Solving Design of Pressure Vessel Engineering Problem Using a Fruit Fly Optimization Algorithm

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Abstract — **We investigate a fruit fly optimization algorithm (FOA) to solve constrained structural engineering design optimization problems. In our work, we compared PSO and QAFSA, and found the FOA to be more valid to search for the optimal solution of three typical functions. As an application, optimal results provided by FOA concerning design of pressure vessel optimization problem are reported, and our result demonstrates that the best solution yielded by FOA is superior to those of state-of-the-art algorithms in the literature.**

Keywords - Fruit fly optimization algorithm, design of pressure vessel, nonlinear constraint.

I INTRODUCTION

During the past several decades there has been a growing interest in nonlinear and constrained problem solving systems based on principles of evolution and hereditary: such as systems maintain a population of potential solutions, they have some selection process based on fitness and individuals. One type of such systems is a class of evolution strategies algorithms ([22]1994), which imitate the principles of natural evolution for parameter optimization problem. The heuristic techniques contain genetic algorithm (GA), simulated annealing (SA), Tabu search (TS), particle swarm optimization (PSO), quantum artificial fish swarm algorithm (QAFSA), artificial bee colony (ABC) etc. [1, 5, 11, 12, 15, 31]. Fruit fly optimization algorithm (FOA) is one of the meta-heuristic approaches firstly considered by Wen-Tsao Pan (2011)[26]. Such an optimization algorithm has advantages such as simple computational process, ease of transformation of such concept into program code and ease of understanding, etc. Very recently, Pan (2011) [26] and QuanKe Pan et al. (2014) [25] have respectively applied FOA into some unimodal and multimodal functions to obtain the approximate optimal results.

However, some structural engineering design problems are general large scale, nonlinear and constrained optimization problems. So as to settle these design optimization problems, there are a growing number of concentrations on artificial intelligence heuristic algorithm, which has been developed with an aim to carry out global search with three purposes: solving problem faster, solving large scale problem, and obtaining robust algorithms. In earlier papers, Deb and Goyal (1986) [7] have presented a method combined binary and real coded GAs to settle the mixed variables. In 2005, Tsai (2005) [29] has put forward a method to solve nonlinear fractional programming problems involved with engineering design optimization. In this paper, we propose the FOA by incorporating the information of the global average maximum, the maximum value, standard deviation and best solution into the search strategy to solve the structural engineering design problem. Based on this, not only can FOA be applied to resolve these simple problems but also can be utilized to handle the structural engineering design optimization problems. The present paper shows that the computation results by FOA are better than those conventional heuristic methods.

An outline of this paper is as follows. In Section 2, the detailed processes of FOA are described to search for global optimal solution for general optimization problems. Three typical functions among PSO, QAFSA and FOA are compared by simulation experiment in Section 3. Section 4 applies FOA to solve design of pressure vessel optimization problem and compared with some other algorithms' results while research conclusions drawn are discussed in Section 5.

II. FRUIT FLY OPTIMIZATION ALGORITHM

The FOA ([26] 2011) is a new approach for searching global optimization, which is based on two main foraging processes: smell the food source by osphresis organ and towards the relevant location to find food by using sensitive vision. The fruit fly on sensory perception is superior to other species, particularly in osphresis and vision, as shown in Fig. 1 ([20] 2013). The osphresis organs of fruit flies can find all kinds of scents floating in the air. When it gets close to the food location, uses its sensitive vision to find food and the companions flocking location, after that fly towards that direction.In this algorithm, based on the food searching behavior of fruit fly, it consists of several essential steps as follows:

Step1. Initialize the fruit fly swarm location randomly and parameters, including maximum number of generations and population size.

$$X\_axis = Value \times rand() \text{ and } Y\_axis = Value \times rand()$$

Step2. Produce the random direction and distance to the search of food depending on osphresis for an individual fruit fly.

