Options Game Modelling for Investment Time and Scale Decision-Making for Distributed Wind Projects

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Abstract — **The options game model for the investment decision-making in distributed wind projects is established, and the contribution margin threshold, optimal investment scale, and the option value and expected waiting time of delayed investment are obtained. The model establishment applies real options theory, based on the assumption that contribution margin varies randomly, and considers that distributed power generation first serves self-use as well as having its own economic characteristics. A case example is presented to validate: i) the conclusions of this study, and ii) the variation patterns of the influences of different parameters on the equilibrium state. It is concluded that for distributed wind projects with: i) limitation on investment time, ii) contribution margin threshold, iii) optimal investment scale, and iv) expected waiting time, are all positively related to the expected increase and variation rate of contribution margin. Meanwhile, the option value of delayed investment has different correlations with these two rates, and the significances of the influences are different. In a certain condition and range of contribution margin, the option value of delayed investment and the corresponding waiting time increases and decreases, respectively, with the increase of the proportion of generated wind power for self-use or the initial contribution margin. However, the proportion of generated wind power for self-use does not influence the optimal investment scale, and the initial contribution margin does not influence the investment threshold. The decision-maker's optimal decision should take both contribution margin threshold and optimal investment scale into consideration.**

Keywords - Distributed wind projects; Real options; Contribution margin; Investment threshold

I. INTRODUCTION

With China's policy of "large-base and distributed development of wind power", inland small-and-medium scale distributed wind power generation projects have become the interest of investment in recent year. The "12th Five-Year Development Plan of Renewable Energy" outlined a target of installed wind power capacity of 100 million kW, which indicates that by 2015, the installed capacity of distributed wind power will reach 30 million kW. Assessment shows that to build wind farms in low-wind speed and high-altitude areas, the cost of equipment will be about 8000 yuan/kW. This means the 30 million kW target indicates an investment of more than 240 billion yuan solely on equipment in the next five years. Following distributed photovoltaic, distributed wind power has become another power generation mode China's promoting [1]. Different from large-scale centralized wind power, distributed wind power farms generally have smaller scale, unstable output, and high access cost. They are more suitable for inland areas near electricity load centers, thus should be locally grid-connected and locally consumed. In addition, wind power generation first serves self-use and than for sales to grid companies, and the profit depends on the cost and revenue in self-use and sales. On the other hand, distributed power generation technology, power storage technology, and smart grid technology are now in the stage of promotion and application, thus grid connection standards and on-grid electricity price implementation rules as well as government support measures are still evolving, yet in the real world, these issues are particularly important [2]. Among various decision-making-related problems in the investment in distributed wind projects, the choice of optimal investment time and scale is an important research topic.

Since distributed wind projects require lump-sum investment, long construction period, and large portion of sunk cost, and the electricity price and contribution margin are facing great uncertainty, the theory of irreversible investment under uncertainty, i.e. real options (ROs), is applicable to the analysis of such projects [3]. Real option developed from the concept of financial option, which is a contract that grants its holder the right to buy or sell a certain quantity of a financial asset (called potential financial asset or underlying asset) within the prescribed period at the price agreed by both parties. Real option is the extension of financial option in the physical investment field. The core idea of real option is that uncertainty could increase the value of enterprise investment, and the option holder could delay their decision on whether to further invest in these assets, in order to minimize the risk of depreciation of the assets and maintain stable benefit from the physical assets. The classic reports on the modeling of real options are written by Titman [4] and Williams [5], the discussion of the former being discrete and that of the latter continuous. Later on, Quigg [6] and Bulan *et al.* [7] performed empirical research and confirmed the value of waiting option. McDonald and Siegel [8] and Pindyck [9] also contributed similar classic studies.

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These studies are general studies done for related investment decision-making problems using real options.

In recent years, some scholars have applied real options theory to the decision-making process of investment in renewable energy-based power generation in uncertain environments. For example, in [10] the price of natural gas and the decision-making for investment in micro-grid of distributed power generation from natural gas in uncertain conditions are studied. In [11], the choosing of investment time and installed capacity in small-scale hydropower projects under electricity price uncertainty is analyzed. Reference [12] discusses the decision-making for investment in power generation projects in electricity market with uncertain policies on the restriction of greenhouse gas emissions in terms of fuel price, market electricity price and price of CO2 emission. In [13], with the uncertainty of grid wind power price, the investment and operation costs of wind farms, the investment policies, and the investment time considered, a wind project investment decision-making model fit for the investment environment of wind power in China is established. In [14], based on real options theory, the investment compensation problem in photovoltaic grid-connected power generation projects is investigated under uncertain cost and grid electricity price. In [15], aiming at the impact of the uncertainty of natural gas price on the investment and upgrade of distributed power generation from natural gas, and based on physical option model, the threshold price of investing in distributed power generation at different levels of natural gas price fluctuation is studied.

In the above studies, the investment opportunity is similar to a call option without definite expiration date, thus the investment decision-making is similar to deciding whether to exercise this call option, while the exercise price is the investment cost. This is the basic idea of modeling in existing literature as well as in this study. In general, the available literature does not include many studies of decision-making of investment in distributed wind projects, and the mainstream studies have long ignored the discussion of the reachability of investment time. In addition, the feature of distributed power generation that its priority is the private electricity use of users directly impacts the value of the investment, and this issue is often overlooked in the literature. Therefore, in this paper, combining the analysis of the economics of wind power, the decision-making on optimal investment time and scale in distributed wind projects is investigated. The remainder of the paper is organized as follows. Section 2 is dedicated to the model establishment of the research problem. The model's equilibrium is solved and analyzed in section 3. We set the model's parameters in section 4. Section 5 presents a numerical example to illustrate the availability of the proposed model. Finally, conclusions are drawn in Section 6.

