Dynamic Damage Constitutive Model of Rock Based on Piecewise Functions

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***Abstract* — By using RMT-301 rock and concrete mechanics test system, uniaxial compression tests of dry and wet sandstone were performed in the laboratory. For the reason of compaction effect of sandstone sample, the predicted results of stress-strain curves by traditional damage constitutive models are greatly deviated from the test results. Based on the deformation characteristics of sandstone sample, the stress-strain curve is divided into linear elastic stage at the pre peak and the damage stage in the post peak. The micro unit strength is described by the power function and Weibull distribution function respectively, the predicted results of the two models show that piecewise damage constitutive model can well describe the characteristics of test curve, and the power function is better than Weibull distribution function in forecasting precision. The research results provide guidance for the design and construction of underground engineering.**

Keywords - sandstone; constitutive model; damage; power function; Weibull distribution function

I. INTRODUCTION

The strength of rock and its deformation performances are the main research contents of rock mechanics, and accurate constitutive model and its parameters are the key steps in the analysis of stress and strain. It should be noted that damage mechanics is an important method to study the constitutive model of rock. There are two methods to study the damage mechanics of rock, the first is the phenomenological method, namely the continuum damage mechanics, and the damage is described by internal variable from macroscopic. The second is to use the concept and statistics method, assuming that the micro unit strength of the rock is subject to random distribution, so the damage evolution equation of the rock is deduced [1]. In previous studies, the distribution equations of micro unit strength are not the same, including Weibull distribution [2-4], power function distribution [5] and normal distribution [6-8], etc. But for the compaction effect of sandstone sample, the predicted results of stress-strain curves by traditional damage constitutive models are greatly deviated from the test results. In this paper, two new piecewise damage constitutive models are proposed. The micro unit strength is described by the power function and the Weibull distribution respectively, and the predicted results of the two models are compared with the test results. The research results are beneficial to the design and construction of underground engineering.

II. PIECEWISE DAMAGE CONSTITUTIVE MODEL

The mechanical tests of sandstone samples were performed by the RMT-301 rock and concrete mechanics test system, as shown in Fig.1.Test samples are shown in Fig.2. Under uniaxial compressive condition, the stress-strain cruves of dry and wet sandstone samples are shown in Fig.3. For the reason of compaction effect of sandstone, the predicted results of stress-strain curves by traditional damage constitutive models are greatly deviated from the test results in the post peak, so the piecewise damage constitutive model is introduced to describe these characteristics.

In damage mechanics, the constitutive models must satisfy the following assumptions: (1)The maximum pressure strain of sandstone sample is 2%; (2) In the three-piecewise model, the ending of linear elastic is 75% peak strain; (3) Assuming that the damage is extended in the last paragraph, the others are elastic deformation paragraph with initial damage, and accord with Hooke's law; (4) The rock is isotropic material in macroscopically, and the expanding of damage is isotropic also; (5) The micro unit strength accords

Figure 1. RMT-301 rock and concrete mechanics test system

Figure 2. Standard sandstone samples

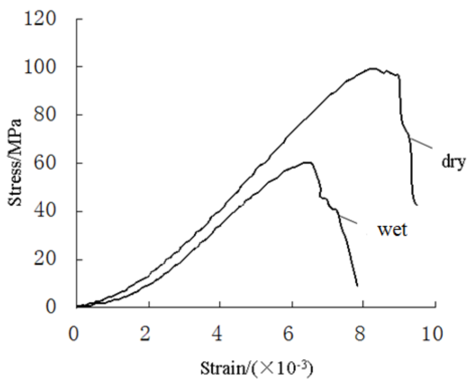


Figure 3. Stress-strain cruves of sandstone samples

with Hooke's law before the failure of rock; (6) When the load reaches a certain value and the damage begins to expand, the micro unit strength accords with power function or Weibull distribution respectively.

