A Study on the Stress Intensity Factors of a Square Defect in Infinite Body

Baoliang LIU 1,2, Guangping ZOU 1

1College of Aerospace and Civil Engineering, Harbin Engineering University, Harbin 150001, China
2Science college, Heilongjiang Institute of Science and Technology, Harbin 150022, China
3Research Laboratory on Composite Materials, Harbin Institute of Technology, Harbin 150001, China

Abstract — This paper deals with a type of surface crack problem with an approximately same depth by using the hybrid displacement discontinuity method (a boundary element method) proposed recently by the author. Based on surface rectangular crack in infinite solid in tension and using the hybrid displacement discontinuity method, a numerical approach is presented. By changing the defect geometry parameters, the effect of geometry parameters (of the square defect in infinite body in tension) on the Stress Intensity Factors, SIFs, is revealed in detail. It is illustrated that the boundary element method is simple yet accurate for calculating the SIFs of complex crack problems in finite plate.

Keywords - Stress Intensity Factors, SIF; Hybrid displacement discontinuity method; Surface crack

I. INTRODUCTION

In aircraft structures, fatigue failures usually occur from the initiation and propagation of cracks from notches or defects in the material that are either embedded, on the surface, or at a corner. These cracks propagate with elliptic or near-elliptic crack fronts. To predict crack-propagation life and fracture strength, accurate stress-intensity factor solutions are needed for these crack configurations. Very few exact solutions for three-dimensional cracked bodies are available in the literatures. One of these, an elliptical crack in an infinite solid subjected to uniform tension, was derived by Irwin [1] using an exact stress analysis by Green and Sneddon [2]. Kassir and Sih [3], Ahah and Kobayashi [4], and Vijayakumar and Atluri [5] have obtained closed-form solutions for an elliptical crack in an infinite solid subjected to non-uniform loadings.

This paper deals with such a kind of surface crack problem with an approximately same depth. As an example, shown in Fig.1 is a schematic of a pair of cracks emanating from a surface square defect in infinite body in tension, which is also called a surface defect crack problem. Based on the previous investigations [6-9] on internal rectangular crack and surface rectangular crack in infinite solid in tension and a hybrid displacement discontinuity method (a boundary element method) proposed recently by Ref. [10], a numerical approach for the liked-plane crack problem in hand is presented. Numerical examples are given to illustrate the numerical approach is simple, yet accurate for calculating the SIFs of a liked-plane crack. Specifically, a surface hole crack problem shown in Fig.1 is investigated in detail.

By the way, it is pointed out that a surface crack is usually treated with as a three-dimensional problem to analyze. For this, many results have been obtained, see the documented Ref. [1]. Especially, Newman and his coauthors [11-13] used the finite element method and Murakami and his coauthors [14] and Isida and his coauthors [15-16] used the body force method to calculate the SIFs of surface cracks. To our knowledge, the solution to the crack problem under consideration in this paper has not been obtained.

In addition, here, it is pointed out that finite element simulations [17-18] when used to analyze crack problems have to face largely computational problems connected with the discretization of the continuum into finite elements, particularly when some cracks propagate, thus changing the interior boundaries of the solids. Recently, it was found from the previous investigation [19-24] that the hybrid displacement discontinuity method has huge robustness in analyzing complex plane elastic crack problems, including a multiple-hole-crack interaction problem, a multiple crack interaction problem.

II. BRIEF DESCRIPTION OF THE HYBRID DISPLACEMENT DISCONTINUITY METHOD

In this section, the hybrid displacement discontinuity method presented by Ref. [23] is described briefly. It consists of the constant displacement discontinuity element presented by Crouch and Starfield [24] and the crack-tip displacement discontinuity elements.
A. Constant Displacement Discontinuity Method

The displacement discontinuity $D_i$ is defined as the difference in displacement between the two sides of the segment [24] (see Fig. 1):

$$D_x = u_x(x,0_+) - u_x(x,0_-),$$
$$D_y = u_y(x,0_+) - u_y(x,0_-).$$

(1)

The solution to the subject problem is given by Crouch and Starfield [32]. The displacements and stresses can be written as

$$u_x = D_x[(2-2v)F_1(x,y) - yF_2(x,y)] + D_y[-(1-2v)F_1(x,y) - yF_2(x,y)],$$
$$u_y = D_y[(2-2v)F_2(x,y) + yF_1(x,y)] + D_x[-(1-2v)F_2(x,y) + yF_1(x,y)],$$

and

$$\sigma_{xx} = 2GD_x[2F_1(x,y) + yF_2(x,y)] + 2GD_y[-F_1(x,y) + yF_2(x,y)],$$
$$\sigma_{yy} = 2GD_y[-yF_1(x,y)] + 2GD_x[-F_2(x,y) - yF_1(x,y)],$$
$$\sigma_{xy} = 2GD_x[-yF_2(x,y)] + 2GD_y[-F_1(x,y) + yF_2(x,y)].$$

(2)

Function $f(x, y)$ in these equations can be written as:

$$f(x, y) = \frac{1}{4\pi(1-v)} \left[ \frac{y(\arctan \frac{y}{x-a} - \arctan \frac{y}{x+a})}{x-a} \ln \left[ (x-a)^2 + y^2 \right] + (x+a) \ln \left[ (x+a)^2 + y^2 \right] \right]$$

(4)

$G$ and $v$ in these equations are shear modulus and Poisson’s ratio, respectively. Functions $F_2$ through $F_7$ are described in Ref. [23]. Eqs (2) and (3) are used by Crouch and Starfield [28] to set up a constant displacement discontinuity boundary element method.