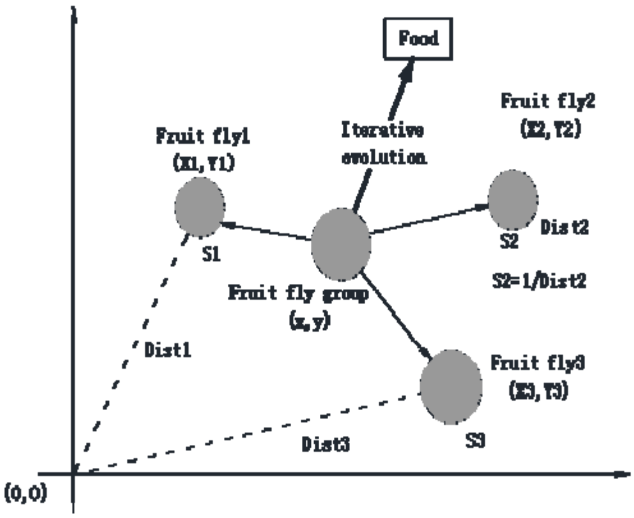

Figure 1. Food finding iterative process of fruit fly swarm.

$$X_i = X\_axis + RandomValue \text{ and } Y_i = Y\_axis + RandomValue$$

Step3. Smell-based searching process: since the food location cannot be known, the original distance is thus estimated first (*Dist*), then the flavor concentration judgment value (*S*) which is the reciprocal of distance is computed. After that, randomly generate several fruit flies around the fruit fly group to set up a population.

$$Dist_i = \sqrt{X_i^2 + Y_i^2}$$
$$S_i = 1/Dist_i$$

Step4. Substitute flavor concentration judgment value (*S*) into the smell concentration judgment function (fitness function) so as to yield the smell concentration of individual location of fruit fly.

$$Smell_i = Function(S_i)$$

Step5. Seek out the maximal smell concentration and note down the location index of the maximal value among the fruit fly swarm.

$$[bestSmell \;\; bestIndex] = \max(Smell)$$

Step6. Keep the best smell concentration value and the location coordinate, then let the fruit fly group fly to the location.

$$Smellbest = bestSmell$$
$$X\_axis = X(bestIndex)$$
$$Y\_axis = Y(bestIndex)$$

Step7. Initiate the iterative optimization and repeat the **Step2-6**, and judge the smell concentration whether is superior to previous iteration, if so, carry out **Step6**.

Step8. Substitute the X_axis and Y_axis from **Step6** into the flavor concentration judgment value (S_i), and then put S_i into the constraints. Afterwards, substitute S_i which meets the constraints into the $Smellbestf_i$ (or called fitness function) and lead to the best optimization solution.

$$Smellbest\_f = Smellbestf(end)$$
$$X\_best = S(end)$$

In the present paper, FOA is applied to cope with design of pressure vessel optimization problem.

III. SIMULATION EXPERIMENT AND ANALYSIS

To verify the feasibility and better convergence precision of the FOA, three typical test functions are proposed and the results obtained by PSO, QAFSA, and FOA are compared. The functions are expressed as follows:

Ex1:

$$\max f_1(x) = \sin\left(\sqrt{\sum_{i=1}^{n} x_i^2}\right) / \sqrt{\sum_{i=1}^{n} x_i^2} + \exp\left(\left(\sum_{i=1}^{n} \cos(2\pi x_i)\right)/2\right) - 2.71289$$
$$s.t. -2 \le x_i \le 2$$

Ex2:

$$\max f_2(x_1, x_2) = \frac{1}{4000}(x_1^2 + x_2^2) - \cos(x_1)\cos\left(\frac{x_2}{\sqrt{2}}\right) + 1$$
$$s.t. -5 \le x_1, x_2 \le 5$$

Ex3:

$$\max f_3(x_1, x_2) = -0.5 - \frac{\sin^2\sqrt{x_1^2 + x_2^2} - 0.5}{[1 + 0.001(x_1^2 + x_2^2)]^2}$$
$$s.t. -100 < x_1, x_2 < 100$$

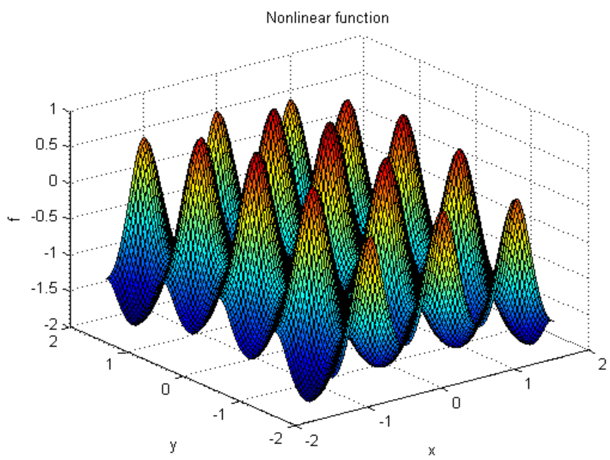

Figure 2. The diagram of $f_1(x)$.