II. MODEL ESTABLISHMENT

A. *Problem Description and Model Assumptions*

In general, decision makers have two strategies to choose after obtaining the right of investment of a distributed wind farm, immediate investment and delayed investment. To choose from the two strategies is to weigh the benefits expected from them. Delayed investment equals to holding a call option with indefinite expiration date, but the decision-maker will have to face the uncertainty of electricity power and wind power generation costs. Accordingly, immediate investment is equivalent to exercising the option, of which the exercise cost is the construction and power generation costs. The modeling in this section aims to answer following questions: How should the decision-maker choose between the two strategies? How is the optimal investment scale decided? What is the boundary (threshold) between the areas of continue and stop waiting? What factors would impact this boundary? Is this boundary reachable, and what are the probability of reachability and the passage time?

The modeling to solve the problem requires some assumptions, including that:

- the investment is completely irreversible and the decision-making can be delayed;
- the profit function of the decision-maker maximizes the net present value (NPV) of the option;
- the contribution margin of distributed wind power generation follows geometric Brownian motion;
- the construction of power plants is instantaneous and the plant generates cash flow immediately;
- the decision maker who has the right of investment has only one choice, which is to construct the distributed wind farm; and
- the decision-maker had no burden of taxation or other rent-seeking costs.

B. *Basic Variables and Their Standardization*

1) Electricity price: Generally, the price of wind power is the most important uncertain factor affecting the decision-making of investment in distributed wind projects. The uncertainty of future wind power price results from three aspects, including short-term price fluctuation, appearing as fluctuation of price near the mean value; long-term price deviation (change), and price leaps after the announcement or implementation of new policies.

Considering that although short-term price fluctuation exists in market conditions, it is usually near the mean value and does not impact the decision-making process, such short-term changes are ignored in the model developed in this study. Long-term price deviations are described as geometric Brownian motions [11, 16-17]. The new policy or mechanism changes encouraging distributed wind power generation often cause wind power price leaps. As a preliminary research, this work does not take such price leaps into consideration.

At this stage, we assume retail electricity price $p_1(t)$ as the average price users pay, which is determined by grid power companies based on electricity purchasing cost and its reasonable profit, grid electricity price $p_2(t)$ is the government guidance price. Provincial grid power companies often purchase electricity at price higher than the local standard price of electricity from thermal power, when the government guidance price is higher than the standard price, grid power companies enjoy corresponding subsidies.

2) Investment scale: At invariant output, we assume the investment scale of a distributed wind power farm is weighed by its annual power generation q (in the unit of kW·h) after completion of construction. The generated power can satisfy self-use (Situation 1), and spare electricity can be sold to grid companies (Situation 2). The energy loss in this process is ignored. The electricity for self-use accounts for λ of the generated power, and the electricity for sales accounts for $1-\lambda$.

3) Costs of wind power generation: The costs of wind power generation include constant cost independent of the investment scale; constant costs are considered to be a part of investment costs in this paper, because after a distributed wind farm is built, it is definitely not optimal to close it, and this means there must be constant costs. Variable costs related to the investment scale, including fuel cost, equipment operation, maintenance and upgrade cost, grid connection fee, selling expense, labor cost and cost of capital. The variable costs can be divided into self-use costs and selling expenses; self-use costs in distributed wind power generation mainly refer to the operation and maintenance costs of wind turbines and wind farm. The selling expenses include, besides self-use costs, the grid connection fee, power transmission and sale costs, etc.

4) Contribution margin: The contribution margin of distributed wind power farm mainly depends on the wind power price and the variable power generation cost, the difference between which is thus used to define contribution margin, i.e.

$$\chi_i(t) = p_i(t) - c_i(t), i = 1,2 \tag{1}$$

where $i = 1,2$ indicate the situations where electricity is for self-use and selling, respectively, $c_i(t)$ is the marginal power generation cost, and $\chi_i(t)$ is the corresponding contribution margin. Considering the uncertainty of wind power price, this paper treats the margin contributing process of distributed wind power generation as a random process controlled by both internal and external factors. Without loss of generality, we assume the contribution margin follows geometric Brownian motion, thus

$$\begin{cases} \mathrm{d}\chi_i(t) = \mu_\chi \chi_i(t)\mathrm{d}t + \sigma_\chi \chi_i(t)\mathrm{d}z(t) \\ \text{s.t.}\chi_i(0) = \chi_i \geq 0 \end{cases} \tag{2}$$

where μ_χ is the deviation (i.e. the expected growth rate of $\chi_i(t)$), σ_χ is the rate of change, $\mathrm{d}z(t)$ is the increment of a standard Wiener process and follows a normal distribution with mean of 0 and standard deviation of $\sqrt{\mathrm{d}t}$, and $\mathrm{d}t$ is the length of a single time step, which is defined as 1 year in the following investigation.

5) Opportunity cost for delayed investment: Assuming that the decision-maker is risk-neutral, the expected risk-adjusted return rate of distributed wind projects is r, and according to capital asset pricing model (CAPM),

$$r = r_0 + \varepsilon\rho\sigma_\chi \tag{3}$$

in Equation (3), r_0 is the risk-free interest rate, ε and ρ are positive constants indicating the correlation coefficient between market price and market return and the correlation coefficient between project return and market return, respectively. Considering that if $\mu_\chi \geq r_0$, then delayed investment is always the better strategy for the decision-maker, and the optimal solution does not exist, we only consider the situation where $\mu_\chi < r_0$. With $\xi = r - \mu_\chi$ to express this difference, ξ can be considered the convenience yield rate, i.e. the opportunity cost for delaying investment and keeping the investment option.

6) Value function: Different from the assumption in real options theory that there is no limitation on investment time, in this study we considered the situation where there is investment time limit in distributed wind power farms. In fact, distributed wind power generation is a newly emerging industry in China, and the economic lifetime of such projects is often very short. The assumption of limited investment time is more realistic.