B. Crack-Tip Displacement Discontinuity Elements

By using the Eqs (2) and (3), recently, Yan [10] presented crack-tip displacement discontinuity elements, which can be classified as the left and the right crack-tip displacement discontinuity elements to deal with crack problems in general plane elasticity. The following gives basic formulas of the left crack-tip displacement discontinuity element.

For the left crack-tip displacement discontinuity element (see Fig.2), its displacement discontinuity functions are chosen as

$$D_x = H_s \left( \frac{a_{tip}}{a_{tip}} \right)^{\frac{1}{2}},$$
$$D_y = H_s \left( \frac{a_{tip}}{a_{tip}} \right)^{\frac{1}{2}}.$$

(5)

where $H_s$ and $H_n$ are the tangential and normal displacement discontinuity quantities at the center of the element, respectively, $a_{tip}$ is a half length of crack-tip element. Here, it is noted that the element has the same unknowns as the two-dimensional constant displacement discontinuity element. But it can be seen that the displacement discontinuity functions defined in (5) can model the displacement fields

Function $f(x, y)$ in these equations can be written as:

$$f(x, y) = \frac{1}{4\pi(1-v)} \left[ \frac{y(\arctan \frac{y}{x-a} - \arctan \frac{y}{x+a})}{x-a} \ln \left[ (x-a)^2 + y^2 \right] + (x+a) \ln \left[ (x+a)^2 + y^2 \right] \right]$$

(6)

around the crack tip. The stress field determined by the displacement discontinuity functions (5) possesses $r^{-1/2}$ singularity around the crack tip.

Based on the Eqs (2) and (3), the displacements and stresses at a point $(x, y)$ due to the left crack-tip displacement discontinuity element can be obtained,

$$u_x = H_s \left[ (2-2v)B_1(x,y) - yB_2(x,y) \right] + H_n \left[ -(1-2v)B_3(x,y) - yB_4(x,y) \right],$$
$$u_y = H_n \left[ (2-2v)B_2(x,y) + yB_1(x,y) \right] + H_s \left[ -(1-2v)B_4(x,y) + yB_3(x,y) \right],$$

(6)

and

$$\sigma_{xx} = 2GH_s \left[ 2B_2(x,y) + yB_1(x,y) \right] + 2GH_n \left[ -B_3(x,y) + yB_4(x,y) \right],$$
$$\sigma_{yy} = 2GH_s \left[ -yB_1(x,y) \right] + 2GH_n \left[ -B_4(x,y) - yB_3(x,y) \right],$$
$$\sigma_{xy} = 2GH_n \left[ -B_3(x,y) + yB_4(x,y) \right] + 2GH_s \left[ -yB_2(x,y) \right].$$

(7)

where functions $B_2$ through $B_7$ are described in Ref. [23].

C. Computational Formulas of the Stress Intensity Factors
The objective of many analyses of linear elastic crack problems is to obtain the SIFs \( K_I \) and \( K_{II} \). Based on the displacement field around the crack tip, the following formulas exist

\[
K_I = \frac{\sqrt{2\pi G H_I}}{4(1-\nu)\sqrt{a_{tip}}} , \quad K_{II} = \frac{\sqrt{2\pi G H_{II}}}{4(1-\nu)\sqrt{a_{tip}}}. \tag{8}
\]

III. NUMERICAL APPROACH

A surface crack is usually treated with as a three-dimensional problem to analyze, which undoubtedly is very complex. This paper is concerned with such a kind of surface crack with an approximately same depth, which is called a liked-plane crack. By using the solution of the liked-plane crack problem shown in Fig.3, in this section, we try to present a numerical approach to treat with a liked-plane crack problem.

We imagine that a plane elasticity crack body is separated from the three-dimensional surface crack body (Fig.3): it has the same form crack as that of the three-dimensional surface crack body and is subjected to the same form load as that of the three-dimensional surface crack body. Further it is assumed that there is a force \(-k_{ui}\) acted on crack surface shown in Fig.4 (b), where the spring constant \(k\) will be identified later.