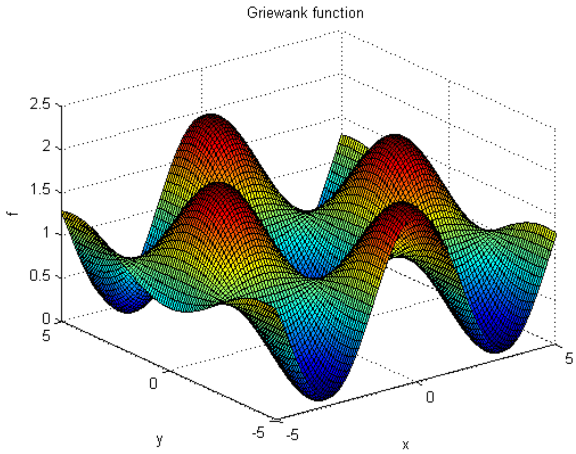

Figure 3. The diagram of $f_2(x)$.

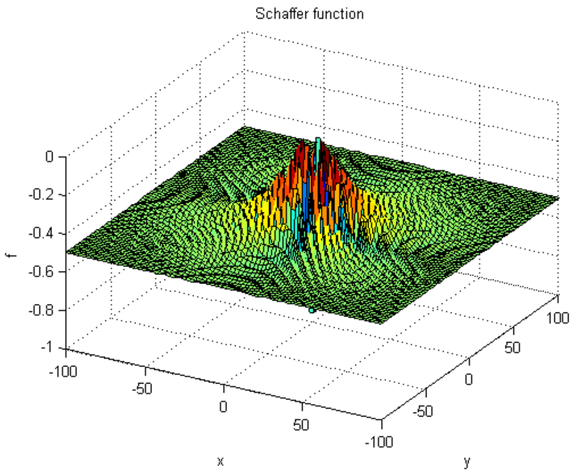

Figure 4. The diagram of $f_3(x)$.

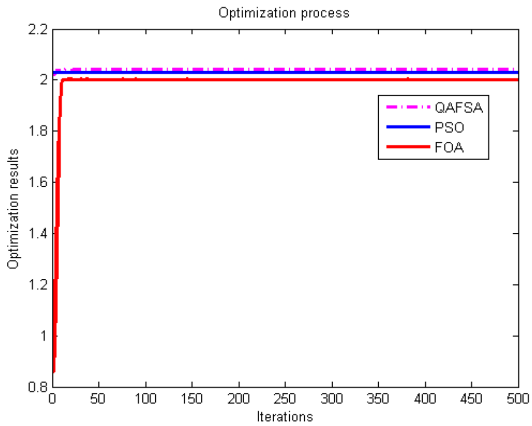

Figure 5. The optimization process of $f_1(x)$.

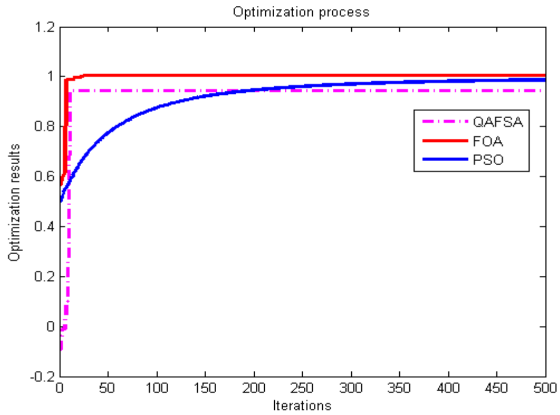

Figure 6. The optimization process of $f_2(x)$.

We can find the optimal values of the three functions are 1, 2 and 0 through the analysis and the function diagrams shown in Fig.2, 3 and 4. The theoretical maximum values of these three multidimensional functions are also 1, 2 and 0 in the definition domain. Using PSO, QAFSA and FOA, we get maximum value of three different functions in Fig. 5, 6, and 7, respectively. The maximum iteration is set to be 500 times. All algorithms run 80 times from different random initial population respectively. The local average maximum value, the global maximum value and standard deviation are to be as a measurement of the performance of algorithm, the results are shown in Table 1.

