Dimension Optimization of Parallel Robots Based on Expert Knowledge

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***Abstract* — Based on performance index analysis and expert knowledge, a new dimension optimization algorithm of parallel robot is designed. First, the global indices for: velocity, dexterity, acceleration, stiffness and payload of three degree-of-freedom (3-DoF) parallel robot with redundant actuation are analyzed and the optimal dimension parameter under performance index is established. Then we take the 3-DoF parallel robot as an example to illustrate the application process of the dimension optimization algorithm, which considers the robot's engineering process as the objective function, and corresponding optimal dimension parameters are derived. The analytical results provide the theoretical basis for the optimal design of parallel robots.**

Keywords - parallel robot; dimension optimization; performance index; expert knowledge

I. Introduction

Parallel robot typically consists of a fixed base, a moving platform and several limbs, which form one or multiple closed chains. Parallel robots possess advantages over their serial counterparts in terms of speed, accuracy, stiffness and load to weight ratio [1]. Since Stewart's proposal to employ a parallel robot as a flight simulator in 1965 [2], over the last three decades, parallel robots have been successfully applied in various field of multi-dimensional dynamic simulation, heavy manipulation and micromanipulation, such as the space docking prototypes, the flight training devices, the vibration and earthquake simulators, the mining products, the tunnel shields, the forging manipulators, the minimal-invasive surgery devices and so on [3-8]. Parallel robots are knowledge-intensive mechanisms because of their complex structures and coupling motions. The design of parallel robots is a challenging problem in the machinery products design process, consisting of structure design and mechanism design which contains two important parts: evaluating characteristics and synthesis [9].

The synthesis of a parallel robot can be divided into two main topics, the definition of the mechanical architecture, and the determination of the dimension of its elements [10]. The first is related to the number, type and relative position of joint axes of the parallel robot, which can be obtained by several systematic approaches [3, 11]. Even though the pattern of movement, the actuators and degrees of freedom of the parallel robot are defined during the structural synthesis, the incorrect dimensioning of its elements will result in a mechanism with poor performance [10]. Therefore, dimensional synthesis is an important issue in the development of parallel robots.

Dimensional synthesis is primarily concerned with the determination of the dimensions of the geometric parameters and the range of the actuated joint variables, leading to, in general, optimizing a nonlinear cost function, subject to a set of appropriate constraints [12]. Since 1980s, there have been numerous studies on the dimension synthesis of parallel robots. Various performance indices were proposed to characterize properties of a parallel robot and were then used to formulate optimal design problems. The numerous kinematically optimal design problems of parallel robots were formulated as optimization problems by taking one index or a mixed index as the objective, while some other indices as design constraints [13]. Furthermore, there are only a few papers on the optimum design of parallel manipulators with redundant actuation. In fact, their methods for the optimum design of parallel manipulators with redundant actuation are similar and traditional ones [14].

In this paper, considering the importance of each performance index in the application of parallel robot, a new dimension optimization algorithm was designed. The paper is divided into four main sections. Next, in Section 2, the velocity global index, dexterity global index, acceleration global index, stiffness global index and payload global index are respectively discussed, and a new dimension optimization algorithm is designed In Section 3, the three degree-of-freedom (DOF) hydraulic parallel robot is used to illustrate the application process of the dimension optimization algorithm. Finally, some conclusions are made in Section 4.

II. Dimension Optimization Algorithm

The dimension optimization of parallel robot is a multi objective optimization problem. In general, the sub objective may be conflicting. Therefore, it is not possible to design the dimension parameters which ensure that all sub objectives are the best. In practical engineering application, the dimension optimization of parallel robot is to select the best dimension parameters within the scope of the application requirements. So, the dimension optimization of parallel robot mainly solves the following problems: 1) problem 1: how to determine the weight of each performance index? 2) problem 2: how to determine the dimension optimization equation ?

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In this paper, it mainly considers the velocity global index, dexterity global index, acceleration global index, stiffness global index and payload global index. The five global indices can be defined as following.

