

Influence of Risk Group Assessment and Decision-Making for Power Supply Companies Based on Catastrophe Theory and Two-Tuple Linguistic

Cunbin Li

e-mail: lcb999@263.net

Wei Dong

e-mail: dongwei@126.com

Zhangyi Pan

email: pan_zhangyi@163.com

School of Economics and Management
North China Electronic Power University
Beijing, China

Abstract — Risk assessment has been playing an increasingly important role in the development of power supply companies with the deepening of electricity market reform. Thus, for the expert group assessment and decision-making problems of these companies, a risk group assessment and decision-making model based on catastrophe theory and two-tuple linguistic is proposed. In this method, linguistic assessments of experts are transformed into two-tuple linguistic first. Catastrophe theory is expanded from real numbers to two-tuple linguistics based on its definition. Then, combined with the experts' linguistic subjected weights, an eclectic weighting method is presented based on the similarity and deviation, and the order of schemes can be listed by comparing all of the total catastrophe two-tuple values of schemes. Finally, the rationality and effectiveness of this approach are illustrated.

Keywords - power supply company; risk assessment; group decision-making; catastrophe theory; two-tuple linguistic

I. INTRODUCTION

With the constant promotion of the reform in industrial system of power market, the power grid in China has developed from a vertical monopoly model with the integration of power generation, transmission, supply and distribution, to the separation of power plants and power grids with the restructuring of power generation and grids. Thus, the power supply departments have turned to be the enterprises, which have to independently face the fierce market competition, rather than the seller's market with the supply failing to meet the demand. And the risk they have to face has also become complicated and changeable. However, the effective evaluation and measurement can help to identify the existing key risk elements in the company operations, and provide a foundation for the targeted control measures and the risk decision emergency plan.

At present, the risk assessment on power system has become an increasing concern among the professional scholars, but the studies aiming at the risk of power supply companies are still not many. Zhang built a multi-level risk evaluation index system for power supply companies and used analytic hierarchy process(AHP) to further study [1]. Cheng considered the uncertainty in the assessment so that fuzzy comprehensive evaluation model was used to evaluate the power supply safety risk for nuclear power plants [2]. Liu analyzed the relationship of risk factors and used AHP and fuzzy evaluation method, combined with grey correlation method to study the investment risk of power plants [3]. These studies make some certain achievements but still have limitations and deficiencies. The weights of

risk indicators are all determined by subjectivity, which are baseless and have poor reproducibility. And using the exact real numbers or certain fuzzy membership functions to describe the risks reduces the fuzziness and uncertainty of the risks needed in the analysis.

Based on the above content, this paper identifies and analyzes the risk sources of the power supply enterprises, and describes the fuzziness and randomness of risk by quantitative linguistic variables to get rid of the limit of designing the index weight. Besides, it combines catastrophe theory with two-tuple linguistic and presents a risk assessment and decision-making method. In order to improve the reliability, group decision-making is introduced to integrate into the method and then an eclectic weighting method is proposed based on the similarity degree and the deviation of the subjective weight from the experts in linguistic form. Finally, this method is tested by an example.

II. FUNDAMENTAL THEORIES

A. Catastrophe Theory

According to catastrophe theory, the critical points of each state in system are classified by the potential function $V(x)$, and every potential function determines a catastrophe. The set of the points, making the first derivative of $V(x)$ zero, is called equilibrium surface M , which can integrally describe the whole process of the system catastrophe. Based on the difference of the control variables of $V(x)$, generally, there are 7 basic catastrophe models: fold, cusp, swallowtail, butterfly, elliptic umbilical, hyperbolic umbilical and parabolic umbilical catastrophe.

The features of the first 4 commonly used models are shown in Table I [4-6].

