A Study on Financial Subsidies of Small Town Public Transportation

Jianzhao Peng[1], Hantao Zhang[1], Chanjie Luo[*1]

[1]*School of Civil Engineering and Mechanics,* Huazhong University of Science and Technology, Wuhan, Hubei 430074, China

***Abstract* — Urban public transportation is a special industry due to its nonprofit nature, which needs financial subsidies to support its operation. How to calculate the public transport financial subsidies in a scientific method is a very important topic. The paper forecasts the number of buses in demand, and the quantities of buses in a selected small town based on the population size in the future. Then, the utility function is used to establish the game model for residents' income, bus enterprises operating costs and government financial subsidies. Passengers are classified into different types according to their income, and given different amount of subsidies. At last, taking Jiangcheng County, Pu'er City, Yunnan Province as an example, the method is verified and applied.**

Keywords—public transport; financial subsidy; utility function; demand forecasting

Ⅰ. INTRODUCTION

Public transportation is an important support to build energy efficient and environmental friendly society. Urban public transportation is nonprofit industry and public welfare is its basic attribute. As an important part of urban infrastructure, urban public transport has the responsibility to service for the society which is the government functions in urban passenger transportation. Public transportation providing a comfortable running service must keep in low fares, in this regard, the government has given a lot of financial subsidies to support. But in practice, due to various reasons, it has not formed the subsidy measure and implementation of effective countermeasures. The losses caused by low ticket price control, social welfare, policy-based traffic lines and so on are difficult to measure reasonable and compensate. Subsidies for how to deal with the relationship between the government and the public transport enterprises, sources of public transport subsidies funding, objects of subsidies and the effect of research are still at the early stage. Therefore, to determine the scientific and reasonable calculation model of public transport subsidies and stimulate bus companies make active efforts to reduce operating costs and improve service quality, reduce the government financial burden, reduce the cost of taking by bus, achieve the government subsidies for transit enterprises maximize the utility , has very important practical significance.

In this paper, we use gray prediction theory, utility function, the tripartite game to establish bus subsidies calculation model. Taking Jiangcheng County, Pu'er City, Yunnan Province as an example, we analysis and verify the practical of the established model through a large amount of data investigation, measuring the scientific and reasonable bus subsidies.

Ⅱ. LITERATURE REVIEW

Prior research in the field of public transport financial subsidies has contributed the modeling of public transport financial subsidies problem.

S. Glaister and D. Lewis [1] developed a quantitative method which is applied to the best available evidence on peak and off-peak bus, rail and private car models in Greater London. S. Schmidt [2] made a lot of analysis based on the population, population density and bus operating miles. They considered that financial subsidies for public traffic enterprises would put incentive effect on them, but it was not taken into account when they accessed the influence of subsidies on transportation. P. Tisato [3] considered that the service of public transport industry was uncertain, which affected the user's cost. At meanwhile, cost was the key factor of optimizing the financial subsidies of public transportation industry. Therefore, the amount of subsidies calculation should base on the quality of service. N. Fearnley, J. T. Bekken, and B. Norheim [4] put forward a new model based on the Norwegian public transport white paper. Formulating subsidies through this model could stimulate enterprises to improve service levels and public transport service. Methods of public transport subsidies were studied respectively in both developed and developing countries by T. Serebrisky, A. Gómez - Lobo, N. Estupiñán, and R. Muñoz - Raskin [5]. M. G. Karlaftis and P. McCarthy [6] found that based on the state of Indiana fixed-line data analysis, changing the allocation plan had not significantly influence on performance of transport, but changing the system size had a significant influence when the total subsidy level was invariable. Taking maintaining the proportion of commuting cost of income in a reasonable state as the prerequisite, the model was constructed by D. O. A. Osula [7] to estimate the amount of urban public transport development subsidies in developing countries. J. A. Van Ristel, M. A. Quddus, M. P. Enoch, C. Wang, and P. Hardy [8] built a multilevel model to analysis the impact of diminishing the school transport subsidies on students which would be implemented by British government. In the absence of authority financial subsidies, the usage of school bus would reduce drastically, and the importance of financial subsidies for the development of public transportation would be certificated from the side. Taking Japan as an example, W. Q. Zou and S. Mizokami [9] found that financial subsidies of public transportation had become a huge burden for local government. In order to reduce the burden, they put forward a method of encouraging bus companies to reduce cost and achieve the win-win strategy for operators and governments based on Laffont-Tirole model. Taking Plymouth as an example, R. Bentley and J. Lynch [10] proposed a network of city bus, offering financial subsidies by means of bid invitation and loans. In this way, a part of subsidies released from the network could be used to develop the other areas, and realize sustainable development.