To find the optimal investment strategy for a distributed wind farm within a limited time span, the value function and investment costs after the wind farm is built need to be found. In the following analysis, we first define the contribution margin of wind power generation and the function of investment scale, and then describe investment costs as a function of investment scale.

With the basic assumptions and parameter configuration of this study, the value function of a distributed wind farm is

$$\max_{q\geq 0, \chi_i(t)\geq 0} V\left[q, \chi_i(t)\right] = qE\int_0^T \mathrm{e}^{-rt}\left[\lambda\chi_1(t) + (1-\lambda)\chi_2(t)\right]\mathrm{d}t \tag{4}$$

where $E(\cdot)$ is the expectation value operator, T is the investment time limit of the wind farm, and $V(\cdot)$ is the value function of the wind farm.

7) Investment costs: Investment costs of distributed wind farms include the costs of wind turbines, import tariffs, interconnection and transmission engineering, communications, necessary civil works, land acquisition, preliminary expenses, management and supervision costs, insurance, preparation costs, foreign exchange risks, interests during construction, etc. Different investment scales require different investment costs.

The relationship between investment costs and scale of distributed energy projects has been investigated in the literature. In [11, 18], it is found that each small-scale hydropower project has a limited largest investment scale, and as investment scale approaches this limitation, marginal investment costs increase. On this basis, it is demonstrated that the investment costs of small-scale hydropower plants can be expressed as an exponential function of the annual power generation. This is essentially in accordance with the investment costs model established in [19-20]. In this study, we adopt the same finding and define the investment costs of distributed wind farms as

$$I(q) = \kappa \, \mathrm{e}^{\alpha q} \tag{5}$$

where κ and α are investment costs of distributed wind farms and their values are positive influencing factors, and $I(\cdot)$ is the investment costs of distributed wind farms and it is a convex function of investment scale.

C. Target Function

Individual distributed wind farms can supply limited amount of electricity, thus are not entitled to argue the electricity price with power grid companies. Therefore, they are the recipients of wind power trade price. However, once the investment scale is determined, as long as the wind power price is higher than the marginal power generation cost, a wind farm is profitable if it is always generating power.

It can be seen in Equation (4) that the random variation of contribution margin causes the uncertainty of a distributed wind farm's value. With such uncertainty, the net present value (NPV) decision-making standard is

$$\begin{cases} \max\limits_{q\geq 0, \chi_i(t)\geq 0} \mathrm{NPV} = V\left[q, \chi_i(t)\right] - I(q) \\ \text{s.t.} \mathrm{d}\chi_i(t) = \mu \chi_i(t)\mathrm{d}t + \sigma \chi_i(t)\mathrm{d}z(t) \\ \chi_i(0) = \chi_i \geq 0 \end{cases} \tag{6}$$

If the investment opportunity is treated as an American call option, the investment decision-making is equivalent to deciding the time and price to exercise this option. Therefore, investment decision-making can be considered an option pricing problem, which can be solved by dynamic planning method. Multiple methods are available for solving maximization problems (6), for example contingent claim method, dynamic planning method, and optimal stopping time method. Since the problem is a duration American option problem without expiration date or termination payoff, the basic equation of continuous time dynamic planning method with infinite time, i.e. the Bellman equation without immediate profit, to solve the problem [21]. The real option decision-making rule for distributed wind farms is

$$F\left[\chi_i(t)\right] = \max_{q\geq 0, \chi_i(t)\geq 0} \mathrm{NPV} \tag{7}$$

where $F(\cdot)$ is the value of the investment time, i.e. option value of the investment, and the constraint condition is Equation (2).

Since all parameters of the model are constants independent of time, and the decision-maker faces the same game in any time period within the limited time span, the strategy can be limited as static strategy and its equilibrium is static feedback equilibrium.

III. Equilibrium Solution and Analysis

In the decision-making process for distributed wind projects, there are two key questions to answer, one it when to invest, i.e. investment time, the other is how much to invest, i.e. investment scale. In this section, these two questions are discussed based on the option pricing model of investment opportunity, in order to obtain the optimal investment decision.

A. Optimal Investment Scale

To deduce investment time and scale for distributed wind farms, the optimal investment scale under given external impact should first be obtained. With initial value of χ_i, according to Ito stochastic integral, Equation (2) has the following solution

$$\chi_i(t) = \chi_i \, \mathrm{e}^{\left(\mu_\chi - \frac{\sigma_\chi^2}{2}\right)t + \sigma_\chi z(t)} \tag{8}$$

For any value of t, this is a lognormal random variable, thus its expectation is

$$E\left[\chi_i(t)\right] = \chi_i \, \mathrm{e}^{\left(\mu_\chi - \frac{\sigma_\chi^2}{2}\right)t} E\left[\mathrm{e}^{\sigma_\chi z(t)}\right] = \chi_i \mathrm{e}^{\mu_\chi t} \tag{9}$$

The uncertainty of the contribution margin of distributed wind power generation leads to investment risk. Although the above equation gives the expectation function of contribution margin, the real-time implementation value of contribution margin may be significantly higher or lower than the expected value in the investment time limit. As we can obtain $p_1(t) > p_2(t), c_1(t) < c_2(t)$ by the definitions of electricity price and marginal power generation cost, so it is easily proved $\chi_1(t) > \chi_2(t)$ in Equation (1). For the convenience of discussion, we assume that a positively proportional relationship exist between these two values, i.e.

$$\chi_1(t) = (1+\eta)\chi_2(t) \tag{10}$$

where $\eta > 0$ is a constant. Substituting Equation (10) in (4), after simplification and organization,

$$\max_{q\geq 0,\chi_i(t)\geq 0} V\left[q,\chi_i(t)\right] = q(1+\lambda\eta)E\left\{\int_0^T \left[\chi_2(t)\mathrm{e}^{-rt}\right]\mathrm{d}t\right\} \quad (11)$$