Now consider the formulation of the plane elastic crack problem shown in Fig.4 (b). When three is not the force \(-k_{ui}\) acted on crack surface shown in Fig.4 (b), boundary element equations can be written as [10]

\[
\sum_{j=1}^{N} A^{ij}_{ss} D^{j}_s + \sum_{j=1}^{N} A^{ij}_{sn} D^{j}_n = \sigma^{j}_s , \tag{9}
\]

\[
\sum_{j=1}^{N} A^{ij}_{ns} D^{j}_s + \sum_{j=1}^{N} A^{ij}_{nn} D^{j}_n = \sigma^{j}_n , \tag{9}
\]

where \(N\) is the number of total boundary elements, \(A_{ijmn}\) etc. are the influence coefficients, which are how to
be calculated, to constant displacement discontinuity elements and crack-tip elements, are described respectively in Refs [32,10], and \( D^j_s \) and \( D^j_n \) are displacement discontinuities. When considering the acted force \(-kui\), see Fig.4 (b), boundary element equations of the plane elastic crack problem shown in Fig.4(b) can be represented as

\[
\begin{align*}
\sum_{j=1}^{N} A^i_{ss} D^j_s + \sum_{j=1}^{N} A^i_{sn} D^j_n &= \sigma^i_s + k\alpha^i_s, \\
\sum_{j=1}^{N} A^i_{ms} D^j_s + \sum_{j=1}^{N} A^i_{mn} D^j_n &= \sigma^i_n + k\alpha^i_n,
\end{align*}
\]

(10)

By substituting the displacement discontinuity \( D_i \) for the displacement \( u_i \) in Eqs (10), Eqs (10) can be written as

\[
\begin{align*}
\sum_{j=1}^{N} A^i_{ss} D^j_s + \sum_{j=1}^{N} A^i_{sn} D^j_n &= \sigma^i_s - 0.5kD_s, \\
\sum_{j=1}^{N} A^i_{ms} D^j_s + \sum_{j=1}^{N} A^i_{mn} D^j_n &= \sigma^i_n - 0.5kD_n,
\end{align*}
\]

(11)

Further Eqs (11) can be rewritten as

\[
\begin{align*}
(A^i_{ss} + 0.5k)D^j_s + \sum_{j=1}^{N} A^i_{sn} D^j_n &= \sigma^i_s, \\
\sum_{j=1}^{N} A^i_{ms} D^j_s + \sum_{j=1}^{N} A^i_{mn} D^j_n &= \sigma^i_n + (A^i_{ss} + 0.5k)D^j_n
\end{align*}
\]

(12)

IV. NUMERICAL RESULTS AND DISCUSSIONS

Shown in Fig.3 is a pair of cracks emanating from a surface square hole with a depth \( h \) subjected to uniform stress \( \sigma \) at infinity. For this crack problem, the symmetry condition can be used. The following geometry parameters are considered

\[
a/c = 1.00, 1.01, 1.02, 1.04, 1.06, 1.08, 1.10, 1.15, 2.0, 5.0, 10.0 \\
h/a = 1.0, 1.1, 1.2, 1.3, 1.4, 1.6, 1.8, 2.0
\]

Regarding discretization, number of boundary elements on a branched crack and a quarter of the square hole is denoted by \( n \) and \( n_c \) respectively. The calculated SIFs normalized by \( \sigma \sqrt{ma} \) are listed. Fig.5 shows variation of the normalized SIFs at the crack tip \( A \) with \( a/c \) and \( h/a \) for three cases: \( h/a = 1.0, 1.4, \) and 2.0. In order to well reveal the effect of a surface hole the SIFs of crack(s) emanating from the surface hole; here, the SIFs of the surface hole crack problem and the surface rectangular crack are denoted by \( K_{ls_1}(a/c, h/a) \) and \( K_{ls_1}(h/a) \), respectively. Their ratio is denoted by \( F_{ls_1}(a/c, h/a) \), i.e.,

\[
F_{ls_1}(a/c, h/a) = K_{ls_1}(a/c, h/a)/K_{ls_1}(h/a)
\]

(13)

which is also called a normalized SIFs. The normalized SIFs corresponding to \( h/a = 1.3 \) are shown in Fig.6.
decrease slowly and $F_{Ishc}$ almost equals 1 (i.e., $F_{Ishc}$ almost equals $K_{Ishc}$) when $a_c$ is large enough.

After introducing the dimensionless parameters $a_{cc}$, $a_{cm}$ and $F_{Ishcm}$, it is found that: As $a_c < a_{cc}$, the surface square hole has a shielding effect on the cracks emanating from the surface hole. And the closer the size of the surface square hole is to that of the surface crack, the stronger the shielding effect is. As $a_c > a_{cc}$, the surface square hole has an amplifying effect on the SIFs of the surface crack and the amplifying effect is the most obvious at $a_c = a_{cm}$.

As $a_c$ is large enough, i.e., the size of the surface square hole is small enough relative to that of the surface crack, the effect of the surface square hole on the SIFs of the surface crack is almost neglected. It can be seen from Table 8 that the dimensionless parameters $a_{cc}$, $a_{cm}$ and $F_{Ishcm}$ are almost independent on $h/a$.

V. CONCLUSIONS

A numerical approach for square defect in infinite body in tension was presented in this paper. Numerical results showed that the numerical approach is simple, yet accurate for calculating the SIFs of square defect in infinite body in tension. The SIFs of a pair of cracks emanating from a surface square defect in infinite body in tension are analyzed in detail. It is found that there are the dimensionless parameters $a_{cc}$, $a_{cm}$ and $F_{Ishcm}$ almost independent on $h/a$, which can be used to reveal the effect of a surface square defect on cracks emanating from the surface defect.

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