In the 90s of last century, the study of public transport subsidy model started relatively late in China. After learning from foreign experience, the main research started through determining the subsidies object, formulating the methods of subsidies, examining the effect of subsidies and so on. According to the characters of traveler, H. Y. Wang, R. Yu, G. X. Wang, and J. Y. Zheng [11] proposed the new ideas of measuring financial subsidies by classification based on the utility function through taking the Nanjing City as an example. J. Zhao, J. X. Hao, W. Zhou, H. F. Huang, and H. Z. Guan [12] used the utility function to get the demand functions by public transport users, and to establish a calculation model of game operational subsidies, aiming at making government subsidies utility maximization.

No difficult to see, previous researches for calculation model of public transport financial subsidies were primarily based on two premises. The first was to increase the use of public transportation and reduce externalities, such as reducing greenhouse gas emissions and traffic congestion. The second was to reduce the burden of people traveling. The calculation was based on the service quality, passenger revenues, number of passengers, etc., and to assess the effect of the subsidy was to see whether the subsidies can achieve two basic premises described above. The majority of case studies concentrate in large or medium-size cities, little attention has been paid to calculation issues of public transport subsidies in small town. Therefore, this research focus on the solving of financial subsidies of public transportation in small town.

Ⅲ. POPULATION FORECAST MODEL IN SMALL TOWN

With the help of gray prediction theory, we established GM (1,1) prediction model, using the previous two decades of demographic data to predict the population in small town in next five years.

A. Model assumptions

In order to present the main idea conveniently and not lack of universality, this paper puts forward the following basic assumptions:

1) The data collected is true and reliable, which can reflect the demographic changes in small towns accurately;

2) No incidents can make dramatic changes in population size, such as war, immigration and so on;

3) Change of floating population quantity follows the law of nature, without the influence of artificial factors.

B. Model preparation

Taking 2014 as a node, we select the previous a_1 year (including 2014) demographic data as historical series, and test the initial data sequence.

We get the expression $x^{(0)} = \left(x^{(0)}(1), x^{(0)}(2) \cdots x^{(0)}(n)\right)$ known as the reference data set, and calculate the number of columns Pole Ratio

$$\lambda(k) = \frac{x^{(0)}(k-1)}{x^{(0)}(k)}, k = 2,3,\cdots,n \tag{1}$$

If the ratio may fall in $\left(e^{\frac{-2}{n+1}}, e^{\frac{2}{n+1}}\right)$, the number of columns can be used directly as the gray forecast model data, or the number of columns $x^{(0)}$ need to make the necessary conversion process, until it falls in the above range.

We take appropriate constant c for the number of columns $x^{(0)}$ to do the translation transform to get new series $y^{(0)}$.

$$y^{(0)}(k) = x^{(0)}(k) + c, k = 1,2,\cdots,n \tag{2}$$

Then we make the new series meet the extreme ratio requirements.

C. Modeling

The reference number of columns $x^{(0)}$ used to be done a second accumulation (AGO), then number of columns $x^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right)$ are generated, among them, $x_i^{(1)}(k) = \sum_{j=1}^{k} x_i^{(0)}(j), (k = 1,2,\cdots,n)$. We average the number of columns $z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 2,3,\cdots,n$.

Then we get $z^{(1)} = \left(z^{(1)}(2), z^{(1)}(3), \cdots, z^{(1)}(n)\right)$. So we establish gray differential equation

$$x^{(0)}(k) + az^{(1)}(k) = b, k = 2,3,\cdots,n \tag{3}$$

where,

a ---the development of factor;

b ---the amount of gray effect.