1) Global index 1: velocity global index can be calculated as:

$$\begin{cases} \varsigma_{\dot{\theta}_{\max}} = \dfrac{\int_w \left\| \dot{\boldsymbol{\theta}}_{\max} \right\| dw}{\int_w dw} \\ \varsigma_{\dot{\theta}_{\min}} = \dfrac{\int_w \left\| \dot{\boldsymbol{\theta}}_{\min} \right\| dw}{\int_w dw} \end{cases} \tag{1}$$

Where w is the workspace of parallel robot, $\left\| \dot{\boldsymbol{\theta}}_{\max} \right\|$ and $\left\| \dot{\boldsymbol{\theta}}_{\min} \right\|$ are respectively the maximum singular value and minimum singular value of forward kinematics Jacobian matrix $\boldsymbol{J}_f$, $\varsigma_{\dot{\theta}_{\max}}$ and $\varsigma_{\dot{\theta}_{\min}}$ are the maximum velocity global index and minimum velocity global index.

2) Global index 2: dexterity global index can be calculated as:

$$\eta_J = \frac{\int_w \frac{1}{\mathrm{Con}(\boldsymbol{J}_f)} dw}{\int_w dw} \tag{2}$$

Where Con($\boldsymbol{J}_f$) is the condition number of forward kinematics Jacobian matrix $\boldsymbol{J}_f$.

3) Global index 3: acceleration global index can be calculated as:

$$\eta_a = \frac{\int_w \frac{1}{K_{J_f+H}} dw}{\int_w dw} \tag{3}$$

Where $K_{J_f+H} = b\left\|\boldsymbol{J}_f\right\| \cdot \left\|\boldsymbol{J}_f^+\right\| + (a^2+2a)\left\|[\boldsymbol{H}]\right\| \cdot \left\|([\boldsymbol{H}])^{-1}\right\|$, $\boldsymbol{H}$ is Hessian matrix, a and b are the error coefficients of input velocity and input acceleration respectively.

4) Global index 4: stiffness global index can be calculated as:

$$\begin{cases} \varsigma_{(\boldsymbol{D}_{ext})_{\max}} = \dfrac{\int_w \left\| (\boldsymbol{D}_{ext})_{\max} \right\| dw}{\int_w dw} \\ \varsigma_{(\boldsymbol{D}_{ext})_{\min}} = \dfrac{\int_w \left\| (\boldsymbol{D}_{ext})_{\min} \right\| dw}{\int_w dw} \end{cases} \tag{4}$$

Where $\left\|(\boldsymbol{D}_{ext})_{\max}\right\|$ and $\left\|(\boldsymbol{D}_{ext})_{\min}\right\|$ are respectively the maximum singular value and minimum singular value of matrix $\boldsymbol{J}_f[\boldsymbol{J}_f]^{\mathrm{T}}$, $\varsigma_{(\boldsymbol{D}_{ext})_{\max}}$ and $\varsigma_{(\boldsymbol{D}_{ext})_{\min}}$ are the maximum deformation value and minimum deformation value.

5) Global index 5: payload global index can be calculated as:

$$\begin{cases} \varsigma_{(\boldsymbol{F}_{ext})_{\max}} = \dfrac{\int_w \left\| (\boldsymbol{F}_{ext})_{\max} \right\| dw}{\int_w dw} \\ \varsigma_{(\boldsymbol{F}_{ext})_{\min}} = \dfrac{\int_w \left\| (\boldsymbol{F}_{ext})_{\min} \right\| dw}{\int_w dw} \end{cases} \tag{5}$$

Where $\left\|(\boldsymbol{F}_{ext})_{\max}\right\|$ and $\left\|(\boldsymbol{F}_{ext})_{\min}\right\|$ are respectively the maximum singular value and minimum singular value of matrix $(\boldsymbol{J}_E)^{\mathrm{T}}$ which is the transpose of inverse kinematics Jacobian matrix, $\varsigma_{(\boldsymbol{F}_{ext})_{\max}}$ and $\varsigma_{(\boldsymbol{F}_{ext})_{\min}}$ are the maximum load capacity and minimum load capacity.

According to the application requirements of parallel robot, expert knowledge and practical experience of engineer, the dimension optimization algorithm can be designed as following.

Step 1. Normalization of the global indices which sure the changing trends of all indices are consistent.

Step 2. Determine the weight of each global index according to the expert knowledge and practical experience of engineer.

Step 3. Determine the dimension optimization equation.

