Lateral Impact Forces of Oscillating Anchor Systems: A Simulation Study using Finite Difference Methods

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***Abstract*: The two main indices that appraise the anchorage properties are the anchorage state and anchorage force. The anchorage state refers to the anchoring length, free length and compactness of anchoring agent. The anchorage force refers to the critical drawing load in the course of pull-out test in situ after construction. Utilizing the acoustic wave's characteristics, we can detect the interior structure and quality of the anchor system, and judge the anchorage state by beating the anchor bar's top with a hammer. This paper has built up the theoretical model of transverse vibration anchor system using Euler-Bernoulli beam theory, and several formulas for calculating integrated anchor system are derived by finite difference method.**

Keywords*: *finite difference method, Anchor Bar, Lateral Impact Load, Vibration

I. INTRODUCTION

Anchor system is a member that transfers the anchorage force to the base rock. Anchoring technique is widely used in geotechnical engineering and mining projects and it plays an important role in the maintenance of stability and safety of slopes, underground chambers and the mine shafts. At the same time, new detecting techniques were urgently required [1]~[4].

Pull-out test is the traditional detective method which is based on the determining of t anchorage force to evaluate the quality of the anchor system. However, traditional evaluation method of anchoring quality is not comprehensive in practice and it should be further improved. New techniques such as moving test method are emerging in the 1980s [1]~[4]. But most of those techniques were limited to measure the length of the anchor bar and could not accurately reflect the quality of the anchor system. To evaluate the stability of anchor system, the simulation tests of vibration of anchor bar should be carried out.

A rigorous mathematical method is difficult to simulate these phenomena accurately as the pile-soil system often presents a non-linear slippage. Using the finite element method, compatible units represent soil and pile to simulate the continuity of soil. This method can describe the propagation of stress wave in soil and reflect different properties of different soil[5]~[9]. Compared with the finite difference method, boundary element method describes the quality of soil with the boundary surface between the soil and pile. The pile is dispersed as pole units to save a part of the computational amount [10]~[14],but this method is only applied to linear analysis.

II. MATERIALS and METHODS

A. Euler-Bernoulli beam model

Traditional calculation methods of analysis for the bending of the beams are based on simple bending theory, that is the Euler-Bernoulli theory[15][16]. And it assumes that the cross section of the beam maintains as a flat during the course of its bending.

As Fig.1 shows, the length of the unit is dx; the boundary flat is perpendicular to the axis of the unit; the internal force and the bending moment on the unit are as shown in illustration. The shear forces are V and $V+(\partial V/\partial x)dx$; the bending moments are M and $M+(\partial M/\partial x)dx$; the lateral load is pdx; the inertial force is $f_2=(mdx)\partial^2u/\partial^2x$; m is the quality of the unit; $p=p(x, t)$ is the load of the unit .

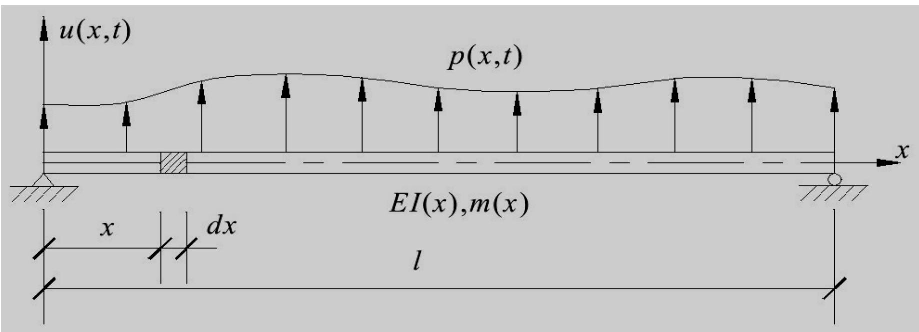

Fig.1 Simple beam with distributed mass and load

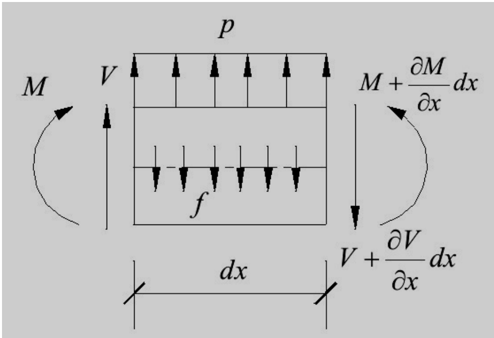

Fig.2 cell's force

If the deflection of the beam is small, the inclination of the beam with no load is small too. The summation of forces of isolated cell (Fig.2) in x-direction must be zero. So, the motion equations in x-direction can be derived are as follows:

$$V-(V+\frac{\partial V}{\partial x}dx)+p(x,t)dx-mdx\frac{d^2u}{dt^2}=0$$ Simplified:

$$\frac{\partial V}{\partial x}+m\frac{d^2u}{dt^2}=p(x,t)$$

As the simple bending theory shows:

$$M=EI\frac{d^2u}{dx^2}$$

$$V=\frac{\partial M}{\partial x}$$

E is the elastic modulus, I is the area moment of inertia about central axis.

For uniform beam:

$$V=\frac{d^3u}{dx^3}$$

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{d^2u(x,t)}{dx^2}\right]+m(x)\frac{d^2u(x,t)}{dt^2}=p(x,t) \quad (1)$$

Differential equation of vibration above is under simplest initial conditions.

B. Building of the bending vibration equation

1). Bending vibration equation of the beam with viscous damping

In the process of the vibration of beam, external damping refers to environmental impedance, like the soil's impedance to vibrating displacement. And internal damping refers to repeated deformation of several layers of fibrous tissue. As the internal damping is relatively smaller, we shall consider only the external damping.

The external damping is directly proportional to the vibrating velocity.

$$f_D=c(x)\frac{\partial u}{\partial t}$$

If we introduce equation above into equation (1), we can conclude equation (2):

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{d^2u(x,t)}{dx^2}\right]+m(x)\frac{d^2u(x,t)}{dt^2}+c(x)\frac{\partial u(x,t)}{\partial t}=p(x,t) \quad (2)$$

Equation (2) is the bending vibration equation considering viscous damping.

2). Bending vibration equation of the beam with elastic base

Assume that the beam is placed on the base with stiffness coefficient (***k***). ***k*** refers to reaction that the base produces when moving through a distance of one unit length vertically. The vertical reaction ***f(x, t)***, produced by unit elastic base, is directly proportional to the vibration displacement. ***k*** is a constant.

$$f(x,t)=ku(x,t)$$

Considering the vertical reaction produced by elastic base, equation (2) can be transformed as follows:

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{d^2u(x,t)}{dx^2}\right]+m(x)\frac{d^2u(x,t)}{dt^2}+c(x)\frac{\partial u(x,t)}{\partial t}+k(x)u(x,t)=p(x,t) \quad (3)$$

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3). Bending vibration equation of the beam with axial force

Besides the vertical load, axial load is also added to the beam. We assume the axial load is not changed with time, then equation (3) can be transformed as follows:

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{d^2u(x,t)}{dx^2}\right]+m(x)\frac{d^2u(x,t)}{dt^2} +c(x)\frac{\partial u(x,t)}{\partial t}+k(x)u(x,t) +N\frac{d^2u(x,t)}{dx^2}=p(x,t) \quad (4)$$