TABLE I. THE PRIMARY CATASTROPHE MODEL WITH ONE STATE VARIABLE

Type	Control dimension	Potential function $V(x)$	Equilibrium surface M	Normalized formulas
Fold	1	$x^3 + ax$	$3x^2 + a = 0$	$x_a = \sqrt{-a}$
Cusp	2	$x^4 + ax^2 + bx$	$4x^3 + 2ax + b = 0$	$x_a = \sqrt{-a}, x_b = \sqrt[3]{-b}$
Swallowtail	3	$x^5 + ax^3 + bx^2 + cx$	$6x^4 + 3ax^2 + 2bx + c = 0$	$x_a = \sqrt{-a}, x_b = \sqrt[3]{-b}, x_c = \sqrt[4]{-c}$
Butterfly	4	$x^6 + ax^4 + bx^3 + cx^2 + dx$	$6x^5 + 4ax^3 + 3bx^2 + 2cx + d = 0$	$x_a = \sqrt{-a}, x_b = \sqrt[3]{-b}, x_c = \sqrt[4]{-c}, x_d = \sqrt[5]{-d}$

In Table I, the state variable stands for the behavior state of the system. The coefficients are the control variables of each state variable with decreasing importance, which stand for the interactional factors in the system. The potential function denotes the relationship between state variable and control variables. And the normalized formula is used to obtain the catastrophe value of each control variable, which is also called catastrophe progression.

B. Two-Tuple Linguistic

Two-tuple linguistic is a representation model presented by a Spanish scholar, Herrera, in 2000, for the decision problem of linguistic assessment information. It uses tuple to represent linguistic assessment information and carry out computation by the translation of symbols, which can effectively avoid information loss and distortion in the integration and operation process, and lead to the improvement in the accuracy of expression and computation result. Its definition and the introduction of computation operators are as follows [7-8].

III. RISK GROUP ASSESSMENT MODEL

In general, due to the differences of subjective cognition and experience among experts, obtaining the score of each scheme according to all experts' subjective weights can't ensure the consistency between individual opinion and group minds. Thus, to improve the reliability, this paper uses the similarity degree with the average of group information and the deviation of the subjective weight aggregation result to determine weights.

A. A Weight Determining Method Based on Similarity

Determine weights by the similarity degree between individual opinion and the average level of group. That is to say, the smaller the difference is, the more similar the opinion of this expert with group minds, and the bigger the weight of him [9]. Combining the above principle and gray relation coefficient, this paper defines the similarity degree and presents a corresponding weight determining method.

Definition 5: Let x_{ij}^k be the linguistic assessment information of the scheme f_i for the risk r_j from the expert

e_k , and \bar{x}_{ij} be the average information of f_i for r_j from the group \bar{e} . The similarity degree between x_{ij}^k and \bar{x}_{ij} can be

$$S_x(x_{ij}^k, \bar{x}_{ij}) = \frac{\xi(x_{ij}^k, \bar{x}_{ij})}{\sum_{k=1}^l \xi(x_{ij}^k, \bar{x}_{ij})} \tag{1}$$

In it, $1 \leq k \leq l$, $1 \leq i \leq m$ and $1 \leq j \leq n$ (l, m, n respectively are the numbers of experts, schemes and risks indexes). $\xi(x_{ij}^k, \bar{x}_{ij})$ is the gray relation coefficient, and its formula is

$$\xi(x_{ij}^k, \bar{x}_{ij}) = \frac{\min_i \min_j D(x_{ij}^k, \bar{x}_{ij}) + \rho \max_i \max_j D(x_{ij}^k, \bar{x}_{ij})}{D(x_{ij}^k, \bar{x}_{ij}) + \rho \max_i \max_j D(x_{ij}^k, \bar{x}_{ij})} \tag{2}$$

The parameter ρ is a distinguishing coefficient and usually has the value 0.5. Then, obviously, the larger $S_x(x_{ij}^k, \bar{x}_{ij})$ is, the more consistent between the expert e_k and the group minds in the scheme f_i under the risk r_j .

Definition 6: With $S_x(x_{ij}^k, \bar{x}_{ij})$, the similarity degree between the expert e_k and the group \bar{e} is

$$S_e(e_k, \bar{e}) = \frac{\sum_{i=1}^m \sum_{j=1}^n S_x(x_{ij}^k, \bar{x}_{ij})}{\sum_{k=1}^l \sum_{i=1}^m \sum_{j=1}^n S_x(x_{ij}^k, \bar{x}_{ij})} \tag{3}$$

In this formula, $0 \leq S_e(e_k, \bar{e}) \leq 1$, and $\sum_{k=1}^l S_e(e_k, \bar{e}) = 1$.