III. Application Analysis

A. *Three Degree-of-freedom Parallel robot with Redundant Actuation*

The three degree-of-freedom hydraulic parallel robot shown in Figure 1 includes a moving platform, a fixed base platform, four spherical joint-prismatic joint-spherical joint (SPS) type active legs with the linear hydraulic actuators, and one spherical joint type constrained passive leg [15]. Here, the constraint passive leg is perpendicular to the fixed base platform, and connects the moving platform with the fixed platform by a flange plate on the fixed base platform at A_5 and a spherical joint on the moving platform at B_5. Each of the SPS-type active legs connects the moving platform to the fixed platform by a spherical joint at B_i (i=1, 2, 3, 4), an active leg with a prismatic joint, and a spherical joint at A_i (i=1, 2, 3, 4), respectively. Quadrilateral $B_1B_2B_3B_4$ is a rectangle with the side $B_1B_2=l_1$, $B_2B_3=l_2$, and a center point B_5. Quadrilateral $A_1A_2A_3A_4$ is also a rectangle with the side $A_1A_2=l_3$, $A_2A_3=l_4$, and a center point A_5. The length of constraint passive leg is h.

B. *Performance Index Analysis of 3-DOF Parallel Robot with Redundant Actuation*

According to the reference [16] and [17], the Jacobian matrix and Hessian matrix can be established. Considering its applied background, the structure parameters of robot can be described as follows: 1) h=4.7m is unchanged, only calculate the influence of l_1, l_2, l_3 and l_4 on performance indices. 2) The area of moving platform and fixed base platform can not be more than 4m×5m. 3) Considering the installation dimension of spherical joints and branched chain, the variation range of l_1, l_2, l_3 and l_4 are $0.5 \le l_1 \le 5.0$, $0.5 \le l_2 \le 4.0$, $0.5 \le l_3 \le 5.0$ and $0.5 \le l_4 \le 4.0$ respectively. 4) The search step of variation is set 0.5m, search 6400 parallel robot configurations, and 1000 sampling points can be calculated in each configuration.

According to the equation (1), the velocity global index atlas shown in Figure 2 and Figure 3 can be calculated by the program. The following information can be obtained from Figure 2 and Figure 3 as: 1) The better variation range of maximum velocity global index is $0.5 \le l_1 \le 0.6$, $0.5 \le l_2 \le 0.6$, $0.5 \le l_3 \le 0.6$ and $0.5 \le l_4 \le 0.6$. 2) The better variation range of minimum velocity global index is $0.5 \le l_1 \le 1.0$, $0.5 \le l_2 \le 1.0$, $4.0 \le l_3 \le 5.0$ and $3.5 \le l_4 \le 4.0$. 3) The maximum velocity global index and minimum velocity global index are in the interval $[0, +\infty)$, and the bigger they are, the better its velocity performance is.