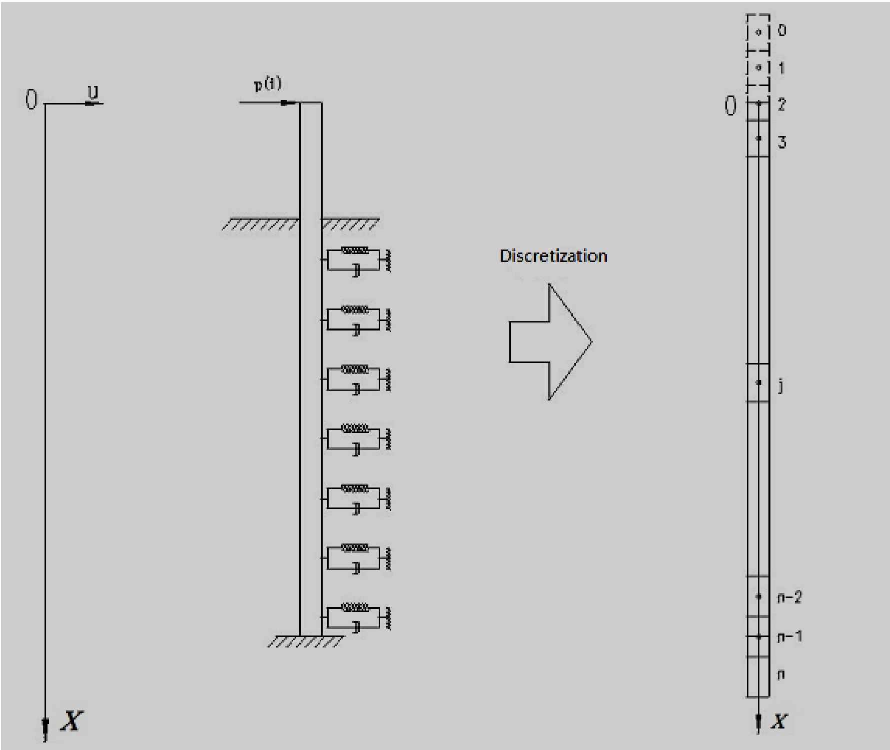

Fig.3 Discrete model of anchor system.

C. Discretization and Boundary Conditions of the Vibration Equation

In general terms, it is difficult to obtain the general solution of equation (4). To simplify the procedure for solving this mathematical problem, several suitable assumptions are made as follows:

1) Medium around anchor system is uniform, isotropic, linear elastic and continuous. Stiffness coefficient and damping coefficient are constants.

2) The cross section of anchor bar is circular, and the rod is simplified as linear elastic Euler beam. Only the deformation caused by horizontal vibration is considered.

3) Only the horizontal displacement is considered.

4) There is not relative slippage between anchor bar and medium around.

5) The axial force N on the beam is not changed with time.

6) The excitation force is loaded on the top of the anchor bar.

Then equation (4) can be simplified as follows:

$$EI\frac{d^4u(x,t)}{dx^4}+m\frac{d^2u(x,t)}{dt^2} +c\frac{\partial u(x,t)}{\partial t}+ku(x,t) +N\frac{d^2u(x,t)}{dx^2}=p(x,t) \quad (5)$$

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Here, $u(x, t)$ is horizontal displacement of the unit; E is elastic modulus; I is rotational moment of inertia; m is quality of the unit, $m=\rho*S$, ρ is density of the anchor bar, S is cross section area of the anchor bar; k is the stiffness coefficient of medium around and c is the damping coefficient of medium.

To simplify boundary conditions and the finite different method, we add two virtual units on top of anchor bar, and one virtual unit at the bottom of anchor bar. The partial derivative of u to x is dispersed by the central difference method (difference units of the anchor bar are shown as Fig.3)：

$$(\frac{\partial u}{dx})_j=\frac{1}{2h}(-u_{j-1}+u_{j+1})$$

$$(\frac{\partial^2 u}{dx^2})_j=\frac{1}{h^2}(u_{j-1}-2u_j+u_{j+1})$$

$$(\frac{\partial^3 u}{dx^3})_j=\frac{1}{2h^3}(-u_{j-2}+2u_{j-1}-2u_{j+1}+u_{j+2})$$

$$(\frac{\partial^4 u}{dx^4})_j=\frac{1}{h^4}(u_{j-2}-4u_{j-1}+6u_j-4u_{j+1}+u_{j+2})$$

For the *i*-th period, the partial derivative of u to t is dispersed by Newmark-β integral equation as [17][18]:

$$(\frac{\partial u}{\partial t})_i=\frac{\gamma}{\beta s}(u_i-u_{i-1})+(1-\frac{\gamma}{\beta})(\frac{\partial u}{\partial t})_{i-1}$$
$$+(1-\frac{\gamma}{2\beta})s(\frac{\partial^2 u}{dt^2})_{i-2}$$

$$(\frac{\partial^2 u}{\partial t^2})_i=\frac{1}{\beta s^2}(u_i-u_{i-1})-\frac{1}{\beta s}(\frac{\partial u}{\partial t})_{i-1}$$
$$-(\frac{1}{2\beta}-1)(\frac{\partial^2 u}{\partial t^2})_{i-1}$$

β and γ are constants to control numerical stability, $\beta=1/4$, $\gamma=1/2$. s is time step. u_i is the horizontal displacement of anchor bar for a *i*-th time. The process of this numerical analysis is unconditionally stable.