It has been turned out that the theoretical maximum values of these functions are 1, 2, and 0 in the definition domain respectively. PSO, QAFSA and FOA are used to find the maximum value of three different functions, severally. The maximum iteration is set to be 500 times. All algorithms are run 80 times from different random initial population. The global average maximum value, the global maximum value and standard deviation are used as a measurement of the performance of algorithm, the results are shown in Table **I**:

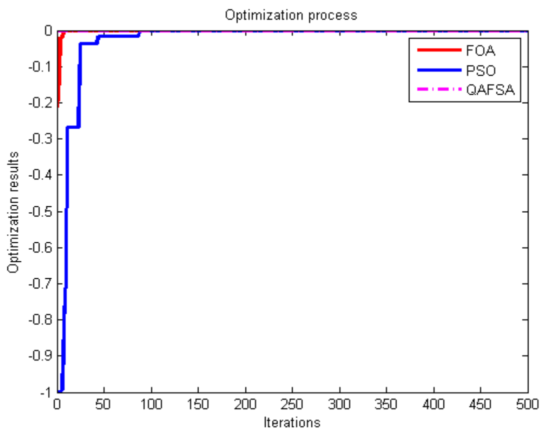

Figure 7. The optimization process of $f_3(x)$.

TABLE I. THE RESULTS OF SIMULATION WITH PSO, QAFSA AND FOA.

	PSO			*QAFSA*			*FOA*		
fun	$A\max$	$G\max$	σ	$A\max$	$G\max$	σ	$A\max$	$G\max$	σ
$f_1(x)$	0.982	1.412	0.624	0.92	0.99	6.01	0.999	1.001	0.015
$f_2(x)$	2.104	2.213	3.391	2.121	2.20	4.951	2.002	2.105	0.103
$f_3(x)$	-0.972	1.124	5.887	-0.998	0.015	2.844	0.000	0.018	0.205

where *fun* denotes function, $A\max$ is global average maximum, $G\max$ is global maximum value, σ is standard deviation, respectively.

This paper uses PSO, QAFSA and FOA for 80 times, gains the average of the evolutionary curve and draws the Fig 5-7. In the Table **I**, the global average maximum value, the global maximum value of the three functions have been achieved and number of iterations by using PSO, QAFSA and FOA. In each Figure, the ordinate is denoted as optimal results of the function, and the abscissa is expressed by evolution generation, three lines with different colors demonstrate the changing tendency of maximum value obtained by PSO, QAFSA and FOA with the increase of the iteration times. From Table **I**, it is clearly indicated that each functions' average optimization result and the best optimal result of FOA are much better than those solved by PSO and QAFSA. Furthermore, the precision of the optimization results are also greater than PSO and QAFSA, most of standard deviations of FOA are nearly 0. Moreover, the stability of FOA in the figures is better than those of PSO and QAFSA by comparing the standard deviation.

IV. DESIGN OF PRESSURE VESSEL OPTIMIZATION PROBLEM

A pressure vessel design model is as shown in Fig. 8, which involves four decision variables: x_1 is defined thickness of the pressure vessel T_s , x_2 stands for thickness of the head T_H , x_3 represents inner radius of the vessel R , and x_4 is on behalf of length of the vessel barring head L , the total variables described as $x=(x_1,x_2,x_3,x_4)$. The objective function of the problem is to minimize the total cost, including the cost of material, forming, and welding. So the general pressure vessel design optimization model can be expressed as:

$$\textit{Minimize} \quad f(x)=0.6224x_1x_3x_4+1.7781x_2x_3^2+3.1661x_1^2x_4+19.84x_1^2x_3$$

$$\textit{s.t.} \quad g_1(x)=-x_1+0.0193x_3\le 0$$

$$g_2(x)=-x_2+0.00954x_3\le 0$$

$$g_3(x)=-\pi x_3^2x_4-\frac{4}{3}\pi x_3^3+1296000\le 0$$

$$g_4(x)=x_4-240\le 0$$

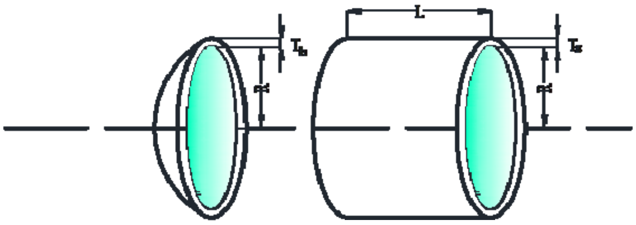

Figure 8. Design of pressure vessel problem.