Substituting Equation (9) in (11), then

$$V\left[q,\chi_i(t)\right] = \chi_i q(1+\lambda\eta)\frac{1-\mathrm{e}^{-\xi T}}{\xi} \quad (12)$$

The optimal investment scale should result in the wind power generation that maximizes the NPV at given contribution margin. Solving the right side of Equation (6), which is a first-order condition of q, it is known that at the optimal investment scale of a distributed wind farm, the marginal value of wind power equals to the marginal investment costs, i.e.

$$\frac{\partial V\left[q,\chi_i(t)\right]}{\partial q} = \frac{\partial I(q)}{\partial q} \quad (13)$$

Substituting the mentioned value function in Equation (13), then

$$\kappa\alpha\,\mathrm{e}^{\alpha q} = \chi_i(1+\lambda\eta)\frac{1-\mathrm{e}^{-\xi T}}{\xi} \quad (14)$$

Thus the optimal investment scale under given external impact is

$$q^* = \frac{\ln\left[\chi_i(1+\lambda\eta)\dfrac{1-\mathrm{e}^{-\xi T}}{\xi\kappa\alpha}\right]}{\alpha} \quad (15)$$

It is not difficult to see that the optimal investment scale of a distributed wind farm is a monotone increasing function of the contribution margin of its wind power generation.

B. Optimal Investment Time

Optimal investment time refers to the time point when the contribution margin, since the initial time point, first reaches or exceeds the threshold investment. This means that there is an investment threshold χ_2^*, and when contribution margin $\chi_2(t)\geq\chi_2^*$, the decision-maker is in a stopping time zone, and his/her optimal investment strategy is to invest immediately. Therefore, the optimal investment time can be expressed as

$$t^* = \inf\left[t\geq 0 \,\middle|\, \chi_2(t)\geq\chi_2^*\right] \quad (16)$$

If contribution margin is lower than the investment threshold, it means the market demand is low and the decision-maker continues waiting. During the waiting period, the decision-maker holds the option to invest in the future. Although no cash flow is generated, the decision-maker is entitled to the capital profit and loss brought by option changes (In the subsequent discussion, time t is not written for convenience).

Since the investment opportunity $F(\chi_2)$ does not generate cash flow before the time point of the investment's exercise, the only return for holding it is its capital appreciation. The Bellman equation for continuous time periods is [22]

$$rF(\chi_2)\mathrm{d}t = E\left[\mathrm{d}F(\chi_2)\right] \quad (17)$$

Applying Itô lemma to expand $\mathrm{d}F(\chi_2)$, the following second-order differential equation can be obtained

$$\mathrm{d}F(\chi_2) = F_\chi^{'}(\chi_2)\mathrm{d}\chi_2 + \frac{F_\chi^{"}(\chi_2)}{2}(\mathrm{d}\chi_2)^2 \quad (18)$$

Substituting Equation (2) in (17), the expectation is

$$E\left[\mathrm{d}F(\chi_2)\right] = \left[\chi_2\mu_\chi F_\chi^{'}(\chi_2) + \frac{\sigma_\chi^2\chi_2^2}{2}F_\chi^{"}(\chi_2)\right]\mathrm{d}t \quad (19)$$

Then, substituting the above $E\left[\mathrm{d}F(\chi_2)\right]$ in Equation (17), the original Bellman equation can be rewritten as

$$\frac{\sigma_\chi^2\chi_2^2}{2}F_\chi^{"}(\chi_2) + \mu_\chi\chi_2F_\chi^{'}(\chi_2) - rF(\chi_2) = 0 \quad (20)$$

According to the standard real option analysis method of Dixit and Pindyck [22], the investment option value of distributed wind farms is in the form of

$$F(\chi_2) = A_1\chi_2^{\beta_1} + A_2\chi_2^{\beta_2} \quad (21)$$

where A_1 and A_2 are undetermined constants, $\beta_1>1$, $\beta_2<0$ and they both are non-linear functions of r, μ_χ, and σ_χ, and satisfy the following equation

$$L(\beta) = \frac{\sigma_\chi^2}{2}\beta^2 + \left(\mu_\chi - \frac{\sigma_\chi^2}{2}\right)\beta - r = 0 \quad (22)$$

Since the discriminant of the above quadratic equation with one unknown is greater than 0, the equation must have two different solutions, and in addition,

$$\begin{cases} \beta_1 = \dfrac{1}{2} - \dfrac{\mu_\chi}{\sigma_\chi^2} + \sqrt{\left(\dfrac{\mu_\chi}{\sigma_\chi^2} - \dfrac{1}{2}\right)^2 + \dfrac{2r}{\sigma_\chi^2}} \\ \beta_2 = \dfrac{1}{2} - \dfrac{\mu_\chi}{\sigma_\chi^2} - \sqrt{\left(\dfrac{\mu_\chi}{\sigma_\chi^2} - \dfrac{1}{2}\right)^2 + \dfrac{2r}{\sigma_\chi^2}} \end{cases} \tag{23}$$

For further analysis, on the basis of satisfying Equation (19), $F(\chi_2)$ should also satisfy the three boundary conditions as follows [23]:

First, initial value must be 0, i.e. $F(0) = 0$. This means when $\chi_2 = 0$, the investment option will not be exercised, thus the option value is 0, and thereby ensuring the solution of the differential equation is economically meaningful. Therefore, only $\beta_1 > 1$ is used, and Equation (21) can be written as

$$F(\chi_2) = A_1 \chi_2^{\beta_1}, \beta_1 > 1 \tag{24}$$

The other two conditions are considerations of the optimal investment, aiming to ensure that no opportunity exists for arbitrage during the exercise of the option, and that the option value is not only continuous, but also smooth at the threshold point.

