Super Resolution Sparse Reconstruction with a High and Low Resolution Dictionary for Single Remote Sensing Image

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***Abstract* — Super resolution remote sensing image is important for the improvement of the target recognition rate, but there is a large demand of training sample in the super resolution reconstruction process, which result in the problem of low computing efficiency and poor reconstruction resolution, so here proposed Super resolution sparse reconstruction with the high and low resolution dictionary for single remote sensing image. Firstly, the training set of samples with high and low resolution is obtained from the high resolution remote sensing image based on the existing high resolution remote sensing image; Secondly, here designed the sparse dictionary joint training method, and deal with the high and low resolution feature of the sample training set, which could get the dictionary with high and low resolution; Finally, the high resolution remote sensing image is reconstructed based on the acquired high and low resolution dictionary, and then the complexity of the algorithm is analyzed. The simulation results show that the algorithm has less requirements on the dictionary training sample, and can achieve a good result.**

Keywords - high and low resolution dictionary; sparse representation; super resolution; remote sensing image; reconstruction

I. INTRODUCTION

The shooting quality of digital image is determined by the performance of imaging device on one hand, on the other hand, it is more sensitive to the shooting condition, especially in the collection of remote sensing image, the resolution of image is not high generally, which has caused the blurring remote sensing image. However, more precise detection effect can be attained from remote sensing image with higher resolution and can improve the practical application value. For example, in remote sensing image, relatively small targets in images can be identified based on images with high resolution. Therefore, study of how to improve the resolution and definition of remote sensing image is with very practical value [1~2].

Most of former super resolution reconstructions of remote sensing image adopted similarity index to construction, for example, in literature [3-4], the writer made use of global similarity property of image to construct the super resolution algorithm for image; in literature [5-6], the writer adopted the similar local properties to make super resolution algorithm design for image. Besides, in literature [7-8], the writer constructed super resolution algorithm for image based on image sparsity and global similarity property of image. In addition, there is also study of reconstruction of remote sensing image with adoption of advanced interpolation, but this algorithm needs to clarify the reasons for decreasing of image quality first and the calculation for this algorithm is huge.

At present, the main study directions for super resolution algorithm of remote sensing image include: way of example [9], neighborhood embedding method [10] and sparse dictionary method [11] etc. The basic design ideas is: study training is made based on priori knowledge to attain the relationship between remote sensing images with high and low resolution respectively, and then realize the reconstruction of remote sensing image with low resolution input based on study model. In literature [9], the writer has adopted example image reconstruction method, which requested extremely big training set of examples and caused high complexity of reconstruction process. In literature [10], the writer has adopted neighborhood embedding method, which required selecting training sample manually during operation and caused excessive and lacking fitting case easily. At present, the reconstruction method of remote sensing image with adoption of sparse learning has gained more study and attention, which is caused by its good reconstruction performance and universal applicability. In literature [11], the writer adopted sparse image reconstruction algorithm, which has problem of large amount of calculation and can't ensure the constancy of sparse coefficient expressed by high and low resolution image dictionary. On the basis of this, literature [12-13] designed dictionary joint learning and dictionary coupled learning to attain high and low resolution image dictionary and improve algorithm performance. However in literature [14-15], the writer weakened the hypothesis of sparse coefficient constancy existed in high and low resolution image dictionary, which is helpful to improve the stability of the reconstruction process. At present, deep research has been made on reconstruction of sparse super resolution image, but there are two shortages: (1) the dictionary learning is with great freedom, which has caused great demand of training samples for high and low resolution image dictionary; (2) the more complicated calculation in dictionary training needs to be improved.

This will be greatly restricted in engineering application. The first is in engineering application, shooting of large number of samples material requires a lot of time and cost; the second is the algorithm with high complexity can't meet the demand of real-time. It proposes super resolution reconstruction algorithm of high and low resolution image

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dictionary learning for the shortages existed in the current reconstruction process of super resolution image and by making use of characteristics of low freedom degree and high efficiency of sparse dictionary and combining the design ideas of dictionary joint training [12]. Its basic design idea is to attain different high and low resolution dictionaries expressed by sparse based on the joint training of sparse dictionary, based on which to realize the high resolution reconstruction of remote sensing image. Compared with the existing algorithms, the proposed algorithm has advantages of less demand of samples and high-efficient calculation, at the same time; it can improve the resolution quality of remote sensing image effectively.