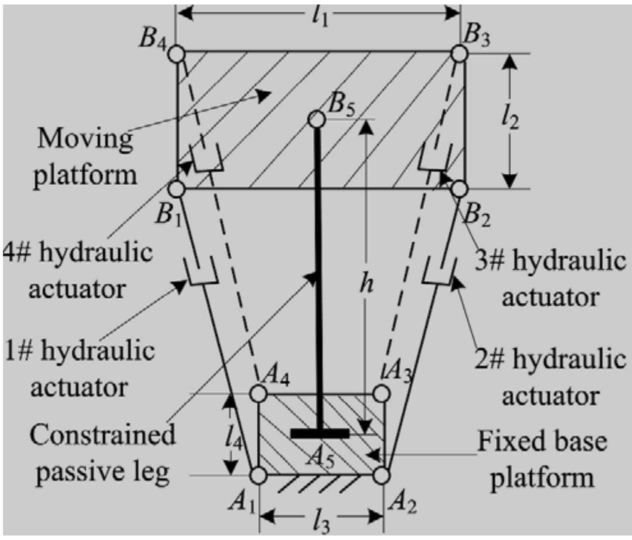

Figure 1. Structure diagram of 3-DOF parallel robot with redundant actuation

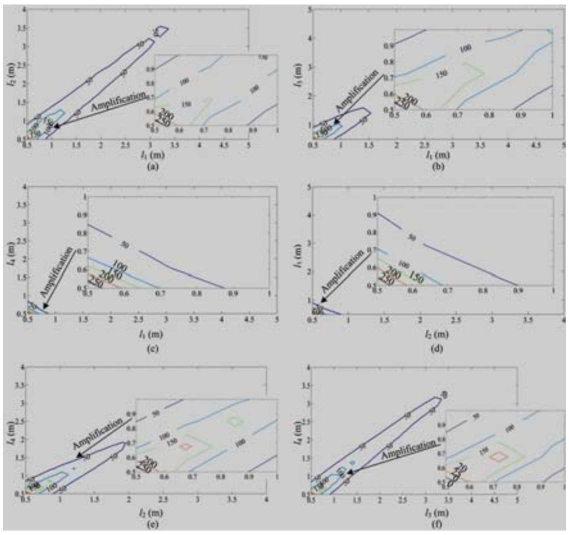

Figure 2. Maximum velocity global index atlas

According to the equation (2), the dexterity global index atlas shown in Figure 4 can be calculated by the program. The following information can be obtained from Figure 4 as: 1) The better variation range of dexterity global index is $3.5 \le l_1 \le 4.5$, $3.5 \le l_2 \le 4.0$, $4.5 \le l_3 \le 5.0$ and $0.5 \le l_4 \le 1.0$. 2) The dexterity global index is in the interval [0, 1], and the bigger it is, the better its dexterity performance is.

According to the equation (3), let $a=b=2\%$, the acceleration global index atlas shown in Figure 5 can be calculated by the program. The following information can be obtained from Figure 5 as: 1) The better variation range of acceleration global index is $2.5 \le l_1 \le 3.0$, $3.5 \le l_2 \le 4.0$, $0.5 \le l_3 \le 1.0$ and $2.5 \le l_4 \le 3.0$. 2) The acceleration global index is in the interval [0, 1], and the bigger it is, the better its acceleration performance is.

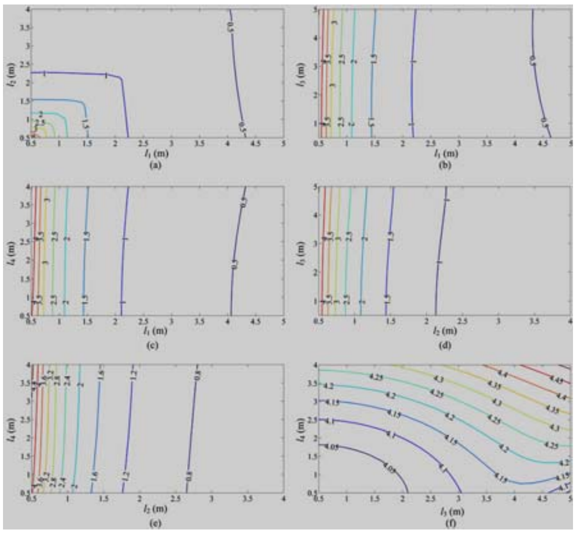

Figure 3. Minimum velocity global index atlas

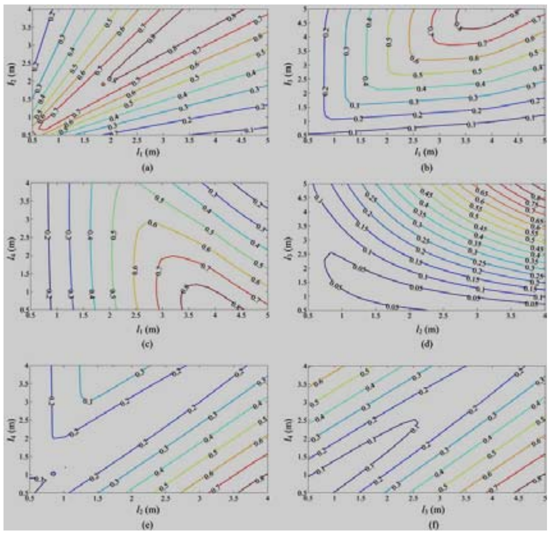

Figure 4. Dexterity global index atlas

According to the equation (4), the stiffness global index atlas shown in Figure 6 and Figure 7 can be calculated by the program. The following information can be obtained from Figure 6 and Figure 7 as: 1) The better variation range of maximum deformation global index is $3.5 \le l_1 \le 5.0$, $3.5 \le l_2 \le 4.0$, $3.5 \le l_3 \le 5.0$ and $0.5 \le l_4 \le 1.0$. 2) The better variation range of minimum deformation global index is $4.5 \le l_1 \le 5.0$, $0.5 \le l_2 \le 1.5$, $4.0 \le l_3 \le 5.0$ and $0.5 \le l_4 \le 1.5$. 3) The maximum deformation global index and minimum deformation global index are in the interval $[0, +\infty)$, and the smaller they are, the better its stiffness performance is.