First of all, let us consider the range of the four decision variables as follows:

$$\text{Region I}: 0.0625\le x_1,x_2\le 99\times 0.0625;\ 10\le x_3,x_4\le 200$$

Many researchers have obtained different results by various algorithms in this region. For example, in early papers, an augmented Lagrangian multiplier method ([16] 1994), genetic adaptive search ([6] 1997), and a branch and bound approach ([28] 1988) have been come up with to conduct that problem. In recent years, in order to dispose of pressure vessel design problem fundamentally, many methods were rapidly blossomed, including a GA-based co-evolution model ([4] 2000), a feasibility-based tournament selection scheme ([5] 2002), cuckoo search ([10] 2003), a co-evolutionary particle swarm optimization ([11] 2007), and an evolution strategy ([23] 2008) and so on. Very recently, hybrid algorithm based on particle swarm optimization with passive congregation ([17] 2009), improved ant colony optimization ([18] 2010), and quantum behaved PSO ([3]2010) etc. have been employed into that problem. The best solution acquired by FOA in the present paper is $f(x)=5896.94890$, the relevant decision variable $x=(x_1,x_2,x_3,x_4)=(0.780956,0.386318,40.433956,198.504889)$, and the corresponding constrains $\left(g_1(x),g_2(x),g_3(x),g_4(x)\right)$

$$=(-0.00058110,-0.00057841,-464.60452458,-41.49511095)$$

.The all solutions from different approaches in Region I about this problem have been compared in Table II, which can be concluded that FOA is of superior searching method for this problem.

TABLE II. COMPARISON OF THE BEST SOLUTION FOR PRESSURE VESSEL IN REGION I BY DIFFERENT METHODS.

Method	Design Variables				$f(x)$
	x_1	x_2	x_3	x_4	
Sandgren ([28] 1988)	1.125000	0.625000	47.700000	117.701000	8129.1036
C.Zhang et al. ([30] 1993)	1.125000	0.625000	58.290000	43.6930000	7197.7000
Kannan et al.([16] 1994)	1.125000	0.625000	58.291000	43.690000	7198.0428
K.Deb et al.([6] 1997)	0.937500	0.500000	48.329000	112.67900	6410.3811
Coello ([4] 2000)	0.812500	0.437500	40.323900	200.000000	6288.7445
Coello et al.([5] 2002)	0.812500	0.437500	42.097398	176.654050	6059.946
X.H.Hu et al. ([14] 2003)	0.812500	0.437500	42.098450	176.6366000	6059.131296
Gandomi et al. ([10] 2003)	0.812500	0.437500	42.0984456	176.6365958	6059.7143348
S.He et al. ([12] 2004)	0.812500	0.437500	42.098445	176.6365950	6059.7143
K.S.Lee et al. ([19] 2005)	1.125000	0.625000	58.278900	43.75490000	7198.433
Montes et al. ([24] 2007)	0.812500	0.437500	42.098446	176.6360470	6059.701660
Q.He et al. ([11] 2007)	0.812500	0.437500	42.091266	176.746500	6061.0777
Montes et al. ([23] 2008)	0.812500	0.437500	42.098087	176.640518	6059.7456
Cagnina et al.([2] 2008)	0.812500	0.437500	42.098445	176.6365950	6059.714335
A.Kaveh et al. ([17] 2009)	0.812500	0.437500	42.103566	176.573220	6059.0925
A.Kaveh et al. ([18] 2010)	0.812500	0.437500	42.098353	176.637751	6059.7258
L.S.Coelho ([3] 2010)	0.812500	0.437500	42.098400	176.6372000	6059.7208
B.Akay et al. ([1] 2012)	0.812500	0.437500	42.098446	176.636596	6059.714339
Present study	**0.780956**	**0.386318**	**40.433956**	**198.504889**	**5896.94890**

In Region I , due to the upper bound of x_4 is 200, at this point the fourth constraint condition is satisfied automatically. In order to take all constraints into account, the upper bound of x_4 is adjusted to 240 ([12] 2007). In the light of this, the region is denoted as Region II .

$$\text{Region II}: 0.0625 \le x_1, x_2 \le 99 \times 0.0625; 10 \le x_3 \le 200; 10 \le x_4 \le 240$$

TABLE III. COMPARISON OF THE BEST SOLUTION FOR PRESSURE VESSEL IN REGION II BY DIFFERENT METHODS.