The corresponding equations albino gray model $GM(1,1)$ for

$$\frac{dx^{(1)}}{dt} + ax^{(1)}(t) = b \tag{4}$$

We set $\mu = (a,b)^T, Y = \left(x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(n)\right)^T, \mathrm{B} = \begin{pmatrix} -z^{(1)}(2) & \cdots & 1 \\ \vdots & \ddots & \vdots \\ -z^{(1)}(n) & \cdots & 1 \end{pmatrix}$, by the least squares method, to make

$J(\hat{\mu}) = (Y - B\hat{\mu})^T (Y - B\hat{\mu})$ the minimum value of $\hat{u} = (a,b)^T = (B^T B)^{-1} B^T Y$. In summary, the predict solution is

$$x^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}, k = 1,2,\cdots,n-1 \quad (5)$$

and $\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}(k), k = 1,2,\cdots,n-1$.

D. Model checking

1) Residual test: setting residuals as $\varepsilon(k)$, we calculate

$$\varepsilon(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}, k = 1,2,\cdots,n \quad (6)$$

If $\varepsilon(k) < 0.2$, it can achieve the general requirements; if $\varepsilon(k) < 0.1$, it indicates a higher requirement is achieved.

2) Pole ratio deviation test: we calculate the pole ratio $\lambda(k)$ by the reference data $x^{(0)}(k-1), x^{(0)}(k)$, then use development index a to get corresponding pole ratio deviation.

$$\rho(k) = 1 - \left(\frac{1-0.5a}{1+0.5a}\right)\lambda(k) \quad (7)$$

If $\rho(k) < 0.2$, it indicates general requirements are achieved, if $\rho(k) < 0.1$, it is considered higher requirements are achieved.

Ⅳ. THE BEST CALCULATION MODEL FOR BUS IN SMALL TOWN

This paper selects the bus as the public transportation in small town. Bus as a kind of public service, to maximize meet the demand of residents' traveling, not only considering operating costs. So, supply and demand balance method is used here to predict the best ownership of bus in small town, with resident is the objective demand of bus protection to calculate the reasonable supply of bus.

A. The category of the population

The population in small town can be roughly divided into the permanent population, temporary population and floating population. The temporary population can be divided into long-term temporary population (living in one month or more) and short-term temporary population. In order to analysis conveniently, taking the long-term temporary population and permanent population as a category, and taking short-term temporary population and floating population as a category. Therefore, the population in small town is:

$$A = A_1 + A_2 \quad (8)$$

where,

A_1 ---the sum of permanent population and long-term temporary population;

A_2 ---the sum of floating population and long-term temporary population.

B. The forecast of total travel volume

It can be directly calculated target small town's total travel volume after predicting the amount of population and per capital travel times.

$$Q_{total} = \Sigma(A_i \cdot E_i) \quad (9)$$

where,

A_i ---the population of the i class people;

E_i ---per capita travel times of the i class people.

C. The best number of bus ownership

When the bus ownership happens to meet residents' travel demand, we call it to achieve the balance between supply and demand, so the best bus ownership is the total demand for residents.

$$\text{The best number of bus ownership} = \frac{Q_{total} \times \text{transfer coefficient}}{\text{bus daily capacity}} \quad (10)$$

D. Results analysis

After the best ownership of bus has measured out, there will be out of the three situation:

1) The target bus amount in town equals to the best ownership. This is the most ideal results. In this case, the decision makers should maintain the status and do not make any change.

2) The target town has a greater amount of bus than its theoretical value. In this case, we need to consider the future demand of town on the number of public buses. It will maintain the status if the demand raises sharply and meet or exceed the current ownership in the short time. If there is no big increase in short-term demand, it can be considered to reduce the amount of ownership, which can not only be sold to obtain money but also to reduce operating costs.

3) The target bus in town is less than the best ownership. In this case, we need to purchase new bus to meet the travel needs of residents.

Ⅴ. CALCULATING METHOD OF PUBLIC TRANSPORT FINANCIAL SUBSIDIES IN SMALL TOWN

The interests of government, bus companies, and residents are different but influence each other, this constitutes a game model. Three requirements may not satisfy at the same time, when interests of one side is met, the interests of the other two sides must be damaged. Therefore, there isn't optimal solution, but satisfied solution, calculating the amount of subsidies to make the interests of three sides are within the acceptable range.

A. Model hypothesis

On the premise of people-oriented, the interests of residents are priority considered. Assuming that the utility 'μ' of residents is only related to the two factors, which is the times they take by bus called 'Z' and consumer spending 'J' without containing bus spending.