II. PROBLEM DESCRIPTION

To solve the problems existed in the reconstruction process of super resolution for remote sensing image, firstly, it needs to analyze the understand the shooting process of remote sensing image. Picture 1 is the general shooting process for remote sensing image with low resolution and the nature of reconstruction process of super resolution is the inverse operation of this shooting process.

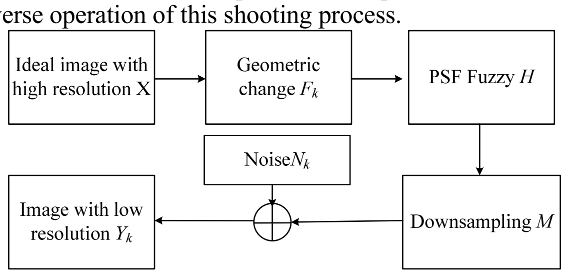


Figure 1. Super resolution reconstruction model

The above reconstruction model can be described based on the following mathematical form:

$$Y_k = DHF_k X + N_k \tag{1}$$

Parameters in above model: k is the number sequence of remote sensing image and the form is $k = 1, 2, \cdots, K$; K is the total number of remote sensing images of low resolution with the same target; X is the remote sensing image with high resolution that meets the requirements; Y_k is the shot remote sensing image with low resolution and the number of this image is k; F_k is the corresponding matrix transformation form of remote sensing image with high resolution for remote sensing image with low resolution and its number is k; N_k is shot noise disturbance; M is downsampling image matrix; H is fuzzy matrix.

In the reconstruction process of remote sensing image with high resolution, if the size of remote sensing image with low resolution Y_k is $N_1 N_2 \times 1$, set the reconstruction coefficient of algorithm as P, and then the size of remote sensing image with high resolution X attained by reconstruction is $PN_1 PN_2 \times 1$. To solve this problem, the nature is similar to solve one ill-posed problem. To make benign treatment for it, it needs to preset priori limiting factor. If there is only 1 input of remote sensing image and the reconstruction coefficient $P > 1$, and then the matrix geometric transformation form of remote sensing image does not exist. The remote sensing image construction process described in formula (1) transformed into the restoration problem of remote sensing image under the condition of $K = 1$, $P = 1$.

In the reconstruction operation of inverse process of remote sensing image, the above parameters need to preset fuzzy matrix H as well as downsampling image matrix M, but the setting precision of these two parameters will affect the reconstruction of remote sensing image and affect the stability of reconstruction process. Therefore, make matching improvement for above parameters based on the learning form of dictionary D, in this way, there is no need to preset parameters and realize the unsupervised learning of parameters.

III. DESCRIPTION OF SUPER RESOLUTION IMAGE ALGORITHM

A. Learning of Sparse Dictionary

For pre-given image $\mathbf{y} \in R^n$ and redundant dictionary whose vector form is $\mathbf{D}=(\mathbf{d}_1,\cdots,\mathbf{d}_m) \in R^{n\times m}$ $(n < m)$, sparse representation problem can be understood as solving sparse form $\mathbf{x}_k$ in given redundant dictionary for vector y and the representation process of this coefficient can be described as:

$$\begin{cases} \min\limits_{\mathbf{x}} & \|\mathbf{y}-\mathbf{D}\mathbf{x}\|_2^2 \\ s.t. & \|\mathbf{x}\|_0 \le T_0 \end{cases} \tag{2}$$

In the formula, T_0 is the sparse representation level of algorithm. There are many solving methods for this problem, such as BPDN algorithm, OMP algorithm and SL0 algorithm etc.

For image sample $\mathbf{Y}=(\mathbf{y}_1,\cdots,\mathbf{y}_K) \in R^{n\times k}$, the main objective of making use of dictionary training is to find suitable redundant dictionary $\mathbf{D} \in R^{n\times m}$ to ensure that sample image can be expressed into sparse matrix form based on this, which can be expressed as $\mathbf{X}=(\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_K) \in R^{m\times K}$, and then the dictionary training process is:

$$\begin{cases} \min\limits_{\mathbf{D},\mathbf{X}} & \|\mathbf{Y}-\mathbf{D}\mathbf{X}\|_F^2 \\ s.t. & \|\mathbf{x}_k\|_0 \le T_0 \quad \forall k \end{cases} \tag{3}$$

The solving of above dictionary training process can be based on MOD method, K-SVD method [20] or online self-learning method [21] etc.