The second boundary condition is the value-matching condition, which means when the contribution margin χ_2 reaches χ_2^*,

$$F(\chi_2^*) = \mathrm{NPV}\left[\chi_2^*, q(\chi_2^*)\right] \tag{25}$$

This means the option value when implementing the optimal investment decision for a distributed wind farm equals to the net present value of immediate investment. This boundary condition reflects the profit and loss at the exercise of the investment option of distributed wind farms.

Simultaneous solve Equations (5), (15) and (24), the investment option value can be expressed as a function of state variable and threshold χ_2^*, and

$$\begin{cases} F(\chi_2, \chi_2^*) = A_1(\chi_2^*)\chi_2^{\beta_1} \\ \qquad = \dfrac{\delta(\chi_2^*)^{1-\beta_1}}{\alpha}\left(\ln\dfrac{\delta\chi_2^*}{\kappa\alpha} - 1\right)\chi_2^{\beta_1} \\ \text{s.t.}\delta = (1+\lambda\eta)\dfrac{1-\mathrm{e}^{-\xi T}}{\xi} \end{cases} \tag{26}$$

The third boundary condition is the smooth-pasting condition or high-order contact condition, which means the option value function $F(\chi_2)$ is continuous and smooth at χ_2^*, i.e.

$$\left.\frac{\partial F(\chi_2)}{\partial \chi_2}\right|_{\chi_2=\chi_2^*} = \left.\frac{\partial \mathrm{NPV}\left[\chi_2, q(\chi_2)\right]}{\partial \chi_2}\right|_{\chi_2=\chi_2^*} \tag{27}$$

This means at the threshold investment χ_2^*, the marginal profit of delayed investment equals to the marginal net present value of inestment. Equation (27) ensures that it is the optimal choice for the decision-maker to exercise his/her option at the threshold.

According to the first-order condition in Equation (26),

$$\chi_2^* = \frac{\kappa\alpha}{\delta}\mathrm{e}^{\frac{\beta_1}{\beta_1-1}}, A_1 = \frac{\kappa}{\beta_1 - 1}\left(\frac{\delta}{\kappa\alpha}\right)^{\beta_1}\mathrm{e}^{-\beta_1} \tag{28}$$

The net present value decision-making rule is a standard for comparing real option decision-making rules. Substituting Equation (15) in (6), and after simplification and organization, there is

$$\mathrm{NPV}(q^*, \chi_2) = \frac{\delta\chi_2}{\alpha}\left(\ln\frac{\delta\chi_2}{\kappa\alpha} - 1\right) \tag{29}$$

Solving the above first-order condition of χ_2, it can be obtained that

$$\chi_2^\times = \frac{\kappa\alpha}{\delta} \tag{30}$$

Apparently, the optimal threshold investment based on real option has enlarged coefficient $\mathrm{e}^{\frac{\beta_1}{\beta_1-1}} > 1$ comparing with the threshold investment under the net present value rule. This coefficient reflects the influence of the irreversibility and uncertainty of investment in distributed wind farms on the optimal investment decision-making rule, and it can be proved that its value increases with the increase of the uncertainty, i.e.

$$\begin{cases} \dfrac{\partial \mathrm{e}^{\frac{\beta_1}{\beta_1-1}}}{\partial \sigma_\chi} = -\dfrac{\mathrm{e}^{\frac{\beta_1}{\beta_1-1}}}{(\beta_1-1)^2}\dfrac{\partial \beta_1}{\partial \sigma_\chi} > 0 \\ \text{s.t.} \dfrac{\partial \beta_1}{\partial \sigma_\chi} < 0. \end{cases} \tag{31}$$

Further analysis shows that as the uncertainty increases, the optimal threshold investment under the real option decision-making rule and the value of distributed wind farm both increase, and at this time the decision-maker would prefer to delay the investment. This indicates that the uncertainty increases the value of waiting of a project. The decision-maker will delay the investment, and eventually invest in greater output scale in order to meet the market demand growth in the future. This is consistent with the conclusion of Dangl [20].

Substituting Equation (28) in Equations (15) and (16), the optimal investment time and optimal investment scale of distributed wind farms are obtained as

$$\begin{cases} t^* = \inf\left[t \geq 0 \middle| \chi_2(t) > \dfrac{\kappa\alpha}{\delta}\mathrm{e}^{\frac{\beta_1}{\beta_1-1}} \right] \\ q^* = \dfrac{\beta_1}{\alpha(\beta_1-1)} \end{cases} \tag{32}$$

C. Reachability of Threshold Contribution Margin

The optimal value of χ_2^* has been obtained in the above discussion, yet the questions have not been fully answered. A reasonable question is whether the state variable χ_2 will eventually reach the threshold value χ_2^*, and if it will, how long is the first passage time. (Obviously, this first passage time refers to the expected value, as it is a random variable.) If χ_2 will not reach the threshold value, χ_2^* is not meaningful in guiding investment. Unfortunately, this issue is not considered in the discussions of investment time under conventional ROs framework. This subsection gives a basic result for the solving of this problem.

Assuming that the first passage time is t^*, then for random processes like in Equation (2), with reference to the theoretical analysis of Rhys *et al.* [24] on the exercise time of real options, the density function $f(\cdot)$ of t^* can be expressed as

$$f(t,\chi_2,\chi_2^*) = \frac{\ln\dfrac{\chi_2^*}{\chi_2}}{\sigma_\chi\sqrt{2\pi t^3}}\mathrm{e}^{-\frac{\left[\ln\frac{\chi_2^*}{\chi_2}-\left(\mu_\chi-\frac{\sigma_\chi^2}{2}\right)t\right]^2}{2\sigma_\chi^2 t}} \tag{33}$$

In the above equation, $\chi_2^*(\chi_2^* > \chi_2)$ is the threshold value that χ_2 needs to reach. Let $s_\chi = \mu_\chi - \frac{\sigma_\chi^2}{2}$, three conditions are considered below to analyze the conditions of reachability of the threshold contribution margin.