A. Piecewise Damage Constitutive Model Based on Power Function

It is assumed that the stress-strain curve obeys Hooke's law at the pre peak and the micro unit strength can be described by power function in the post peak. The probability density function of the micro unit strength can be expressed as follows:

$$P(\varepsilon)=\frac{m}{\varepsilon_0}\left(\frac{\varepsilon-\varepsilon_a}{\varepsilon_0}\right)^{m-1} \tag{1}$$

Where ε is a random variable of the micro unit strength, m and ε_0 are the distribution parameters, ε_a is a constant, in the two-piecewise model $\varepsilon_a=2‰$, in the three-piecewise model $\varepsilon_a=0.75\varepsilon_t$, and ε_t is the peak strain of the sample.

It is assumed that the damage variable D is the ratio of failure unit numbers n to total unit numbers N. So the damage variable can be expressed as follows:

$$D=n/N \tag{2}$$

When the parameter ε reaches ε_a, the micro unit strength begins to meet the damage condition. If the load reaches a specific value, the failure unit numbers will be:

$$n(\varepsilon)=\int_{\varepsilon_a}^{\varepsilon}NP(x)d(x)=N\left(\frac{\varepsilon-\varepsilon_a}{\varepsilon_0}\right)^m \tag{3}$$

So the damage variable can be expressed as follows:

$$D=\left(\frac{\varepsilon-\varepsilon_a}{\varepsilon_0}\right)^m \tag{4}$$

Based on the theory of continuum damage mechanics, the two and three piecewise damage constitutive models are obtained:

$$\sigma=\begin{cases}\alpha kE\varepsilon,\varepsilon\le 2‰\\(1-D)\alpha E[\varepsilon-(1-k)2‰],2‰\le\varepsilon\end{cases} \tag{5}$$

$$\sigma=\begin{cases}\alpha kE\varepsilon,\varepsilon\le 2‰\\\alpha E[\varepsilon-(1-k)2‰],2‰\le\varepsilon\le 0.75\varepsilon_t\\(1-D)\alpha E[\varepsilon-(1-k)2‰],0.75\varepsilon_t\le\varepsilon\end{cases} \tag{6}$$

Based on the above two conditions that the derivative of stress-strain curve in peak point is equal to zero and the values of peak intensity σ_t and peak strain ε_t can be obtained simultaneously, the distribution parameters m and ε_0 can be expressed as follows.

$$m=\frac{\sigma_t}{\alpha E[\varepsilon_t-(1-k)2‰]-\sigma_t}\left(\frac{\varepsilon_t-\varepsilon_a}{\varepsilon_t-(1-k)2‰}\right) \quad 7)$$

$$\varepsilon_0=\exp\left(\ln(\varepsilon_t-\varepsilon_a)-\frac{1}{m}\ln\left(1-\frac{\sigma_t}{\alpha E[\varepsilon_t-(1-k)2‰]}\right)\right) \tag{8}$$

B. Piecewise Damage Constitutive Model Based on Weibull Distribution

It is assumed that the stress-strain curve obeys Hooke's law at the pre peak and the micro unit strength is described by Weibull distribution in the post peak. The probability density function of the micro unit strength can be expressed as follows:

$$P(F)=\frac{m}{F}\left(\frac{F-F_a}{F_0}\right)^{m-1}\exp\left[-\left(\frac{F-F_a}{F_0}\right)^m\right] \tag{9}$$

Where F is a random variable of the micro unit strength, m and F_0 are the distribution parameters, F_a is a constant, in the two-piecewise model $F_a=2‰$, in the three-piecewise model $F_a=0.75\varepsilon_t$, and ε_t is the peak strain of the sample.