Based on the principle mentioned above, the similarity degree between e_k and \bar{e} can be regarded as the weight w'_{e_k} of the expert e_k , i.e. $w'_{e_k} = S_e(e_k, \bar{e})$. Likewise, there are two constraints: (1) $0 \leq w'_{e_k} \leq 1, 1 \leq k \leq l$; (2) $\sum_{k=1}^l w'_{e_k} = 1$.

B. A weight Determining Method Based on Deviation

In order to avoid blindly pursuing the consistency and ignore the impact of some experts on the results, this paper borrows the idea from the reference [10] that using deviation determines the weights. The smaller the deviation between

the expert's decision result and the group minds is, the smaller the amount of information for schemes ranking from this expert is, so that his weight should be small. Following this principle, the definition of the deviation and the corresponding weight determining method are as follows.

Definition 7: Let y_i^k be the assessment result of the expert e_k for the scheme f_i , and z_i be the final result of the whole group for f_i by subjective weights aggregation, then the deviation between the expert e_k and the group e can be

$$H(e_k, e) = \sum_{i=1}^m (y_i^k - z_i)^2 \tag{4}$$

Definition 8: Let $H(e_k, e)$ be the deviation between the expert e_k and the group e , then the weight of e_k can be

$$w_{e_k}'' = H(e_k, e) / \sum_{k=1}^l H(e_k, e) \tag{5}$$

Obviously, $0 \leq w_{e_k}'' \leq 1$, $1 \leq k \leq l$ and $\sum_{k=1}^l w_{e_k}'' = 1$.

Thus, with these two different weight determining methods, an eclectic method is presented to consider both the consistency and the individual contribution:

$$w_{e_k} = \alpha w_{e_k}' + (1 - \alpha) w_{e_k}'' \tag{6}$$

In it, $k=1, \dots, l$, and $\alpha, 1-\alpha$ respectively are the eclectic preference coefficients for these two methods with a constraint $0 \leq \alpha \leq 1$. When $\alpha > 0.5$, it means the decision maker prefers the mean opinion of the group. When $\alpha < 0.5$, the individual opinions are more preferred. And $\alpha = 0.5$ means the same preference to these two.

C. Risk Evaluation and Decision-Making Method

To describe clearly, some symbols and parameters are prescribed at first. The scheme set is represented by $F = \{f_1, \dots, f_m\}$, and $R = \{r_1, \dots, r_n\}$ denotes the risk set to be assessed. The expert group is $E = \{e_1, \dots, e_l\}$ with their respective weights $\lambda_k (k=1, \dots, l)$ determined by their knowledge and experience. The linguistic assessment value of the expert e_k to the scheme f_i under the risk r_j is represented by $x_{ij}^k (k=1, \dots, l; i=1, \dots, m; j=1, \dots, n)$, so that the risk evaluation and decision matrix is $D = (x_{ij}^k)_{m \times n \times l}$. Then, the steps of this method are as follows:

Step 1: According to definition 1, convert the risk information matrix $D = (x_{ij}^k)_{m \times n \times l}$ described qualitatively into two-tuple linguistic decision matrix $D_{2-T} = (x_{ij}^k, 0)_{m \times n \times l}$.

Step 2: Determine the weights of experts.

1) According to definitions 2 and 3, using the following formula to calculate the average opinion of the group.

$$(\bar{x}_{ij}, a_{ij}) = \Delta^{-1} \left(\sum_{k=1}^l \Delta(x_{ij}^k, 0) / l \right) \tag{7}$$

In it, $a_{ij} \in [-0.5, 0.5]$. Then, calculate the similarity degree $S_x \left((x_{ij}^k, 0), (\bar{x}_{ij}, a_{ij}) \right)$ of each scheme between the expert e_k and the group \bar{e} based on definition 3 and formula (5):

$$S_x \left((x_{ij}^k, 0), (\bar{x}_{ij}, a_{ij}) \right) = \frac{\xi \left((x_{ij}^k, 0), (\bar{x}_{ij}, a_{ij}) \right)}{\sum_{k=1}^l \xi \left((x_{ij}^k, 0), (\bar{x}_{ij}, a_{ij}) \right)} \tag{8}$$