- When $s_\chi > 0$, the possibility to reach χ_2^* is 1, and the expected time of reaching this value and the deviation of the expected time are

$$E(t^*) = \frac{\ln\dfrac{\chi_2^*}{\chi_2}}{s_\chi}, Var(t^*) = \frac{\sigma_\chi^2}{s_\chi^3}\ln\frac{\chi_2^*}{\chi_2} \tag{34}$$

- When $s_\chi = 0$, the possibility of reaching χ_2^* is 1, however the expected waiting time and its deviation are both infinite.
- When $s_\chi < 0$, neither the expected time of reaching χ_2^* nor the the deviation of the expected time exist, χ_2^* can be reached at the possibility of

$$Pr = \left(\frac{\chi_2^*}{\chi_2}\right)^{\frac{2\mu_\chi}{\sigma_\chi^2}-1} \tag{35}$$

Similarly, when the initial contribution margin of a distributed wind farm is greater than the contribution margin's threshold boundary, i.e. $\chi_2 > \chi_2^*$, then only when $s_\chi \leq 0$, the contribution margin χ_2^* can be reached at the possibility of 1 within finite time. Otherwise, the expected waiting time and its deviation do not exist. However, the equilibrium solution in this paper has the economic meaning of $\chi_2^* > \chi_2$.

Therefore, it is not assured that χ_2^* can be reached within finite time, and the value of s_χ determines whether it can be reached or not.

IV. Model Parameters Configuration

A. Power Generation Cost

At present, the international common operation and maintenance cost of distributed wind farms is 0.05 yuan/kW·h. For power generation projects from renewable energy in China, the grid connection fee is charged by the length of line. For lines shorter than 50 km, the fee is 0.01 yuan/kW·h, for lines longer than 50 km and shorter than 100 km, it is 0.02 yuan/kW·h, and for lines longer than 100 km, it is 0.03 yuan/kW·h. For distributed wind power generation, besides the grid connection fee, the cost also involves electricity transmission and selling expenses, which have not been clearly specified in China. As in [11], the European Union's official directed price for electricity transmission, released in 2005, is 0.25 euro/MW·h, and the selling expense is 0.31 euro/MW·h. With the difference in line length and grid selling of distributed wind power taken into consideration, we use in this study $c_1 = 0.05$ yuan/kW·h and $c_2 = 0.08$ yuan/kW·h.

B. Investment Costs

Owing to the lack of practical data, the input-output capacity of onshore wind farms is used approximately as the capacity of distributed wind farms. Based on the overall investment, annual electricity generation, and installed capacity of the recent major wind farm development projects in China (Table 1), regression analysis is used to estimate the investment cost coefficients κ and α .

Since the installed capacity of the projects listed in Table 1 are mostly 49.5MW, while the installed capacity of distributed wind farms is generally lower than 50MW. Based on the installed capacity, distributed power plants can be classified into four categories: Micro-scale, with capacity of 5 kW; Small-scale, with capacity of 5 kW to 5 MW; Medium-scale, with capacity of 5 MW to 50 MW; and Large-scale, with capacity of 50 MW to 300 MW. The data was corrected and after regression fitting we got $\kappa = 4.776 \times 10^7$ and $\alpha = 2.686 \times 10^{-8}$.

TABLE I. DATA FROM RECENT MAJOR WIND FARM DEVELOPMENT PROJECTS IN CHINA

Number	Project Name	Total Investment/million yuan	Annual Power Generation/million kW·h	Installed Capacity/MW
1	Guangxi Jinzishan Wind Project	579	95.00	49.5
2	Gansu Jingtai Wind Project of China Power Investment Corporation	369	95.38	49.5
3	Heilongjiang Fangzheng Gaoleng Wind Project	1040	246.00	100.0
4	Dalongtan Wind Project of China Huaneng	480	122.78	49.5
5	Yunnan Huashitou Wind Project	410	118.19	49.5
6	Yunnan Yeniutang Wind Project	423	120.60	49.5
7	Jiancaitang Wind Project Stage I of SDIC	465	99.32	45.0
8	Jiancaitang Wind Project Stage II of SDIC	408	95.04	49.5
9	Jinjiang Jinjing Wind Project	300	77.34	32.0
10	Junxian Kushan Wind Project Stage I of GD Power Development	400	90.00	40.0
11	Inner Mongolia Dabanliang Wind Project	420	133.65	49.5
12	Erenhot Wind Project of Guodian Inner Mongolia Branch	508	178.10	49.5
13	Datang Fengning Wanshengyong Wind Project	1423	332.76	150.0
14	Shanxi Pingyao Zhukeng Wind Project	420	98.00	50.0
15	Chongqing Nantianmen Wind Project	450	81.00	15.0

Data gathered from information related to recent wind farm development projects published on http://wp.china-nengyuan.com/ and sorted by the authors.

C. Contribution Margin of Wind Power Generation

The benefit from distributed wind power generation consists of saved self-use cost and profit from selling electricity to power grid companies. The saved self-use cost is directly related to the retail electricity price, which is determined by the consumption type of local electricity users. For example, in Chongqing, the second level of the city's ladder electricity price standard is that for household electricity consumption at a monthly amount of 200~400 kW·h, the electricity price is 0.57 yuan/kW·h, which is set as the initial retail electricity price.

Sales profit consists of the profit from selling electricity to grid companies, the subsidies, and the benefit from clean development mechanism (CDM). In China, grid companies generally purchase electricity at local standard price of electricity from thermal power, which is often lower than the guidance price of grid-connected wind power. The difference in this price is compensated to wind farms. Since the CDM in China is not complete, and the CDM benefit for small-scale distributed wind projects are hard to obtain, this benefit is not considered in this study. Therefore, the sales profit equals to the guidance grid-connected electricity price. Currently, the grid-connected wind power prices in China are 0.51, 0.54, 0.58, or 0.61 yuan/ kW·h. The median value 0.54 yuan/kW·h is used in this paper as the initial on-grid electricity price. Through calculation with Equations (1) and (9), it is known that the self-use to sales contribution margin proportion coefficient $\eta = 0.13$.