B. Pretreatment and Learning Process

Select high resolution remote sensing image $\{\mathbf{y}_h^j\}, (j = 1,\cdots, J)$ from sample image database, attain low resolution image set of remote sensing image by fuzzy and downsampling mode and then reconstruct $\{\mathbf{z}_l^j\}$ into interpolated low resolution image $\{\mathbf{y}_l^j\}$ with the same size as high resolution image based on cubic interpolation processes. These three differences is a kind of piecewise interpolation, compared with cubic spline interpolation and other modes, interpolation polynomial value at node equals to the value of interpolation function at node, while cubic spline interpolation also need to know the derivative at interpolation polynomial at some nodes, which is difficult to realize in practical application. The process of cubic interpolation is as following:

$$H(x)=\alpha_j(x)f_j+\alpha_{j+1}(x)f_{j+1}+\beta_j(x)f_j'+\beta_{j+1}(x)f_{j+1}' \quad (4)$$

In which the basis function value of cubic interpolation is:

$$\alpha_j(x)=(\frac{x-x_{j+1}}{x_j-x_{j+1}})^2(1+2\frac{x-x_j}{x_{j+1}-x_j})$$

$$\alpha_{j+l}(x)=(\frac{x-x_{j+1}}{x_{j+1}-x_j})^2(1+2\frac{x-x_{j+1}}{x_j-x_{j+1}})$$

$$\beta_j(x)=\left(\frac{x-x_{j+1}}{x_j-x_{j+1}}\right)^2(x-x_j)$$

$$\beta_{j+1}(x)=\left(\frac{x-x_j}{x_{j+1}-x_j}\right)^2(x-x_{j+1})$$

The pretreatment process needs to make filtration for low frequency information in remote sensing image for high resolution image and to ensure the dictionary to make full expression for texture, edge and other information of remote sensing image, for which can adopt image difference record for treatment: $e_h^j=\mathbf{y}_h^j-\mathbf{y}_l^j$. For low resolution remote sensing image, the pretreatment process needs to process its high frequency characteristics, here adopt S group high pass filter to realize the extraction of high frequency characteristics, which can be expressed as $\mathbf{f}_s\otimes\mathbf{y}_l^j$. Through the said pretreatment process, high and low resolution remote sensing image can be decomposed into remote sensing image block with overlapping dimensions of $\sqrt{n}\times\sqrt{n}$, and then attain K remote sensing image block randomly to construct high and low resolution image sample $\{\mathbf{p}_k^h,\mathbf{p}_k^l\}\quad(k=1,\cdots,K)$.

Following form description can be attained based on image sample $\{\mathbf{p}_k^h,\mathbf{p}_k^l\}$ and attained training dictionary $\{\mathbf{A}^h,\mathbf{A}^l\}$ with combination of pretreatment process: $\mathbf{p}_k^h\in R^{n\times1}$, $\mathbf{p}_k^l\in R^{nS\times1}$, first given form $\mathbf{P}^h=\{\mathbf{p}_1^h,\mathbf{p}_2^h\cdots\mathbf{p}_k^h\}$, $\mathbf{P}^l=\{\mathbf{p}_1^l,\mathbf{p}_2^l\cdots\mathbf{p}_k^l\}$, and then the high resolution image dictionary training process is:

$$\begin{cases}\min\limits_{\mathbf{A}_h,\mathbf{Q}} & \left\|\mathbf{P}^h-\mathbf{A}_h\mathbf{Q}\right\|_F^2\\ s.t. & \left\|\mathbf{q}_k\right\|_0\le T_0\quad\forall k\end{cases} \quad (5)$$

Similarly, the low resolution image dictionary training process is:

$$\begin{cases}\min\limits_{\mathbf{A}_l,\mathbf{Q}} & \left\|\mathbf{P}^l-\mathbf{A}_l\mathbf{Q}\right\|_F^2\\ s.t. & \left\|\mathbf{q}_k\right\|_0\le T_0\quad\forall k\end{cases} \quad (6)$$

