1 = \begin{bmatrix} \Phi_{cv} & 0 \\ 0 & \Phi_{cv} \end{bmatrix}, \Phi_{cv} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}.$$

Process noise covariance is given below.

$$G_2 = \begin{bmatrix} G_{cv} & 0 \\ 0 & G_{cv} \end{bmatrix}, \ G_{cv} = n\begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix}.$$

Where the process noise covariance coefficient $n$ of CA is 1. $\mu_1 = \frac{1}{2}$ and $\mu_2 = \frac{1}{2}$ are the initial value of two model probability respectively. The initial input of Markov transition probability matrix is given below.

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}.$$

Using RMSE(root mean square error), We compare the improved IMM algorithm with the conventional IMM algorithm, RMSE is defined as

$$\mathrm{RMSE}_k = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(x_i(k) - \hat{x}_i(k))^2}.$$

Where $x_i(k)$ and $\hat{x}_i(k)$ denotes the state true value and the estimated value of the $i$ Monte Carlo simulation respectively. $M$ denotes the total times of Monte Carlo simulation. the simulation results are shown as Fig.2,3,4,5and Table II.

Fig.2 shows the truth-value and measuring value of target tracking trajectory. Fig 3 and Fig 4 show the mean and standard variance of the X and Y position estimates respectively , Fig 5 and Fig 6 show the mean and standard variance of the X and Y speed estimates respectively. From the target tracking error results, the improved IMM algorithm has better tracking performance that is compared to the conventional IMM algorithm. From table II as well, the improved IMM algorithm improved the time of model transformation probability.

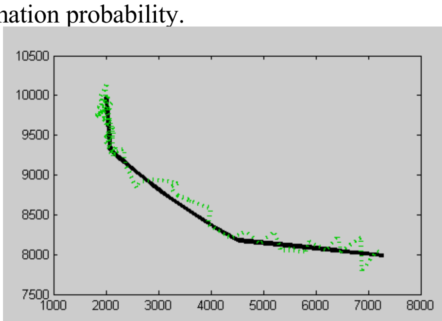

Figure. 2. Target motion trajectory in true and measure value.

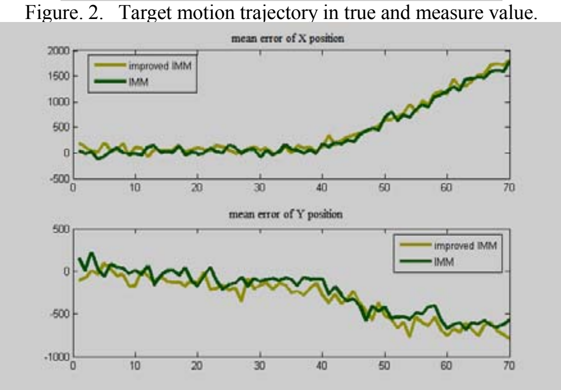


Figure. 3. Target X Y error.

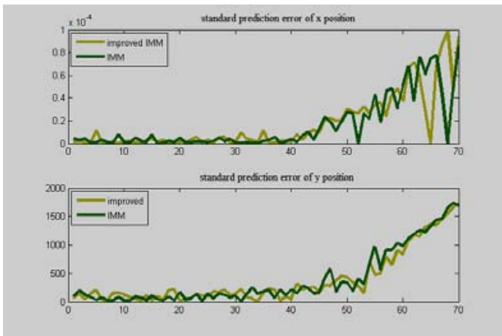


Figure. 4. Target X Y Stand error.


Figure. 5. Target X Y Speed error.

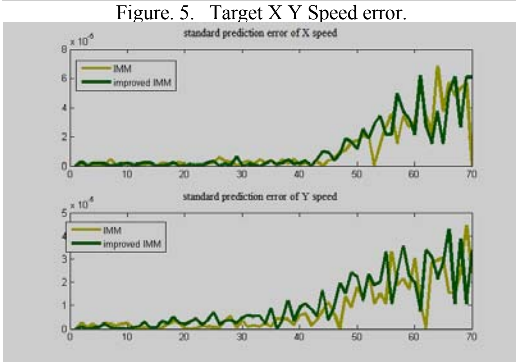


Fig. 6. Target X Y Speed Stand_error.

TABLE II SIMULATION RUNNING TIME

| algorithm | time |
|---|---|
| IMM | 2.349063 |
| Improved IMM | 2.223658s |

## VI. CONCLUSIONS

In this paper, we improve the conventional IMM algorithm, using linear minimum variance calculation based on multi-sensor fusion optimal theory to get the model probability. In order to speed up the model switch and reduce the filtering precision, we propose a time-varying Markov transition probabilities, which the Baum's auxiliary function results in maximizing to estimate the transition probabilities of hidden Markov model. The experiment shows that the improved IMM can cut the computation cost and decrease track loss.

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