D. Other Relevant Parameters

The economic lifecycle of wind farms is generally 20 to 30 years [25]. Considering that the manufacture of wind power generation equipment in China is less advanced than that in western countries, the depreciation of machines and equipment is faster, thus we use the investment time limit of $T = 20$ years.

Risk-free interest is generally the annual deposit interest offered by banks, and in this paper we use $r_0 = 5.8\%$. The correlation coefficient between market risk price and market combination in [26] is used in this study, i.e. $\varepsilon = 0.4$ and $\rho = 0.7$.

In recent years, the equipment cost in wind projects is decreasing year by year, yet the land use fee and other constant costs are increasing gradually. In addition, the long-term contract price of wind power is not publicly available. Thus it is difficult to directly use history data for calculation [25]. In this study, we use the average annual profit rate of increase and rate of change of a thermal power plant as the expected rate of increase and rate of change of contribution margin. It is assumed that μ_χ = 0.8% and $\sigma_\chi = 2.5\%$.

Since the proportion of generated wind power for self-use is different from project to project, no special requirement is appointed to this proportion in this study. Thus the value of this proportion ranges from 0 to 100%.

V. CASE STUDIES

A company plans to invest in a small-scale wind farm in a newly developed community, which is located in a type II resource area. The main purpose of this wind farm is to supply the electricity demand of this community's residents. The project is planned to use domestic wind power generation equipment with direct-driven permanent magnet technology, and the estimated annual full-load utilization time is 1800 hours. It is estimated that as the community is gradually inhabited, 60% of the wind farm's annual power generation will be for self-use. Substituting the parameter values configured in the last section in Equations (15) and (28), we know that the contribution margin threshold is 0.311 yuan/kW·h and the optimal investment scale is 43.814 million kW·h for this distributed wind project. If the initial contribution margin of this project is 0.316 yuan/kW·h, it means the decision-maker should invest immediately, and the corresponding net present value is 30.12 million yuan.

In this scenario, the values of the parameters are constants. However, in practice, the parameters are variable, thus influencing the actual investment decision-making. This section presents the analysis of the sensitivity of investment decision-making to the variation of relevant parameters.

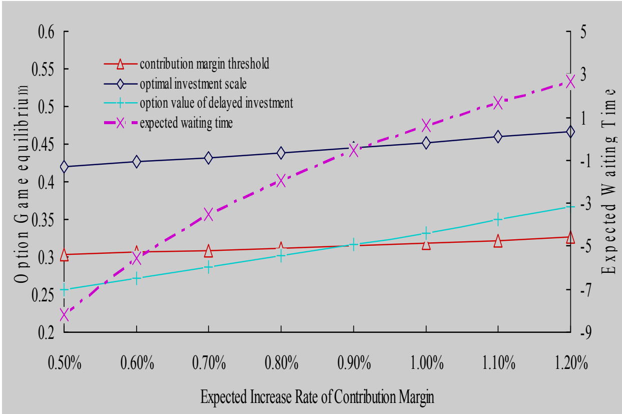

Figure 1. Investment decision-making with changing expected increase rate of contribution margin.

As seen in Fig. 1, with fixed values of other parameters, as μ_χ increases, the contribution margin threshold χ_2^* and the option value $F(\chi_2)$ of delayed investment gradually increase, while the optimal investment scale q^* and the expected waiting time $E(t^*)$ increase successively. For example, when α = 1.0% and the contribution margin threshold is 0.318 yuan/kW·h, the decision-maker should delay the investment in this wind farm, and at this time, the option value is 33.24 million yuan, and the optimal investment scale is 45.199 million kW·h, which equals to a capacity of 25.111 MW. Correspondingly, it can be estimated according to Equation (34) that the expected waiting time before the optimal investment time is 0.635 years. This can be explained as that the decision-maker has higher expectation increase rate of contribution margin for the future, which increases the option value of investment in the wind farm, thus the decision-maker chooses to delay the investment. In addition, once the investment is placed, the decision-maker will increase the annual power generation, in order to obtain the highest net present value.

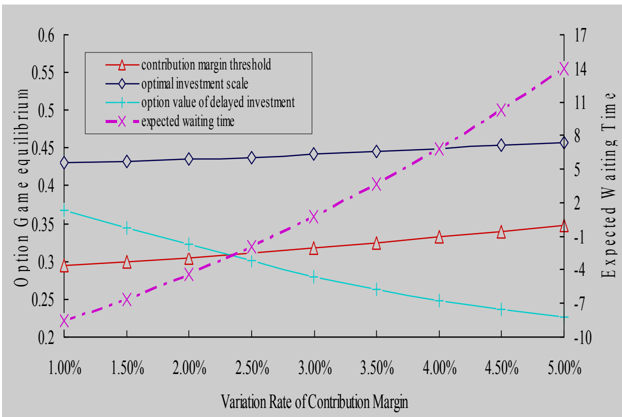

Figure 2. Investment decision-making with changing variation rate of contribution margin.

As seen in Fig. 2, with the increase of the rate of change σ_χ of contribution margin, the contribution margin threshold χ_2^*, optimal investment scale q^*, and expected waiting time $E(t^*)$ all monotonically increase, while the investment option value $F(\chi_2)$ of distributed wind farms is negatively related to σ_χ. This means greater variation rate of contribution margin leads to smaller reachability of the threshold, and the decision-maker increases the threshold contribution margin in order to avoid the consequent high risk. For the decision-maker, immediate investment at this time leads to greater option value. Meanwhile, σ_χ has greater influence on investment option value than on threshold contribution margin and optimal annual power generation.