Combining (4-5) dictionary training process and adopting $\frac{1}{n}$ and $\frac{1}{nS}$ as dictionary training weight, and then the high and low resolution image dictionary training process can be expressed as:

$$\begin{cases}\min\limits_{\mathbf{A}_l,\mathbf{A}_h,\mathbf{x}} \frac{1}{n}\left\|\mathbf{P}^h-\mathbf{A}_h\mathbf{Q}\right\|_F^2+\frac{1}{nS}\left\|\mathbf{P}^l-\mathbf{A}_l\mathbf{Q}\right\|_F^2\\ s.t.\ \left\|\mathbf{q}_k\right\|_0\le T_0\quad\forall k\end{cases}$$

The above formula can be simplified as:

$$\begin{cases}\min\limits_{\mathbf{A},\mathbf{x}} & \left\|\mathbf{P}-\mathbf{A}\mathbf{Q}\right\|_F^2\\ s.t. & \left\|\mathbf{q}_k\right\|_0\le T_0\quad\forall k\end{cases} \quad (7)$$

Parameters in the formula:

$$\mathbf{P}=\begin{bmatrix}\frac{1}{\sqrt{n}}\mathbf{P}^h\\ \frac{1}{\sqrt{nS}}\mathbf{P}^l\end{bmatrix},\quad \mathbf{A}=\begin{bmatrix}\frac{1}{\sqrt{n}}\mathbf{A}^h\\ \frac{1}{\sqrt{nS}}\mathbf{A}^l\end{bmatrix} \quad (8)$$

Making direct solution for formula (8) will cause extremely high freedom of dictionary A solution as well as the increase demand of image sample quantity for dictionary A, which will increase the calculation amount for image reconstruction. To decrease the freedom of dictionary A training process, the adopted method is to make sparse expression for dictionary $\mathbf{A}\in R^{(S+1)n\times m}$. If $\mathbf{A}=\mathbf{A}_0\mathbf{Z}$, $\mathbf{Z}\in R^{m_0\times m}$ represents sparse matrix, in which $\mathbf{A}_0\in R^{(S+1)n\times m_0}$ is the base dictionary and the quantity of non-zero of its matrix column element needs to less than $\mathbf{T}_1$, that is to presume that all the atoms of dictionary can be selected as sparse representation of $\mathbf{A}_0$. Based on above assumption, the dictionary training process described in formula (7) can be expressed as:

$$\begin{cases}\min\limits_{\mathbf{Z},\mathbf{Q}} & \left\|\mathbf{P}-\mathbf{A}_0\mathbf{Z}\mathbf{Q}\right\|_F^2\\ s.t. & \begin{pmatrix}\left\|\mathbf{q}_k\right\|_0\le T_0\ \forall k\\ \left\|\mathbf{z}_j\right\|_0\le T_1\ \forall j\end{pmatrix}\end{cases} \quad (9)$$

In the formula, $\mathbf{z}_j$ matches j Column element of matrix $\mathbf{Z}$ and then the optimized solving process of problem (9) can be made by two steps: (1) fix sparse matrix $\mathbf{Z}$ and update the sparse representation of coefficient matrix $\mathbf{Q}$; (2) fix sparse matrix $\mathbf{Q}$ and update sparse matrix $\mathbf{Z}$.

First, fix sparse matrix $\mathbf{Z}$ and then the optimization process of problem (9) is:

$$\forall k \quad \begin{cases} \min\limits_{\mathbf{q}_k} & \left\|\mathbf{p}_k - \mathbf{A}_0\mathbf{Z}\mathbf{q}_k\right\|_2^2 \\ s.t. & \left\|\mathbf{q}_k\right\|_0 \le T_0 \end{cases} \tag{10}$$

Select OMP method to solve optimization problem of formula (10).

Secondly, fix sparse coefficient $\mathbf{Q}$ and then the optimization process of problem (9) is:

$$\begin{cases} \min\limits_{\mathbf{Z}} & \left\|\mathbf{P} - \mathbf{A}_0\mathbf{Z}\mathbf{Q}\right\|_F^2 \\ s.t. & \left\|\mathbf{z}_j\right\|_0 \le T_1 \ \forall j \end{cases} . \tag{11}$$

Make $\tilde{\mathbf{q}}_j$ as sparse matrix Q element of No. j Column, then

$$\left\|\mathbf{P} - \mathbf{A}_0\mathbf{Z}\mathbf{Q}\right\|_F^2 = \left\|\mathbf{P} - \mathbf{A}_0\sum_{j=1}^{m}\mathbf{z}_j\tilde{\mathbf{q}}_j\right\|_F^2 = \left\|\mathbf{E}_j - \mathbf{A}_0\mathbf{z}_j\tilde{\mathbf{q}}_j\right\|_F^2 \tag{12}$$