The damage variable D is defined the same as Equation (2), when the parameter F reaches F_a, the micro unit strength begins to meet the damage theory. If the load reaches a specific value, the failure unit number will be:

$$n(F)=\int_{F_a}^{F} NP(x)d(x)=N\left\{1-\exp\left[-\left(\frac{F-F_a}{F_0}\right)^m\right]\right\} \tag{10}$$

The damage variable can be defined as follows:

$$D=1-\exp\left[-\left(\frac{F-F_a}{F_0}\right)^m\right] \tag{11}$$

The piecewise damage constitutive models are the same as Equation (5) and Equation (6). Based on the above two conditions that the derivative of stress-strain curve in peak point is equal to zero and the values of peak intensity σ_t and peak strain ε_t can be obtained simultaneously, the distribution parameters m and F_0 can be expressed as follows:

$$m=\frac{(\varepsilon_t-F_a)}{\left[\varepsilon_t-(1-k)2‰\right]\ln\left(\frac{\alpha E\left[\varepsilon_t-(1-k)2‰\right]}{\sigma_t}\right)} \tag{12}$$

$$F_0=\exp\left\{\ln(\varepsilon_t-F_a)-\frac{1}{m}\ln\left(\ln\left(\frac{\alpha E\left[\varepsilon_t-(1-k)2‰\right]}{\sigma_t}\right)\right)\right\} \tag{13}$$

III. Verification of Piecewise Damage Constitutive Model

Based on the uniaxial compression test results of dry and wet sandstone samples, the distribution parameters m and ε_0 in two and three piecewise damage constitutive models are obtained, as shown in Table Ⅰ.

TABLE Ⅰ. DISTRIBUTION PARAMETERS OF POWER FUNCTION

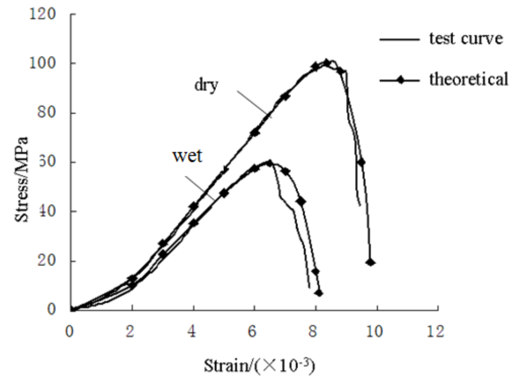
Distribution parameters	Two-piecewise model dry	Two-piecewise model wet	Three-piecewise model dry	Three-piecewise model wet
m	12.6624	7.0161	4.1593	2.5346
ε_0	0.007901	0.006172	0.0040292	0.003908

The distribution parameters m and F_0 in two and three piecewise damage constitutive models are obtained also, as shown in Table Ⅱ.

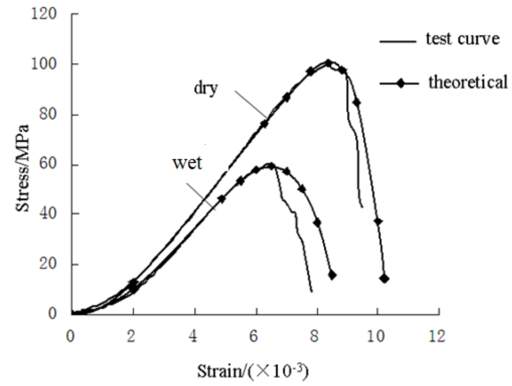
TABLE Ⅱ. WEIBULL DISTRIBUTION PARAMETERS

Distribution parameters	Two-piecewise model dry	Two-piecewise model wet	Three-piecewise model dry	Three-piecewise model wet
m	13.1056	7.4324	4.3036	2.6850
F_0	0.007824	0.006018	0.003911	0.003642

When the micro unit strength is described by power function in the post peak, the piecewise damage constitutive model could be obtained by introducing the calculated parameters. The theoretical curves of dry and wet sandstone samples were comparied with the test curves, as shown in Fig.4.