Top of the anchor bar is free and we can conclude from the boundary conditions：

Shear force of the top:

$$\frac{EI}{V}=(\frac{\partial^3 u}{dx^3})_{i,2}=\frac{1}{2h^3}(-u_{i,0}+2u_{i,1}-2u_{i,3}$$
$$+u_{i,4})=\frac{p(x)}{EI}$$

Bending moment of the top:

$$\frac{M}{EI}=(\frac{\partial^2 u}{\partial x^2})_{i,2}=\frac{1}{h^2}(u_{i,1}-2u_{i,2}+u_{i,3})=0$$

Bottom of the anchor bar is stationary.

Displacement of the bottom:

$u_{i,n-1}=0$

Angular rotation of the bottom:

$$(\frac{\partial u}{\partial x})_{i,n-1}=\frac{1}{2h}(-u_{i,n-2}+u_{i,n})=0$$

In summary, four equations are obtained from the boundary conditions:

$$-u_{i,0}+2u_{i,1}-2u_{i,3}+4u_{i,4}=\frac{2P(t)h^3}{EI}$$

$$u_{i,1}-2u_{i,2}+u_{i,3}=0$$

$$u_{i,n-1}=0$$

$$-u_{i,n-2}+u_{i,n}=0$$

III. RESULTS

For there is only impact force, $P(t)$=0. Four boundary conditions are simplified as follows：

$$u_{i,2}=2u_{i,3}-u_{i,4}$$

$$u_{i,1}=4u_{i,3}-4u_{i,4}+u_{i,5}$$

$$u_{i,n-1}=0$$

$$u_{i,n} = u_{i,n-2}$$

Simplify equation (5) and we shall get the iteration method for the displacement analysis:

$$[\frac{4m}{t^2}+\frac{2}{t}c(x)+k(x)]u_i+[-\frac{4m}{t^2}-\frac{2}{t}c(x)]u_{i-2}$$
$$+[-\frac{4m}{t}-c(x)](\frac{\partial u}{\partial t})_{i-1}-m(\frac{\partial^2 u}{\partial t^2})_{i-1}$$
$$+EI\frac{\partial^4 u}{\partial x^4}+N\frac{\partial^2 u}{\partial x^2}=0$$

Derivative of u to x can be obtained by the central difference method.

The mechanical parameters of anchor system are as follows: length of the anchor bar is 20m; 0.022m in diameter; cross section area is 380.13 mm^2; elastic modulus is 2.0×10^{11}Pa, density ρ=7800kg/m; inertia modulus is 1.15×10^4 mm^4. Elasticity coefficient k=2.61×10^9 N /m^2 and damping coefficient c=2.71×10^4Ns / m^2. [19]

The finite element method model is divided into 200 units (0.1m per unit). Elasticity coefficient k=2.61×10^7 N /m^2 and damping coefficient c=2.71×10^2Ns / m^2.

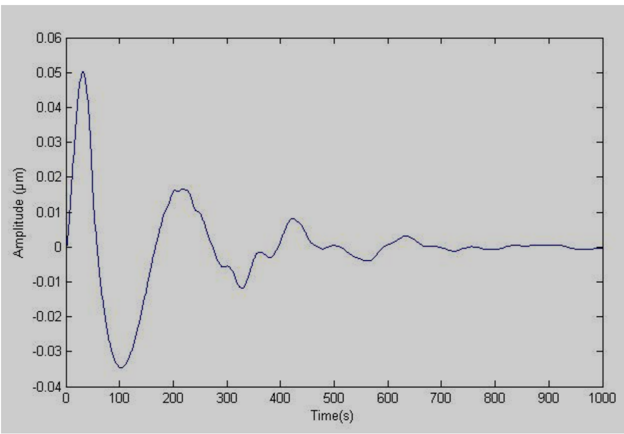

Fig.4 Comparison of finite difference method and theoretical solution

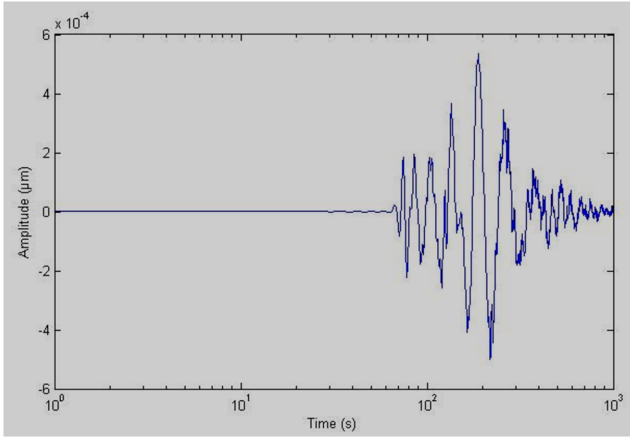

Fig.5 Curve of velocity of the anchor bar's bottom in time domain.