In the formula, $\mathbf{E}_j = \mathbf{P} - \mathbf{A}_0\sum_{k\neq j}\mathbf{z}_k\tilde{\mathbf{q}}_k$ the optimization update process of problem (11) can be expressed as

$$\begin{cases} \min\limits_{\mathbf{z}_j} & \left\|E_j - \mathbf{A}_0\mathbf{z}_j\tilde{\mathbf{q}}_j\right\|_F^2 \\ s.t. & \left\|\mathbf{z}_j\right\|_0 \le T_1 \end{cases} \tag{13}$$

If meet $\tilde{\mathbf{q}}_j\tilde{\mathbf{q}}_j^T = 1$, and then:

$$\begin{aligned} \left\|\mathbf{E}_j - \mathbf{A}_0\mathbf{z}_j\tilde{\mathbf{q}}_j\right\|_F^2 &= Tr\left(\left(\mathbf{E}_j - \mathbf{A}_0\mathbf{z}_j\tilde{\mathbf{q}}_j\right)\left(\mathbf{E}_j - \mathbf{A}_0\mathbf{z}_j\tilde{\mathbf{q}}_j\right)\right) \\ &= Tr(\mathbf{E}_j^T\mathbf{E}_j) - 2Tr(\tilde{\mathbf{q}}_j\mathbf{E}_j^T\mathbf{A}_0\mathbf{z}_j) + Tr(\tilde{\mathbf{q}}_j\tilde{\mathbf{q}}_j^T\mathbf{z}_j^T\mathbf{A}_0^T\mathbf{A}_0\mathbf{z}_j) \\ &= Tr(\mathbf{E}_j^T\mathbf{E}_j) - 2\tilde{\mathbf{q}}_j\mathbf{E}_j^T\mathbf{A}_0\mathbf{z}_j + \mathbf{z}_j^T\mathbf{A}_0^T\mathbf{A}_0\mathbf{z}_j \\ &= \left\|\mathbf{E}_j\tilde{\mathbf{q}}_j - \mathbf{A}_0\mathbf{z}_j\right\|_2^2 + Tr(\mathbf{E}_j^T\mathbf{E}_j) - \tilde{\mathbf{q}}_j\mathbf{E}_j^T\mathbf{E}_j\tilde{\mathbf{q}}_j^T \end{aligned}$$

In the formula, $Tr(\bullet)$ is the trace expression of matrix. Because the $Tr(\mathbf{E}_j^T\mathbf{E}_j) - \tilde{\mathbf{q}}_j\mathbf{E}_j^T\mathbf{E}_j\tilde{\mathbf{q}}_j^T$ value is irrelevant to element $\mathbf{z}_j$, and then under the situation of meeting $\tilde{\mathbf{q}}_j\tilde{\mathbf{q}}_j^T = 1$, the optimization process of problem (13) can be revised as:

$$\begin{cases} \min\limits_{\mathbf{z}_j} & \left\|E_j\tilde{\mathbf{q}}_j - \mathbf{A}_0\mathbf{z}_j\right\|_F^2 \\ s.t. & \left\|\mathbf{z}_j\right\|_0 \le T_1 \end{cases} \tag{14}$$

Formula (14) is sparse coding form and can be solved based on OMP method. Sparse matrix Z can be attained based on above two steps and then attain high and low resolution image dictionary $\left\{\mathbf{A}^h, \mathbf{A}^l\right\}$.

C. High Resolution Reconstruction

Reconstruct high resolution image $\mathbf{X}$ based on low resolution remote sensing image input $\mathbf{Y}$ and with adoption of high and low resolution image dictionary $\left\{\mathbf{A}^h, \mathbf{A}^l\right\}$. First, restore input $\mathbf{Y}$ into low resolution image $\mathbf{Y}_l$ with the same size of high resolution image by making use of cubic interpolation process. And then extract image high frequency characteristics with adoption of S group high pass filter, which can be expressed as $\mathbf{f}_s \otimes \mathbf{Y}_l$, and then decompose $\mathbf{Y}_l$ into overlapping image block with dimensions of $\sqrt{n}\times\sqrt{n}$, attain corresponding high frequency characteristics $\{\mathbf{y}_k\}$ and calculate its sparse coefficient as following:

$$\begin{cases} \min\limits_{\boldsymbol{\alpha}_k} & \left\|\mathbf{y}_k - \mathbf{A}_l\boldsymbol{\alpha}_k\right\|_2^2 \\ s.t. & \left\|\boldsymbol{\alpha}_k\right\|_0 \le T_0 \end{cases} \tag{15}$$