And the distance value of the grey correlation coefficient $\xi(0)$ is $D \left((x_{ij}^k, 0), (\bar{x}_{ij}, a_{ij}) \right) = \left| \Delta^{-1} (x_{ij}^k, 0) - \Delta^{-1} (\bar{x}_{ij}, a_{ij}) \right|$. the weight w_{e_k}' of e_k can be

$$w_{e_k}' = \frac{\sum_{i=1}^m \sum_{j=1}^n S_x \left((x_{ij}^k, 0), (\bar{x}_{ij}, a_{ij}) \right)}{\sum_{k=1}^l \sum_{i=1}^m \sum_{j=1}^n S_x \left((x_{ij}^k, 0), (\bar{x}_{ij}, a_{ij}) \right)}, 1 \leq k \leq l \tag{9}$$

2) According to definitions 2 and 3, for every scheme, use the following formulas to calculate the assessment result of each expert (y_i^k, b_i) and the result of the group (z_i, c_i) .

$$(y_i^k, b_i) = \Delta \left(\sum_{j=1}^n \Delta^{-1} (x_{ij}^k, 0) \right) \tag{10}$$

$$(z_i, c_i) = \Delta \left(\sum_{k=1}^l w_k \sum_{j=1}^n \Delta^{-1} (x_{ij}^k, 0) \right) \tag{11}$$

In these formulas, $1 \leq k \leq l$, $b_i, c_i \in [-0.5, 0.5]$, and w_k is the weight converted from the linguistic expert weight λ_k with the formula $w_k = \Delta^{-1} (\lambda_k, 0) / \sum_{k=1}^l \Delta^{-1} (\lambda_k, 0)$. Then, calculate the deviation between each expert and the group by

$$H(e_k, e) = \sum_{i=1}^m \left(\Delta^{-1} (y_i^k, b_i) - \Delta^{-1} (z_i, c_i) \right)^2 \tag{12}$$

And get the weight w_{e_k}'' based on formula (9):

$$w_{e_k}'' = \frac{H(e_k, e)}{\sum_{k=1}^l H(e_k, e)}, 1 \leq k \leq l \tag{13}$$

3) Calculate the final weight w_{e_k} to consider both the similarity degree and the deviation.

Step 3: Aggregate the assessment information of each expert. For every scheme under each risk indicator, aggregate the assessment information by the formula

$$(X_{ij}, A_{ij}) = \Delta \left(\sum_{k=1}^l w_{e_k} \times \Delta^{-1} (x_{ij}^k, 0) \right) \tag{14}$$

Step 4: According to above equations to calculate the schemes' total catastrophe values.

Step 5: Sort the schemes by the principle “the larger the two-tuple linguistic total catastrophe value is, the higher the risk is, and the worse the scheme is” to obtain the best one.

IV. CASE STUDY

This paper studies risk assessment and decision-making of power companies based on the actual science and technology project of State Grid Corp of China, and refers to the index system in references [1], which built is shown in Table II.

TABLE II. THE RISK ASSESSMENT INDEX SYSTEM

1 st level	2 nd level	3 rd level
The risk of power supply company	Economic risks (R ₁)	Regional power demand risk (R ₁₁)
		Classified power price risk (R ₁₂)
	Policy risks (R ₂)	Peak and valley time-of-use power price policy risk (R ₂₁)
		Interruptible power price policy risk (R ₂₂)
		Local investment attraction policy risk (R ₂₃)
		Large consumers direct-supplying policy risk (R ₂₄)
	Trading risks (R ₃)	Purchase power price risk (R ₃₁)
		Power charges collection risk (R ₃₂)
	Internal management risks (R ₄)	Grid construction risk (R ₄₁)
		Power supply cost control risk (R ₄₂)
		Technology and equipment selection risk (R ₄₃)
	External competition risks (R ₅)	Captive power plant risk (R ₅₁)
		Distribution and retail separation risk (R ₅₂)
		Substitute risk (R ₅₃)

$A = \{A_1, A_2, A_3, A_4\}$ contains 4 power supply companies, and 3 senior experts are invited to assess these risks by reading documentation and communicating with staff. The risk assessment set using qualitative linguistic variables is:

$$S = \{s_0 = VL(\text{Very Low}), s_1 = L(\text{Low}), s_2 = ML(\text{Medium Low}), s_3 = M(\text{Medium}), s_4 = MH(\text{Medium High}), s_5 = H(\text{High}), s_6 = VH(\text{Very High})\}$$

The assessment result is shown in Table III. According to the knowledge accumulation and experience of experts, use qualitative linguistic to determine the experts weights:

$$W_S = \{w_1 = VH(s_6), w_2 = H(s_5), w_3 = M(s_3)\}$$

And based on the proposed method, these companies can be ranked.