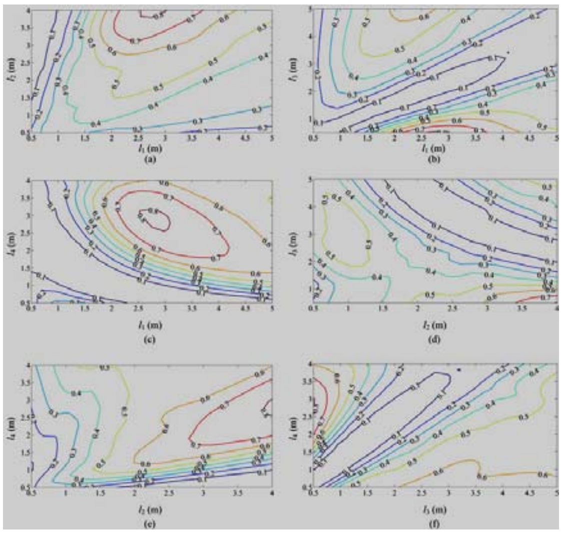

Figure 5. Acceleration global index atlas

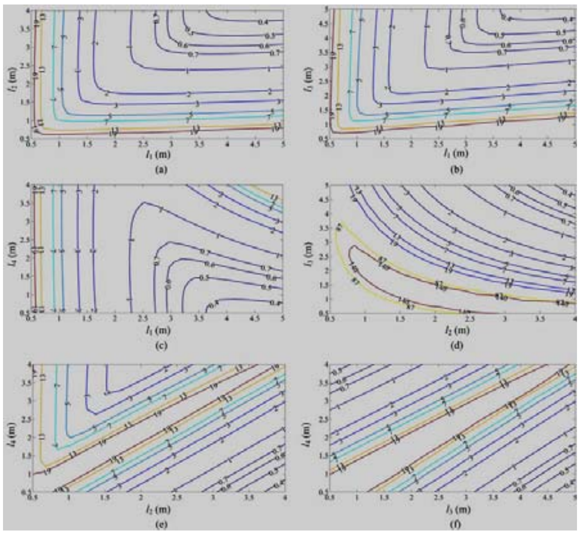

Figure 6. Maximum deformation global index atlas

According to the equation (5), the payload global index atlas shown in Figure 8 and Figure 9 can be calculated by the program. The following information can be obtained from Figure 8 and Figure 9 as: 1) The better variation range of maximum load capacity global index is $4.5 \le l_1 \le 5.0$, $0.5 \le l_2 \le 1.5$, $4.0 \le l_3 \le 5.0$ and $0.5 \le l_4 \le 1.5$. 2) The better variation range of minimum load capacity global index is $3.5 \le l_1 \le 5.0$, $3.5 \le l_2 \le 4.0$, $4.0 \le l_3 \le 5.0$ and $0.5 \le l_4 \le 1.0$. 3) The maximum load capacity global index and minimum load capacity global index are in the interval $[0, +\infty)$, and the bigger they are, the better its payload performance is.