Attain sparse coefficient $\{\boldsymbol{\alpha}_k\}$ based on OMP method and then attain the expression method of high resolution image block $\mathbf{x}_k = \mathbf{A}_h\boldsymbol{\alpha}_k$ with adoption of high resolution image matrix $\mathbf{A}_h$ and sparse coefficient $\{\boldsymbol{\alpha}_k\}$. Combine the attained image blocks, make mean method treatment for overlapping part and then attain the high frequency characteristics of high resolution image. In addition, introduce low resolution image $\mathbf{Y}_l$ attained from interpolation and then the form of reconstructing high resolution image is:

$$\mathbf{X} = \mathbf{Y}_l + \left(\sum_k \mathbf{R}_k^T\mathbf{R}_k\right)^{-1}\left(\sum_k \mathbf{R}_k^T\mathbf{x}_k\right) \tag{16}$$

In the formula, $\mathbf{R}_k$ represents image block characteristics extraction matrix, for example, $\mathbf{R}_k\mathbf{Y}_l$ can be expressed as the image block k extracted from image $\mathbf{Y}_l$.

IV. ANALYSIS OF ALGORITHM PROCESS

A. Algorithm Steps

Based on above analysis, the steps of proposed algorithm are:

Algorithm 1: reconstruction of super resolution image sparse dictionary joint learning

Input: base dictionary $\mathbf{A}_0$, high resolution image $\{\mathbf{y}_h^j\}$, low resolution image $\mathbf{Y}$;

Initialization: under the limitation of $\|\mathbf{z}_j\|_0 \le T_1$, select sparse matrix $\mathbf{Z}$ at random;

Step 1:

1-1 attain high and low resolution image dictionary $\{\mathbf{A}^h, \mathbf{A}^l\}$;

1-2 pretreatment of $\{\mathbf{y}_h^j\}$ to attain high and low resolution image sample $\{\mathbf{p}_k^h, \mathbf{p}_k^l\}$;

1-3 attain sparse matrix $\mathbf{Z}$ based on iterative procedure displayed in formula (10) and (14);

1-4 solve formula (10) to attain sparse coefficient $\mathbf{q}_k$ based on OMP method;

1-5 make normalization treatment $\tilde{\mathbf{q}}_k = \tilde{\mathbf{q}}_k / \|\tilde{\mathbf{q}}_k\|_2$ for sparse coefficient $\mathbf{q}_k$;

1-6 solve formula (14) to attain sparse matrix $\mathbf{Z}$ based on OMP method;

1-7 implement steps (1-3) to (1-6) repeatedly until convergence of reconstruction process;

1-8 attain image matrix $\mathbf{A} = \mathbf{A}_0\mathbf{Z}$ based on sparse matrix $\mathbf{Z}$;

1-9 attain high and low resolution image dictionary $\{\mathbf{A}^h, \mathbf{A}^l\}$ based on formula (8);

Stpe 2: reconstruction of high resolution image $\mathbf{X}$

2-1 pretreatment of $\mathbf{Y}$ to attain high frequency characteristics $\{\mathbf{y}_k\}$;

2-2 solve formula (15) based on OMP method to attain sparse coefficient $\{\boldsymbol{\alpha}_k\}$;

2-3 attain high resolution reconstruction image $\mathbf{X}$ based on formula (16);

Output: reconstruct image $\mathbf{X}$.

Algorithm notes”

(a) $\mathbf{A}_0$ Selection: the selection of base dictionary $\mathbf{A}_0$ needs to ensure the data dictionary can be realized rapidly, such as wavelet dictionary and DCT dictionary etc.

(b) Filter f_s: make first order and second order difference extraction for low resolution image by selecting four groups of filters: $\mathbf{f}_1 = \mathbf{f}_2^T = [1,-1]$, $\mathbf{f}_3 = \mathbf{f}_4^T = [1,2,-1]$

B. Analysis of Complexity

Explore the complexity of dictionary learning in this paper by comparing with K-SVD method. First, analyze the implementation process of algorithm large circulation as well as the complexity comparison between K-SVD methods in this paper.