Method	Design Variables				$f(x)$
	x_1	x_2	x_3	x_4	
Hedar et al. ([13] 2006)	0.7683257	0.3797837	39.8096222	207.2255595	5868.764836
Mahdavi et al. ([21] 2007)	0.75	0.375	38.86010	221.36553	5849.76169
Dimopoulos ([8] 2007)	0.75	0.375	38.86010	221.36549	5850.38306
Gandomi et al. ([9] 2011)	0.75	0.375	38.86010	221.36547	5850.38306
Present study	**0. 73416509**	**0. 36346997**	**38.03065892**	**234.73387898**	**5821.19232**

In this scope, Hedar and Fukushima (2006) [20], Dimopoulos (2007) [12], Mahdavi (2007) [28], and Gandomi et al. (2011) [14] have investigated the problem by multifarious methods. The best outcome found by FOA is $f(x) = 5821.19232$, where $x = (x_1, x_2, x_3, x_4)$

$= (0.73416509, 0.36346997, 38.03065892, 234.73387898)$,

and constraints $(g_1(x), g_2(x), g_3(x), g_4(x))$

$= (-0.00017337, -0.00065749, -983.86311861, -5.26612101)$.

Table III shows a comparison between the traditional methods and FOA, which demonstrates that the solutions gained by FOA are better than those of algorithms.

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TABLE IV. STATISTICAL RESULTS OF DIFFERENT APPROACHES FOR PRESSURE VESSEL (NA MEANS NOT AVAILABLE).

	Method	Best	Mean	Worst	Std-dev
	Sandgren([28] 1988)	8129.1036	NA	NA	NA
	Kannan et al.([16] 1994)	7198.0428	NA	NA	NA
	K.Deb et al([6] 1997)	6410.3811	NA	NA	NA
	Coello ([4] 2000)	6288.7445	6293.8432	6308.1497	7.4133
	Coello et al ([5] 2002)	6059.9463	6177.2533	6469.3220	130.9297
	Gandomi et al.([10] 2003)	6059.714	6447.7360	6495.3470	502.693
	S.He et al.([12] 2004)	6059.7143	6289.92881	NA	305.78
	B.Akay et al.([1] 2012)	6059.714339	6245.308144	NA	205
	Montes et al.([23] 2008)	6059.7456	6850.0049	7332.8798	426.0000
	Kaveh et al.([18] 2010)	6059.7258	6081.7812	6150.1289	67.2418
	AKaveh et al.([17] 2009)	6059.0925	6075.2567	6135.3336	41.6825
	Coello ([3] 2010)	6059.7208	6440.3786	7544.4925	448.4711
	Q.He et al.([11] 2007)	6061.0777	6147.1332	6363.8041	86.4545
	Cagnina et al.([2] 2008)	6059.714335	6092.0498	NA	12.1725
	Present study	**5896.948902**	**5899.605374**	**5929.977026**	**5.840479**
	Hedar et al.([13] 2006)	5868.764836	6164.585867	6804.328100	257.4736
	Mahdavi et al([21] 2007)	5849.7617	NA	NA	NA
	Dimopoulos ([8] 2007)	5850.38306	NA	NA	NA
	Gandomi et al.([9] 2011)	5850.38306	5937.33790	6258.96825	164.5474
	Present study	**5821.192321**	**5824.370489**	**5866.307048**	**4.730923**

According to the statistical simulation results, summed up in Table IV, which can be seen that the average searching ability of FOA is better than those of other algorithms applied in the literature from [8, 9, 13, 21]. Moreover, the standard deviation of the outcome acquired by FOA is smaller comparatively after running 50 trials with MATLAB independently. We can see that the stability of FOA is better than those of other approaches by comparing the standard deviation.

V. CONCLUSION

This paper introduces FOA, which indicates the better searching ability of FOA such as effectiveness and robustness than PSO and QAFSA via three typical functions simulation experiment. Then FOA is applied to solve a design of pressure vessel optimization problem. Since the procedure of FOA is relatively uncomplicated, it is effortless to understand. Furthermore, FOA is more convenient and suitable to deal with some engineering design optimization problems. However, there are some shortcomings of that algorithm, for example, the decision value of the taste concentration is the reciprocal of the distance, which has limited that variables should be positive values, the scope of FOA is narrowed by this limitation, and that may be improved in further study.

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