According to the above steps, convert the linguistic assessment matrix in Table III into two-tuple linguistic decision matrix at first, and then calculate the similarity degree and the deviation with the corresponding weights, as shown in Table IV. Convert the weights of experts $W_S = \{s_6, s_5, s_3\}$ into real numbers $W_S = \{0.428, 0.357, 0.215\}$ by the inverse function of two-tuple linguistic. And when the eclectic preference coefficient $\alpha = \{0, 0.25, 0.5, 0.75, 1\}$, the final experts weights are shown in Table V.

TABLE III. RISK ASSESSMENT RESULT OF EXPERTS

3 rd level	Expert E ₁				Expert E ₂				Expert E ₃			
	A ₁	A ₂	A ₃	A ₄	A ₁	A ₂	A ₃	A ₄	A ₁	A ₂	A ₃	A ₄
R ₁₁	s ₅	s ₄	s ₆	s ₄	s ₅	s ₆	s ₆	s ₅	s ₆	s ₅	s ₄	s ₆
R ₁₂	s ₅	s ₅	s ₃	/	s ₄	s ₅	s ₃	s ₄	s ₅	s ₄	s ₄	s ₃
R ₁₃	s ₁	s ₀	s ₃	s ₂	s ₂	s ₀	s ₁	s ₂	s ₃	s ₂	s ₁	s ₀
R ₂₂	s ₂	s ₂	s ₂	s ₂	s ₀	s ₂	s ₂	s ₂	s ₃	s ₁	s ₁	s ₂
R ₂₃	s ₄	s ₂	s ₃	s ₃	s ₁	s ₂	s ₃	s ₄	s ₃	s ₃	s ₃	s ₂
R ₂₄	s ₃	s ₃	s ₃	s ₂	s ₂	s ₂	s ₁	s ₃	s ₂	s ₂	s ₃	s ₁
R ₃₁	s ₄	s ₄	s ₃	s ₄	s ₅	s ₆	s ₅	s ₄	s ₄	s ₅	s ₃	s ₄
R ₃₂	s ₄	s ₃	s ₃	s ₄	s ₅	s ₄	s ₃	s ₄	s ₄	s ₄	s ₂	s ₂
R ₄₁	s ₂	s ₁	s ₃	s ₂	s ₁	s ₂	s ₁	s ₁	s ₂	s ₁	s ₀	s ₃
R ₄₂	s ₁	s ₀	s ₁	s ₂	s ₂	s ₀	s ₂	s ₀	s ₁	s ₁	s ₀	s ₃
R ₄₃	s ₁	s ₀	s ₀	s ₂	s ₀	s ₀	s ₂	s ₀	s ₀	s ₁	s ₁	s ₀
R ₅₁	s ₆	s ₅	s ₄	s ₅	s ₄	s ₅	s ₄	s ₃	s ₆	s ₅	s ₅	s ₅
R ₅₂	s ₄	s ₅	s ₅	s ₅	s ₃	s ₅	s ₄	s ₄	s ₃	s ₄	s ₃	s ₄
R ₅₃	s ₃	s ₄	s ₃	s ₃	s ₃	s ₄	s ₄	s ₅	s ₃	s ₄	s ₃	s ₄

TABLE IV. THE EXPERTS WEIGHTS OF SIMILARITY AND DEVIATION

Expert	E ₁	E ₂	E ₃
Similarity degree	37.447	35.9687	33.4664
w'_{e_k}	0.3504	0.3365	0.3131
Deviation value	27.1674	35.7014	86.6634
w''_{e_k}	0.1817	0.2388	0.5796