Solve formula (10) and (14) based on the big circulation process of OMP algorithm and the complexity of its calculation process is $O(\tilde{N}\tilde{K}^2)$, $\tilde{N}$ is the quantity of dictionary atom and $\tilde{K}$ is the quantity of non-zero elements in dictionary. It includes K group image sample and then the complexity of dictionary training process mainly include solving process of OMP for formula (10) and (14), its complexity is:

$$T_{JSDL} = O(KmT_0^2) + O(m^2T_1^2) \tag{17}$$

While the complexity of SVD method is $O((S+1)^3n^3)$, so the calculation complexity of K-SVD process is:

$$T_{K-SVD} = O(KmT_0^2) + O(m^3) \tag{18}$$

As meeting $T_1 < m$, the complexity of dictionary learning in this paper is lower than that of K-SVD algorithm that is meeting $T_{JSDL} < T_{K-SVD}$. So adopting dictionary training method can speed up the convergence of algorithm in this paper. Compared with K-SVD method process, the design of image reconstruction process in this paper can decrease the time needed for dictionary training in this paper.

V. RESULT ANALYSIS

Here verify algorithm performance based on the reconstruction process of remote sensing super resolution image and compare algorithm selecting Bicubic image reconstruction method [1] and Yang image reconstruction method [13]. Experiment 1 offers performance comparison of Yang image reconstruction method and image reconstruction method in this paper when the quantities of training image are different; experiment 2 offers experimental results of three image reconstruction methods; experiment 3 is the influence of noise on the performance of image reconstruction method; experiment 4 is the quantity of atom in base dictionary on the performance of image reconstruction method.

Select 20 high resolution remote sensing images and process to attain high and low resolution image database. Because the calculation process of algorithm involves matrix calculation, first, unify the size of sample image into 5×5 and set the pixel of neighboring overlapping image block at 3 and select 50000 sets of images randomly. Set the atom quantity of dictionary $\mathbf{A}$ as 1024, $\mathbf{A}_0$, quantity of atom as 400, that is $\mathbf{A}_0 \in R^{125\times400}$. Iteration limit is 40. Hardware equipment: dual-core CPU, RAM 4Gb ddr3-1600, set zoom ratio of image at 2, the evaluation indexes are peak signal-to-noise (PSNR) and structural similarity (SSIM).

A. Quantity of Image Sample Affects Experiment

Here attain the PSNR index mean value and algorithm implementation time of sample quantity under different situations based on making two times extension super resolution image reconstruction for selected three remote sensing images. Under the situation of noise free, presume the parameters of other algorithms are fixed, the quantity of sample image ranges between 104 and 105. Picture 2-3 lists the PSNR mean value and comparison situation of dictionary learning for 3 remote sensing images changing with the quantity of sample image selected from above three images. It can be learned from picture 2 that the quantity of PSNR index stable point in reconstruction process of this paper is around 30000, while the quantity of PSNR index stable point of Yang method in sample is around 50000. The reason lies in that based on existing high resolution remote sensing image, this paper attains training set of samples with characteristics of high and low resolution through pre-treatment, which is helpful to improve sample selection quality and then realize the ability of decreasing sample quantity. Under the setting of same conditions, it can be learned from picture 3 that the requested time for dictionary training in reconstruction process is less than that of Yang method.

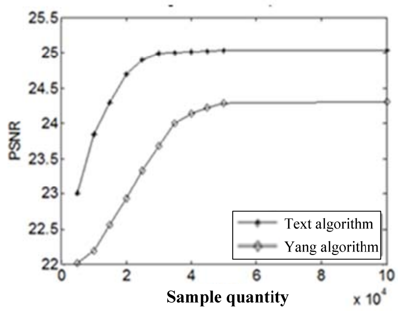


Figure 2. Comparison of average PSNR values

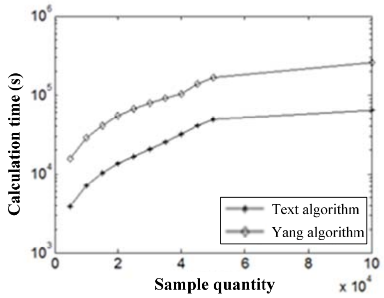
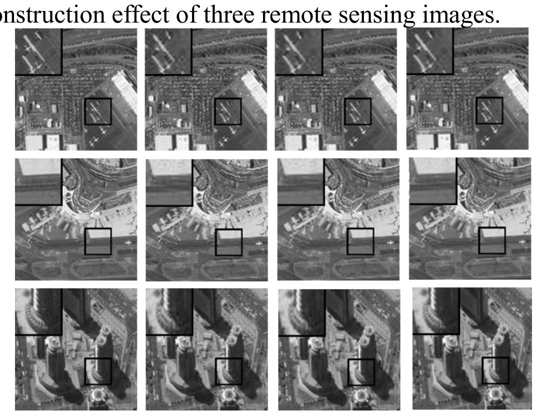