B. Modeling

1) Using utility theory and Cobb-Douglas production function, we get that

$$\mu(Z,J) = (Z - Z_0)^{\alpha} \cdot J^{\beta} \tag{11}$$

where,

Z ---The number that residents take bus every month;

Z_0---The minimum number that residents take bus every month;

α, β ---utility coefficient, $0 < \alpha, \beta < 1$.

2) The budget constraint of residents is

$$M - Z(P_0 - P) = J \tag{12}$$

where,

M ---Monthly income of residents (yuan per month).

3) Establishing K—T equation

$$\mu(Z,J) - \lambda\left[J - M + Z\cdot(P_0 - P)\right] = 0 \tag{13}$$

According to the K—T equation conditions, we can get the conclusion:

$$\begin{aligned} &\frac{\partial\mu}{\partial J} - \lambda = 0 \\ &\frac{\partial\mu}{\partial Z} - \lambda(P_0 - P) = 0 \end{aligned} \tag{14}$$

From the above conditions, we get the conclusion:

$$Z = \frac{Z_0 \cdot \beta}{\alpha + \beta} + \frac{\alpha \cdot M}{\left[(\alpha+\beta)(P_0 - P)\right]} \tag{15}$$

Denote $\gamma = \frac{\alpha}{\alpha+\beta}$, the above equation can make a change like $Z = \frac{\gamma \cdot M}{P - P_0} + (1-\gamma) Z_0$.

4) Establishing utility function of tripartite game

Enterprise income:

$$\mu_{\text{enterprise}} = Z \cdot P_0 = \frac{\gamma \cdot M \cdot P_0}{P_0 - P} + (1-\gamma)\cdot Z_0 \cdot P_0 \tag{16}$$

Government subsidies:

$$\mu_{\text{government}} = Z \cdot P = \frac{\gamma \cdot M \cdot P}{P_0 - P} + (1-\gamma)\cdot Z_0 \cdot P \tag{17}$$

Transportation spending:

$$\mu_{\text{resident}} = Z\cdot(P_0 - P) = \gamma \cdot M + (1-\gamma)\cdot Z_0 \cdot (P_0 - P) \tag{18}$$

C. Model analysis

From Equation (16), Equation (17) and Equation (18), we can see that there are only one unknown parameter $'P'$, so we can adjust the size of $'P'$ to achieve the purpose of adjusting the three utilities. Assuming that companies only need to maintain operations, and consider financial subsidies and residents travel expenses. In order to achieve a balance, we denote $\mu_{\text{government}} = \mu_{\text{resident}}$, so that the value of P can be solved.

However, what we need to pay attention is that the amount of subsidies $'P'$ is not just a number. We classify the people and give subsidies according to the income of different group. So, we need to establish a classification system of subsidies, classifying people in a scientific method and give subsidies according to the actual income level. Meeting the operating costs of bus companies and the transportation demand of residents and reduce the government subsidies to the lowest level, we obtain the win-win-win situation ultimately.

Ⅵ. CASE ANALYSIS

Below analysis is based on the example of Jiangcheng County, Pu'er City, Yunnan Province. The results in TABLE Ⅰ show the changes of population from 2010 to 2014 in Jiangcheng County.

TABLE Ⅰ. THE SITUATION OF POPULATION CHANGE

Year	2010	2011	2012	2013	2014
Total population	121500	122900	124000	124800	125616
Small town population	22700	25000	25600	25900	26110

Due to its remote geographical position, it is considered that the population in small town is permanent population, others are floating population. By fitting analysis on Jiangcheng population size from 2010 to 2014, and verifying on the forecast result, the results are shown in TABLE Ⅱ through the calculation with Matlab software.