Two-piecewise model

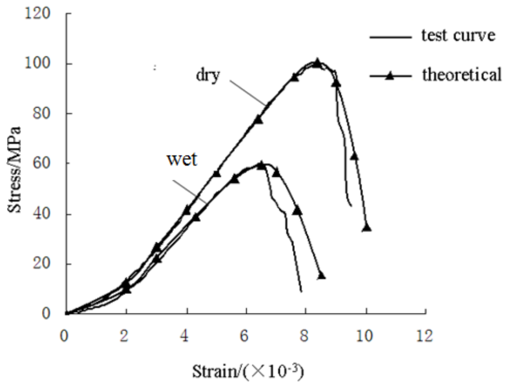


Three-piecewise model

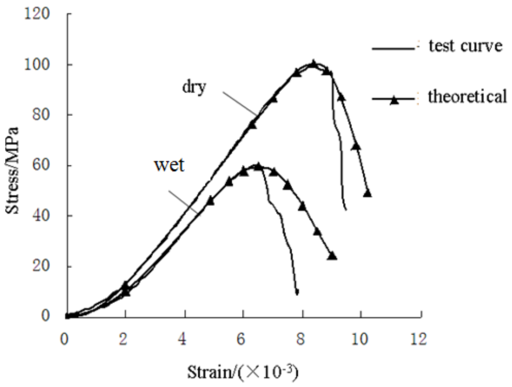
Figure 4. The comparison of theoretical curves and test curves by power function

By the comparison of theoretical curves and test curves of dry and wet sandstone samples, it is known that the theoretical curves of two-piecewise model have a good agreement with test curves, but the predicting precision is less than three-piecewise model at the pre peak. In these two models, the consistent degree between theoretical curve and test curve reduces in the post peak for the increase in humidity of sandstone. For the wet sandstone, the predicting precision of two-piecewise model is better than three-piecewise model, but this trend is not obvious for dry sandstone. Because of the large deviation of the single segment model, the multi-piecewise damage constitutive model could be a better solution.

When the micro unit strength is described by Weibull dis-tribution in the post peak, the piecewise damage constitutive model could be obtained by introducing the calculated para-meters also. The theoretical curves of dry and wet sandstone samples were comparied with the test curves, as shown in Figure 5.



Two-piecewise model



Three-piecewise model

Figure 5. The comparison of theoretical curves and test curves by Weibull distribution

By the comparison of theoretical curves and test curves of dry and wet sandstone samples, it is known that the theoretical curves of two-piecewise model have a good agreement with test curves, and the predicting precision is better than three-piecewise model in the post peak. Similar to power function, the consistent degree of theoretical curve and test curve reduces in the post peak for the increase in humidity of sandstone. For the wet sandstone, the predicting precision of two-piecewise model is better than three-piecewise model, but this trend is not obvious for dry sandstone also. Comparied with the traditional Weibull distribution models, the new model has a better predicting precision on stress-strain relationship.

IV. COMPARISON AND DISCUSSION OF TWO CONSTITUTIVE MODELS

For the piecewise damage constitutive models under the uniaxial compression, the predicting precision of power function is better than Weibull distribution, but this conclusion is contrary to literature [9] under triaxial compression. Under the uniaxial compression, the small deformation in the post peak belongs to the sudden brittle failure, so it is reasonable to describe the damage variable by power function. But under triaxial compression, the subsequent deformation of stress-strain curve is more prominent, so the Weibull distribution is suitable to desceibe this characteristic.

V. CONCLUSIONS

Based on test results of dry and wet sandstone samples, two types of damage constitutive models were established. By the comparison and analysis, the following conclusions are obtained.

Considering the compaction effect of sandstone sample, the damage constitutive model can be described in two or three paragraphs. The predicted results by using piecewise damage constitutive models have a good agreement with test results.

The two and three piecewise damage constitutive models have their advantages and disadvantages, the predicting precision of three-piecewise model is better than two-piecewise model at the pre peak, but worse than two-piecewise model in the post peak. Whether for two-piecewise model or three-piecewise model, the consistent degree of theoretical curve and test curve reduces in the post peak for the increase in humidity of sandstone.

Under the uniaxial compression, the predicting precision of power function is better than Weibull distribution, but this conclusion is opposite to the triaxial compression condition.

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