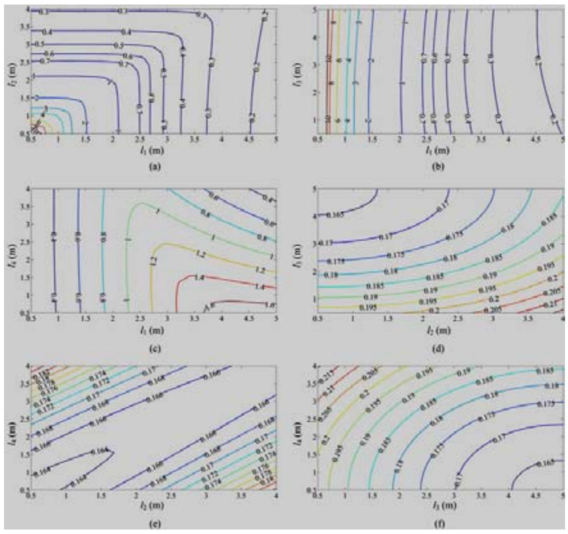

Figure 7. Minimum deformation global index atlas

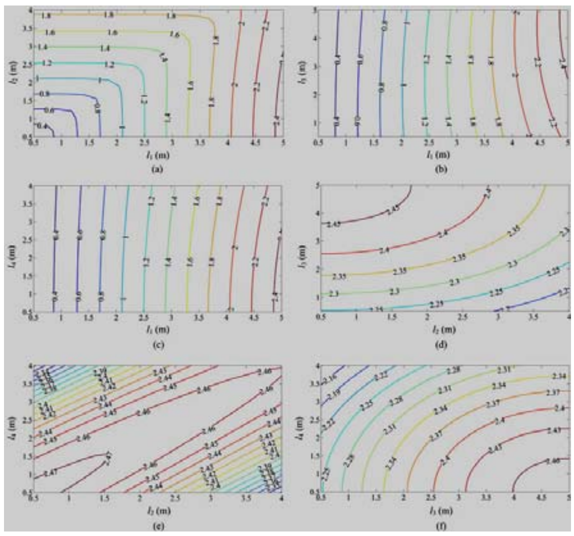

Figure 8. Maximum load capacity global index atlas

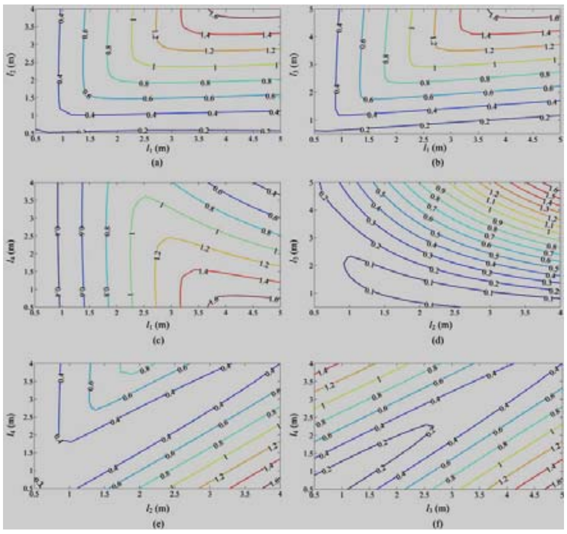

Figure 9. Minimum load capacity global index atlas

C. Dimension Optimization Design of 3-DOF Parallel Robot with Redundant Actuation

According to the performance index analysis of 3-DOF parallel robot with redundant actuation, there is a conflict between the optimization of the performance index. Therefore, the dimension parameter which makes all the performance indices optimal does not exist. The dimension optimization design of 3-DOF parallel robot with redundant actuation can be executed as following.

1) The dimension optimization equation can be designed as:

$$f(x) = W_{\dot{\theta}_{\max}} \varsigma_{\dot{\theta}_{\max}} + W_{\dot{\theta}_{\min}} \varsigma_{\dot{\theta}_{\min}} + W_{\boldsymbol{J}} \eta_{\boldsymbol{J}} + W_a \eta_a - W_{(\boldsymbol{D}_{ext})_{\max}} \varsigma_{(\boldsymbol{D}_{ext})_{\max}} - W_{(\boldsymbol{D}_{ext})_{\min}} \varsigma_{(\boldsymbol{D}_{ext})_{\min}} + W_{(\boldsymbol{F}_{ext})_{\max}} \varsigma_{(\boldsymbol{F}_{ext})_{\max}} + W_{(\boldsymbol{F}_{ext})_{\min}} \varsigma_{(\boldsymbol{F}_{ext})_{\min}} \quad (6)$$

Where $W_{\dot{\theta}_{\max}}$, $W_{\dot{\theta}_{\min}}$, $W_{\boldsymbol{J}}$, W_a, $W_{(\boldsymbol{D}_{ext})_{\max}}$, $W_{(\boldsymbol{D}_{ext})_{\min}}$, $W_{(\boldsymbol{F}_{ext})_{\max}}$, $W_{(\boldsymbol{F}_{ext})_{\min}}$ are the weight of each global index. Furthermore, $f(x) \in [0, +\infty)$, the bigger it is, the better the performance is.