TABLE V. THE FINAL WEIGHTS OF EXPERTS

α	Experts' final weights		
	E ₁	E ₂	E ₃
0	0.1817	0.2388	0.5796
0.25	0.2239	0.2631	0.513
0.5	0.266	0.2876	0.4464
0.75	0.3082	0.3121	0.3797
1	0.3504	0.3365	0.3131

It can be seen from Table IV and Table V that the assessment information of the expert E_1 is most similar to the group minds with the value 37.447. So when $\alpha = 1$, his weight $w_{e_1} = 0.3504$ is the largest. And the largest deviation with the group belongs to the expert E_3 with the value 86.663. So when $\alpha = 0$, his weight $w_{e_3} = 0.5796$ is the largest. In order to give consideration to both methods, this paper chooses $\alpha = 0.5$, i.e. $w_{e_k} = \{0.266, 0.2876, 0.4464\}$.

According to Step 3, the aggregation result of experts' assessment information is shown in Table VI. Then, follow Step 4 to determine the catastrophe type of the 2nd level index. Economic risks and trading risks with 2 sub-indexes belong to cusp catastrophe model, while internal management risks and external competition risks are swallowtail catastrophe models, and the ascription of policy risks is butterfly catastrophe model. Normalize these risk indexes, and attention must be paid to the importance ranking of their control variables. In the index system mentioned above, the 3rd level risk indexes are all sorted by importance except indexes of policy risks. So, taking it as an example, its two-tuple linguistic catastrophe values after normalization are shown in Table VII.

TABLE VI. THE AGGREGATION RESULT OF ASSESSMENT INFORMATION

Index	A ₁	A ₂	A ₃	A ₄
R ₁₁	(s ₅ , 0.446)	(s ₅ , 0.021)	(s ₅ , 0.107)	(s ₅ , 0.18)
R ₁₂	(s ₅ , -0.288)	(s ₅ , -0.447)	(s ₃ , 0.446)	(s ₄ , -0.447)
R ₂₁	(s ₂ , 0.18)	(s ₁ , -0.107)	(s ₂ , -0.468)	(s ₁ , 0.107)
R ₂₂	(s ₂ , -0.129)	(s ₂ , -0.446)	(s ₂ , -0.446)	(s ₂ , 0)
R ₂₃	(s ₃ , -0.309)	(s ₂ , 0.446)	(s ₃ , 0)	(s ₃ , -0.159)
R ₂₄	(s ₂ , 0.266)	(s ₂ , 0.266)	(s ₂ , 0.424)	(s ₂ , -0.159)
R ₃₁	(s ₄ , 0.287)	(s ₅ , 0.021)	(s ₄ , -0.425)	(s ₄ , 0)
R ₃₂	(s ₄ , 0.287)	(s ₄ , -0.267)	(s ₃ , -0.447)	(s ₃ , 0.107)
R ₄₁	(s ₃ , -0.288)	(s ₁ , 0.288)	(s ₁ , 0.086)	(s ₂ , 0.159)
R ₄₂	(s ₁ , 0.288)	(s ₀ , 0.446)	(s ₁ , -0.159)	(s ₂ , -0.129)
R ₄₃	(s ₀ , 0.266)	(s ₀ , 0.446)	(s ₁ , 0.021)	(s ₁ , -0.468)
R ₅₁	(s ₅ , 0.424)	(s ₅ , 0)	(s ₄ , -0.447)	(s ₄ , 0.424)
R ₅₂	(s ₃ , 0.266)	(s ₅ , -0.447)	(s ₄ , -0.181)	(s ₄ , 0.266)
R ₅₃	(s ₃ , 0)	(s ₄ , 0)	(s ₃ , 0.287)	(s ₄ , 0.021)

TABLE VII. THE CATASTROPHE VALUES OF POLICY RISKS AFTER NORMALIZATION

Index	Policy risks			
	R ₂₃	R ₂₄	R ₂₁	R ₂₂
Normalization formula	$x_a = \sqrt{a}$	$x_b = \sqrt[3]{b}$	$x_c = \sqrt[4]{c}$	$x_d = \sqrt[5]{d}$
A ₁	(s ₂ , -0.36)	(s ₁ , 0.391)	(s ₁ , 0.281)	(s ₁ , 0.219)
A ₂	(s ₂ , -0.436)	(s ₁ , 0.347)	(s ₁ , 0.251)	(s ₁ , 0.196)
A ₃	(s ₂ , -0.268)	(s ₁ , 0.442)	(s ₁ , 0.316)	(s ₁ , 0.246)
A ₄	(s ₂ , -0.314)	(s ₁ , 0.416)	(s ₁ , 0.298)	(s ₁ , 0.232)