TABLE Ⅱ. ANALYSIS OF FORECAST TEST

Year	Original Value of Small Town Population	Forecast Value	Residuals	Relative Error	Load Ratio Deviation
2010	22700	22700	0	0	
2011	25000	25112	-111.8437	0.0045	0.0791
2012	25600	25469	131.3879	0.0051	0.0096
2013	25900	25830	69.5909	0.0027	-0.0025
2014	26110	26197	-87.4268	0.0033	-0.0061

It is stated that the model has good practicability after verification which can be used to forecast. The forecasted population in small town should be 26,570 in 2015. According to the historical data, the number of buses taking by everyone every day was 0.0111. Calculating in this way, the total amount for daily bus taking should be 295 times, and the optimal ownership of buses should be 10 vehicles. However, there are 20 vehicles in this county which are two times more than the optimal ownership. It will not only form the waste of resource, but also add the financial expenditures and company operation cost. It is suggested to reduce the quantities of the public buses in order to reduce the cost. The ticket price for the bus has the direct influence on the desire of bus taking. Increasing the public bus ticket price will reduce the payment desire from the residents if their income keep at the same level. Similarly, if the public bus ticket price keeps no change, the desire will be increased along with the increasing resident income. The result from a questionnaire survey in Beijing in 2005 was the accepted cost of monthly transportation by residents will increase along with the increasing income as showed in Figure 1.

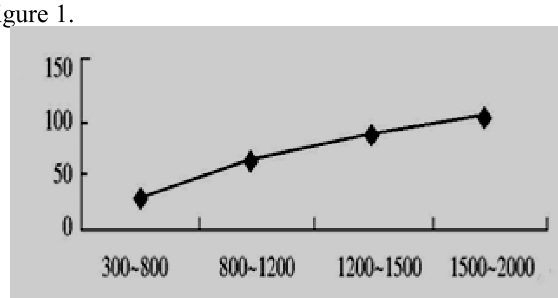

Figure 1. Growth rate between income of residents and accepted cost of monthly transportation.

It is forecasted that disposable income of residents in Jiangcheng county will reach to RMB 26,478 in 2015 based on the historical data from 2010 to 2014. Residents spend annually on transportation will be RMB 794.34 which is supposed to take 3% share of the income.

If the government subsidies are made based on disposable income, it will not efficiently solve the conflict between people's desire to reduce payment and cost guarantee pressure from bus company. So it is suggested to classify the residents and arrange the subsidies based on different income. We divide residents into student, urban residents with the lowest allowance for living, retired employee and the older, common IC card holders and no IC card holders. Free of charge policy should be applied to retired employee and the older who have no accepted expenses of monthly public transportation. TABLE Ⅲ shows the accepted expenses of monthly transportation of different residents group.

TABLE Ⅲ. INCOME FOR DIFFERENT RESIDENTS GROUP

Category	Average Income	Accepted Monthly Expense
Student	300	18.3
Urban Residents With The Lowest Allowance For Living	300	18.3
Retired Employee And The Old	1000	-
Common IC Card Holders	1500	91.5
No IC Card Holders	2200	128.1

According to the survey, there were 20 vehicles in Jiangcheng County in 2014, average yearly public transport passenger was 183,726 with expense 3.46 RMB each passenger. For equation (16), equation (17) and equation (18), taking a=0.3, b=0.7, the subsidies are shown in TABLE Ⅳ.

TABLE Ⅳ. SUBSIDIES AMOUNT OF EACH GROUP

Category	Subsidies Amount
Student	3
Urban Residents With The Lowest Allowance For Living	3
Retired Employee And The Old	3.46
Common IC Card Holders	1.1725
No IC Card Holders	0.2575

To sum up, it will not only reduce the operation cost for the public bus company, but also provide the convenience to the residents. What's more, it will generate and increase people's greater desire to choose public bus for travelling in order to save energy and pollute less which will reduce the great pressure of transportation.

Ⅶ. CASE ANALYSIS

In general, the number of bus is not the optimal. It cannot be the optimal amount of subsidies on the basis of the less than optimal number of bus. Therefore, kinds of hidden troubles including bus ownership, operational status, etc. should be screened before the subsidy determined.

It founds that people who often take the bus own a low income through the research to passengers. Therefore, the government must increase the amount of subsidies to this part of crowd in order to guarantee the need of them. Then in order to reduce the burden, the government should cut subsidies to higher income people in order to meet the needs of people at all levels.

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Finally, the use of game theory is not to make any parties occupy the absolute superiority, or let the three parties to achieve the optimal situation. It is impossible to make the three parties optimal at the same time but only as soon as possible satisfactory when they have differing interests. In urban public transport subsidies game model, due to the interests of the masses of the people should first be guaranteed, the government should increase subsidies and give their interests priority to some extent and finally achieve tripartite balance.

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