Figure 3. Time comparison

B. Comparison of Reconstruction Performance

Here compare the reconstruction effect of selected remote sensing image from the visual perspective. Under the condition of noise free, picture 4 offers the visual reconstruction effect of three remote sensing images.



(a) (b) (c) (d)

Figure 4. comparation of the three algorithms for reconstruction of visual effects (a) the original image; (b) Bicubic algorithm; (c) Yang algorithm; (d) The algorithm of this paper

There is slight difference in reconstruction part of three images as shown in picture 4. The image overlapping part of the first line locates at dark color region and with relatively low contrast ratio; while the image reconstruction part of the second line locates at transition part of dark region and light region with obvious contrast ratio; the difference lies in that there is relatively dark pixel region in reconstruction area. It can be seen from simulation picture 4 that in the reconstruction process of three different images, the image attained from proposed reconstruction algorithm is closest to original image visually.

C. Noise Interference Affects Experiment

Here mainly study the influence of mean variance change of noise on image. In table 1 simulation data, it offers PSNR and SSIM index situation of above comparison of reconstruction process under the conditions of different mean variance values. It can be learned from table 1 that the PSNR and SSIM index of proposed reconstruction process is better that of comparison reconstruction process and is with stronger performance in noise suppression.

Table 1. Comparison of PSNR and SSIM values of three algorithms

σ	Bicubic		Yang		Algorithm in this paper	
	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
5	22.28	0.55	24.15	0.68	24.87	0.74
10	21.67	0.53	23.66	0.63	24.36	0.68
20	20.27	0.41	21.93	0.52	22.66	0.54
30	18.22	0.32	20.11	0.41	20.83	0.45

D. Performance of Base Dictionary Affects Experiment

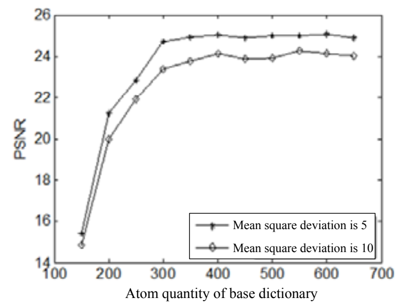


Figure 5. PSNR value changes

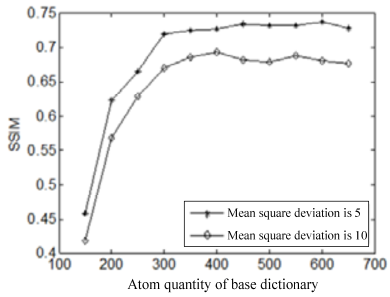


Figure 6. SSIM value changes

Here make experimental comparison for the influence of atom quantity of base dictionary on the reconstruction effect. Set the mean variance values of noise at 5 and 10 respectively to compare the influence of noise with different mean variance values on reconstruction process, while atom quantity changes between 150 and 700 to test the influence of different atom quantities on reconstruction process. Picture 5-6 offer PSNR and SSIM index change situation when adopting dictionaries with different atom quantities. It can be learned from the comparison curve of picture 5-6 that when atom quantity is higher than 400, the reconstruction effect of algorithm is tend to be saturated.

VI. CONCLUSION.

Here propose super resolution sparse reconstruction with the high and low resolution dictionary for single remote sensing image and put forward super resolution sparse reconstruction algorithm with high and low resolution dictionary learning for single remote sensing image. This reconstruction algorithm attains high and low resolution dictionary of image sample based on sparse dictionary training, make reconstruction based on its input images with low resolution to attain images with high resolution, finally, make theoretical analysis for the complexity degree of reconstruction algorithm. The simulation data shows that the reconstruction algorithm of this paper requires the least quantity of training samples and calculation time and with higher image reconstruction effect.

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