2) Selecting ten theory experts and ten engineers to determine the weights shown in Table I, and the average is the weight of each performance index.

3) Calculating the relationship between parallel robot configurations and $f(x)$ shown in Figure 10, where the maximum of $f(x)$ is $f(x)_{\max}$=0.7855, and corresponding dimension parameters are l_1=5.0m, l_2=4.0m, l_3=5.0m and l_4=0.5m.

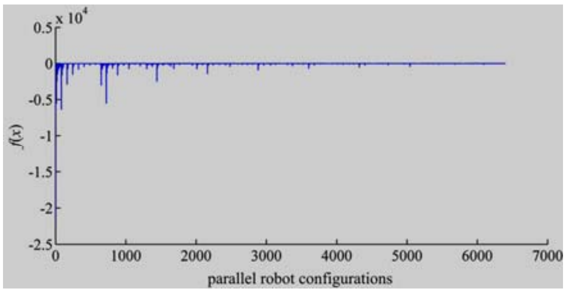

Figure 10. Relationship of *f*(x) and parallel robot configurations

TABLE I. SCORE CHART OF WEIGHT

Weight Referee	$W_{\dot{\theta}_{\max}}$	$W_{\dot{\theta}_{\min}}$	$W_{\boldsymbol{J}}$	W_a	$W_{(\boldsymbol{D}_{ext})_{\max}}$	$W_{(\boldsymbol{D}_{ext})_{\min}}$	$W_{(\boldsymbol{F}_{ext})_{\max}}$	$W_{(\boldsymbol{F}_{ext})_{\min}}$
Expert 1	0.10	0.10	0.10	0.10	0.18	0.12	0.18	0.12
Expert 2	0.10	0.10	0.10	0.10	0.20	0.10	0.20	0.10
Expert 3	0.05	0.10	0.15	0.10	0.15	0.15	0.15	0.15
Expert 4	0.05	0.05	0.15	0.05	0.25	0.10	0.25	0.10
Expert 5	0.05	0.05	0.15	0.15	0.17	0.13	0.17	0.13
Expert 6	0.10	0.15	0.15	0.10	0.15	0.10	0.15	0.10
Expert 7	0.10	0.10	0.10	0.10	0.15	0.15	0.15	0.15
Expert 8	0.05	0.10	0.15	0.10	0.20	0.10	0.20	0.10
Expert 9	0.05	0.05	0.15	0.05	0.25	0.10	0.25	0.10
Expert 10	0.10	0.10	0.15	0.10	0.15	0.10	0.15	0.15
Engineer 1	0.05	0.05	0.15	0.05	0.25	0.10	0.25	0.10
Engineer 2	0.10	0.10	0.10	0.10	0.15	0.15	0.15	0.15
Engineer 3	0.05	0.05	0.05	0.05	0.25	0.15	0.25	0.15
Engineer 4	0.05	0.10	0.10	0.05	0.20	0.15	0.20	0.15
Engineer 5	0.05	0.08	0.10	0.07	0.22	0.13	0.22	0.13
Engineer 6	0.05	0.10	0.15	0.10	0.15	0.15	0.15	0.15
Engineer 7	0.10	0.05	0.10	0.05	0.15	0.20	0.15	0.20
Engineer 8	0.05	0.05	0.05	0.05	0.30	0.10	0.30	0.10
Engineer 9	0.05	0.05	0.05	0.05	0.20	0.20	0.20	0.20
Engineer 10	0.05	0.05	0.15	0.15	0.15	0.15	0.15	0.15
Average	0.0675	0.079	0.1175	0.0835	0.1935	0.1315	0.1935	0.134

IV. CONCLUSIONS

In this paper, the velocity global index, dexterity global index, acceleration global index, stiffness global index and payload global index of parallel robot are respectively discussed. It finds that there is a conflict between the optimization of the performance index in the process of analysis. Therefore, a new dimension optimization algorithm of parallel robot is presented for solving the conflict and illustrated with the three degree-of-freedom parallel robot with redundant actuation. The conclusion that dimension optimization algorithm is effective and easy to implement can be obtained from the results of the case analysis.

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