When the transient impulsive load is applied to the top of anchor, initial value of displacement and velocity is 0, but the acceleration is not equal to 0.

In the process of computation, time step is 0.001μs, sampling points are 2000, that is the total time is 2ms.

Matlab is adopted to simulate vibration of the top and bottom of anchor bar. The vibrating curve of the top is illustrated in Fig.4 and the bottom is shown in Fig.5.

IV. DISCUSSION AND CONCLUSION

Finite difference method is proposed to investigate the dynamic response of the anchor bar under lateral transient loads. A periodic damping vibration is clearly displayed on the top of the anchor bar and amplitude of vibration grows gradually weaker. At 0.8ms, the longer period waveform is superposed by a longer period waveform, which is reflected from the bottom of the anchor bar. Frequency of the reflected wave decreases while period decreases and this is due to dispersion. As Fig.4 shows, the calculated result shows that it fits for actual result and simulates vibrating characteristics accurately.

As is illustrated in Fig.5, there is a nonlinear change of amplitude and frequency of the bottom of anchor bar. This might be the superposition of vibrations of anchor system and surrounding rock.

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REFERENCES

[1] Tadolini S C. Evaluation of Ultrasonic Measurement Systems for Bolt Load Determinations. The U S Bureau of Mines, Denver,CO. 1990

[2] Tadolini S C,Dyni R C. Transfer Mechanics of Full-column Resin-grouted Roof Bolts . The US Bureau of Mines, Denver,CO. 1990

[3] Singner S P. Field Verification of Load-transfer Mechanics of Fully Grouted Roof Bolts. The US Bureau of Mines, Denver,CO. 1990
[4] Wang ming-wu, Wang he-ling. Nondestructive testing of grouted bolts system. Chinese Journal of Geotechnical Engineering. 2001. 23（1）. 109-113
[5] Nogami,T.&Novak,M. Resistance of soil to a horizontally vibrating pile. Earthquake Engineering and Structural Dynamics,1977,5(3):247~261
[6] Novak,M.,Nogami,T.&Aboul-Ella, E.Dynamic soil reactions for plane strain case. Journals of Engineering Mechanics,104(4):953~959
[7] Novak,M.&Aboul-Ella,F. Impedances functions of piles in layered media. Journal of Engineering Mechnics,1978,104(3):643~661
[8] Novak,M.&Aboul-Ella,F.Stiffness and damping of piles in layered media. ASCE specialty Conference on Earthquake Engineering and Soil Dynamics, Pasadena, Calif. ,1978, June, 16.
[9] Takemiya,H.&Yamada,Y.Layered soil-pile-structure dynamic Interaction, Earthquake Engineering and Structural Dynamics,1981,9:437~457
[10] Novak,M. Dynamic stiffness and damping of piles. Canadian Geotechnical Journal,1974, 11(4):574~598
[11] Novak,M.Effect of soil on structural response to wind and earthquake. Earthquake Engineering and Structural Dynamics,1974,3:79~96
[12] Nogami,T.&Konagai,K.Time domain flexural responses of dynamically loaded single piles. Journal of Engineering Mechnics,1988,114(9):1512~1525
[13] Yao,S.&Nogami,T.Lateral cyclic response of a pile in viscoelastic winkler subgrade. Journal of Engineering Mechnics,1994,120(4):758~775
[14] Mylonakis,G.&Gazetas,G.Lateral vibration and internal forces of grouped piles in layered soils.Journal of Geotechnical and Geoenvironmental Engineering,1999,125(1):16~25
[15] Winkler E.Die Lehre von Elastizitat und Festigkeit(on elasticity and fixity).Prague,182
[16] Timoshenko S P.On the correction for shear of the differential equation for transverse vibration of prismatic bars.Philosophical Magazine,1921,41:744
[17] Clough,R.W.&Penzin,J.Dynamic of Structure, McFGraw-Hill,New York,1975;Chapter 3
[18] Clough,R.W.&Penzin,J.Dynamic of Structure, McFGraw-Hill,New York,1975;Chapter 2
[19] Yoshida,I.&Yoshinaka,R.A method to estimate modulus of horizontal subgrade reaction for pile.Soils and Foundations,1972,12(3):1~1