At the same time, it is known from Figs. 1and 2 that μ_χ has greater influence on the decision-making of investment scale q^* than σ_χ does, while σ_χ has greater influence on contribution margin threshold χ_2^* and the option value $F(\chi_2)$ of delayed investment than μ_χ does. Therefore, decision-makers of investment in wind farms pay more attention to the value of σ_χ. Meanwhile, to realize the goal of the government to encourage regulated investment, the expected increase rate of contribution margin is more important than the variation stability of contribution margin.

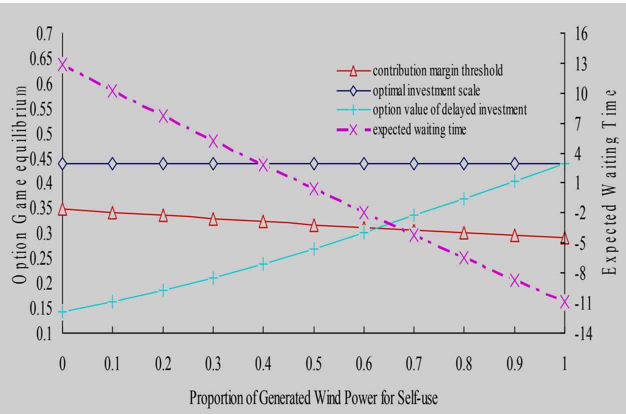

Figure 3. Influence of the proportion of generated wind power for self-use on the investment decision-making.

An important characteristic of distributed power plants is that the generated power is first supplied to meet the self-use demand, and the surplus electricity is then sold to grid companies. In addition, the contribution margin of the sale is often lower than the contribution margin of the self-use portion. Therefore, with fixed values of other parameters, the proportion of generated wind power for self-use has great influence on the equilibrium state of the decision-maker's option game. As seen in Fig. 3, the contribution margin threshold χ_2^* and the expected waiting time $E(t^*)$ decrease with increasing λ, while the investment option value $F(\chi_2)$ is positively related to λ. The optimal investment scale q^*, on the other hand, does not change with the change of λ.

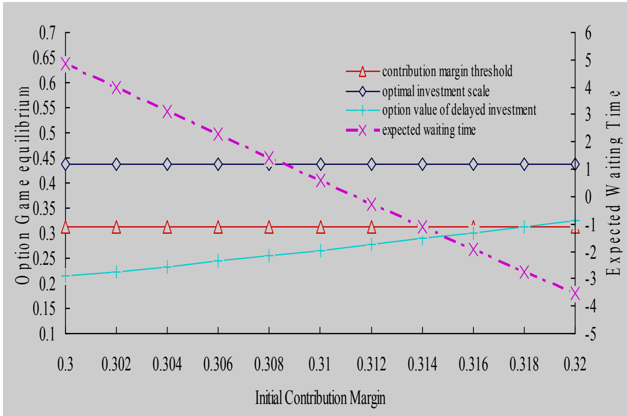

Figure 4. Influence of initial contribution margin on the investment decision-making

Known from Equation (34), $s_\chi > 0$ is the prerequisite for the existence of the expected waiting time $E(t^*)$ of the decision-marker's investment. If the initial contribution margin χ_2 is lower than the contribution margin threshold χ_2^*, the optimal strategy is to delay the investment. As seen in Fig. 4, as χ_2 approaches the threshold contribution margin, 0.311 yuan/kW·h, from the left side, $E(t^*)$ gradually decreases to 0. If the government wants to encourage investments in a distributed wind farm, it should consider how to decrease the expected waiting time of decision-makers. For example, the government should try to increase the expected rate of increase μ_χ of the contribution margin in investing in this wind farm, or to decrease the rate of change σ_χ of the contribution margin.

In addition, the option value $F(\chi_2)$ of the decision-maker's delayed investment is positively related to the initial contribution margin χ_2, while neither the contribution margin threshold χ_2^* nor the optimal investment scale q^* are is related to the change of χ_2.

VI. CONCLUSIONS

In this study, an option game model for determining optimal investment time and scale of distributed wind projects is established based on that distributed power generation first serves self-use, as well as its economic characteristics. The model establishment involves the use of real options theory, and considers the uncertainty of marginal profit increase. The actual decision-making process is divided into two stages. In the first stage, the investment benefit and cost of a wind project is evaluated, and the optimal investment scale and value functions are constructed. In the second stage, the real option value of delayed investment is evaluated, the optimal investment scale is determined, and the contribution margin threshold and its reachability are analyzed. A case study is presented to validate the conclusions of our study and analyze the sensitivity of investment decision-making to the variations of the expected increase and variation rate of contribution margin, the proportion of generated wind power for self-use, and the initial contribution margin.

The results indicate that for distributed wind projects with limitation on investment time, the contribution margin threshold, optimal investment scale, and expected waiting time are all positively related to the expected increase and variation rate of contribution margin. The option value of delayed investment, on the other hand, has different correlations with these two rates, and the significances of the influences are different. In a certain condition and range of contribution margin, the option value of delayed investment and the corresponding expected waiting time increases and decreases, respectively, with the increase of the proportion of generated wind power for self-use or the initial contribution margin. However, the proportion of generated wind power for self-use does not have influence on the optimal investment scale, and the initial contribution margin does not have influence on the investment threshold. The decision-maker's optimal investment decision should take both the contribution margin threshold and the optimal investment scale into consideration.

Although the option game model for investment decision-making developed in this study has practical value, it can be further improved. For example, the average first passage time, possibility of reachability, and the quantitative and qualitative relations between the model parameters are not deduced in detail. In addition, only one type of construction opportunity is assumed, and taxation, rent-seeking cost, and time delay in construction are not considered. In practice, various costs and their variation, the time delay of construction, the diversity and combination of construction opportunities are all meaningful and fruitful research topics. However, one article is far insufficient to discuss all these topics. We will further discuss them in future investigation.

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