In the process of recursive operation for these two-tuple linguistic values of the 3rd level indexes, economic risks follow the complementation principle, while others follow the non-complementation one. Then the catastrophe values of the 2nd level indexes of each scheme and their normalized values can be obtained. And the importance ranking of them is $R_4 \prec R_2 \prec R_3 \prec R_5 \prec R_1$, as shown in Table VIII.

Let the 2nd level indexes follow the complementation principle and obtain the two-tuple linguistic total catastrophe values of all schemes:

$A_1 = (s_1, 0.166)$, $A_2 = (s_1, 0.158)$, $A_3 = (s_1, 0.144)$, $A_4 = (s_1, 0.158)$. According to the principle in Step 5, the ranking result of these 4 companies is $A_1 \prec A_4 = A_2 \prec A_3$, and the risk of A_3 is minimal.

In addition, the corresponding results under other 4 eclectic preference coefficients are displayed in Table IX to show the impact of the coefficient and verify the reliability of the ranking above.

TABLE VIII. THE CATASTROPHE VALUE OF THE SECOND-LEVEL INDICATORS AFTER NORMALIZATION

Index	R ₁	R ₅	R ₃	R ₂	R ₄
Normalization formula	$x_a = \sqrt{a}$	$x_b = \sqrt[3]{b}$	$x_c = \sqrt[4]{c}$	$x_d = \sqrt[5]{d}$	$x_e = \sqrt[6]{e}$
A₁	(s ₂ , 0.046)	(s ₁ , 0.219)	(s ₂ , -0.376)	(s ₁ , 0.144)	(s ₂ , -0.243)
A₂	(s ₂ , -0.024)	(s ₁ , 0.196)	(s ₂ , -0.288)	(s ₁ , 0.065)	(s ₂ , -0.291)
A₃	(s ₂ , -0.01)	(s ₁ , 0.246)	(s ₂ , -0.471)	(s ₁ , 0.021)	(s ₂ , -0.474)
A₄	(s ₂ , 0.003)	(s ₁ , 0.232)	(s ₂ , -0.413)	(s ₁ , 0.212)	(s ₂ , -0.359)

TABLE IX. THE CATASTROPHE VALUE OF THE SECOND-LEVEL INDICATORS AFTER NORMALIZATION

	A₁	A₂	A₃	A₄	Ranking result
$\alpha=0$	(s ₁ , 0.168)	(s ₁ , 0.159)	(s ₁ , 0.135)	(s ₁ , 0.162)	$A_1 \prec A_4 \prec A_2 \prec A_3$
$\alpha=0.25$	(s ₁ , 0.167)	(s ₁ , 0.159)	(s ₁ , 0.140)	(s ₁ , 0.160)	$A_1 \prec A_4 \prec A_2 \prec A_3$
$\alpha=0.75$	(s ₁ , 0.165)	(s ₁ , 0.158)	(s ₁ , 0.148)	(s ₁ , 0.157)	$A_1 \prec A_2 \prec A_4 \prec A_3$
$\alpha=1$	(s ₁ , 0.164)	(s ₁ , 0.158)	(s ₁ , 0.151)	(s ₁ , 0.155)	$A_1 \prec A_2 \prec A_4 \prec A_3$

Table IX shows the ranking result changes with the coefficient α . A mean opinion oriented risk aversion decision maker prefers α close to 1, while a risk appetite decision maker is opposite. Thus, α should be determined by actual situation. Besides, the risk of the company A_3 is always minimal, which confirms the reliability of the result.

V. CONCLUSIONS

This paper studied by the combination of catastrophe theory and two-tuple linguistic to get rid of the limitation of proper weight determination. It expands the application of catastrophe theory from real numbers to uncertain and fuzzy linguistic variables, which provides a new way for the theory.

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