Attitude Control of Space Station based on Linear Extended State Observer

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***Abstract* — Due to strong external disturbances, attitude control and momentum management (ACMM) system needs to be designed to prevent the rapid saturation of the control moment gyros. This paper applies a Linear Extended State Observer (LESO) with gains calculated by LQR to solve the ACMM problem of the space station. LESO is designed to estimate disturbances of the space station, and a unique disturbance compensation strategy is proposed. The stability of the proposed scheme is proved theoretically. The effectiveness of the proposed scheme is demonstrated by comparing its control effect with that of the internal model principle based LQR method via simulation results.**

Keywords - linear extended state observer; attitude control and momentum management; LQR

I. INTRODUCTION

Space stations such as Mir and International Space Station (ISS) usually adopt Control Moment Gyros (CMGs) as main actuators [1, 2]. The space station has a weight of over 100 tons, and the massive external disturbance is up to 1-10Nm. Without satisfactory management, CMGs will saturate within a short time and lead to unloading frequently. Consequently, it will result in great fuel consumption and reduction in the design life of the space station. A great challenge of controlling the space station is to maintain long-term attitude control, meanwhile coping with external disturbance to avoid rapid saturation of the CMGs. It is a combined problem concerning attitude control and momentum management (ACMM).

Internal model principle (IMP) based LQR method [2-7] has been widely used in previous studies to reject the external disturbance of the space station, which is primarily the aerodynamic torque, including bias, cyclic terms at the orbit rate and the twice orbit rate. However, the IMP-based disturbance rejection scheme in ACMM will introduce a fourth-order internal model in the closed-loop system. Consequently, the feedback controller ought to be designed in a □^{3x24} dimension space, leading to intensive calculation.

Another common method for disturbance rejection is the extended state observer (ESO)-based controllers [9-11]. ESO can well estimate the “total disturbance” including external disturbance and internal dynamic (such as coupling part). It works by augmenting the disturbances into a new state variable, and using a special feedback mechanism to establish the disturbance observer. It can be divided into linear extended state observer (LESO) and nonlinear extended state observer (NLESO) based on the function form.

For the excellent performance, ESO has been extensively applied to many practical processes involving flight systems [12], fault diagnosis [13, 14], MEMS systems [15], robotic systems [16], etc. Unfortunately, it has never been applied to ACMM problem of space stations.

In this paper, to avoid augmenting the ACMM system from □^{12} to □^{24} , a third-order LESO (3rd-LESO) with LQR controller design scheme is first proposed to achieve ACMM. The main contributions of our work lie in the following aspects: (i) The LESO is designed in the attitude control passage to estimate the disturbance of each axis. (ii) A novel compensation strategy is proposed based on the influence mechanism of the disturbance on each axis to achieve ACMM. (iii) The stabilities of the 3rd-LESO and the proposed scheme is proved strictly. (iv) Simulation results have been employed to demonstrate the advantages of the proposed scheme against the classical IMP-based LQR method.

This paper is organized as follows: Section 2 presents mathematical models for ACMM of the space station. In section 3, the LESO is designed, and applicable compensation strategy along each axis is presented to achieve ACMM. The stabilities of the designed LESO and closed-loop system are investigated theoretically in section 4. Section 5 carries out simulations of a given space station to verify the proposed approach, and conclusions are drawn in section 6.

II. PROBLEM DEFINITION

A. *Mathematical Models*

The space station is expected to maintain LVLH (local vertical and local horizontal) orientation during normal flight motion. A decoupled time-variant dynamic model is adopted [2-4].

In the normal flight mode, attitude deviations from the LVLH orientation are small and the cross-product inertial terms are negligible. The equations of motion in terms of components along body-fixed control axis are given in the state-space form as follows [2-4].

The pitch model is

$$\dot{\boldsymbol{x}}_y = \boldsymbol{A}_y \boldsymbol{x}_y + \boldsymbol{B}_y u_y + \boldsymbol{w}_{by},$$

$$\boldsymbol{A}_y = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B}_y = \begin{bmatrix} 0 \\ b_y \\ 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{w}_{by} = \begin{bmatrix} 0 \\ -b_y \\ 0 \\ 0 \end{bmatrix} w_y, \tag{1}$$

$$\boldsymbol{x}_y = \begin{bmatrix} x_{y1} & x_{y2} & x_{y3} & x_{y4} \end{bmatrix}^T = \begin{bmatrix} \theta_y & \dot{\theta}_y & \int h_y & h_y \end{bmatrix}^T.$$

The roll/yaw model is:

$$\dot{\boldsymbol{x}}_{xz} = \boldsymbol{A}_{xz} \boldsymbol{x}_{xz} + \boldsymbol{B}_{xz} \boldsymbol{u}_{xz} + \boldsymbol{w}_{bxz},$$

$$\boldsymbol{A}_{xz} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{xz21} & 0 & 0 & 0 & 0 & a_{xz26} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_o \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & a_{xz62} & 0 & 0 & a_{xz65} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\omega_o & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{B}_{xz} = \begin{bmatrix} 0 & 0 \\ b_x & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & b_z \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{w}_{bxz} = \begin{bmatrix} 0 & 0 \\ -b_x & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -b_z \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{w}_{xz},$$

$$\begin{aligned} \boldsymbol{x}_{xz} &= \begin{bmatrix} \boldsymbol{x}_x \\ \boldsymbol{x}_z \end{bmatrix} \\ &= \begin{bmatrix} x_{x1} & x_{x2} & x_{x3} & x_{x4} & x_{z1} & x_{z2} & x_{z3} & x_{z4} \end{bmatrix}^T \\ &= \begin{bmatrix} \theta_x & \dot{\theta}_x & \int h_x & h_x & \theta_z & \dot{\theta}_z & \int h_z & h_z \end{bmatrix}^T, \end{aligned} \tag{2}$$

$$\boldsymbol{u}_{xz} = \begin{bmatrix} u_x \\ u_z \end{bmatrix}, \quad \boldsymbol{w}_{xz} = \begin{bmatrix} w_x \\ w_z \end{bmatrix}.$$

where $(x \quad y \quad z)$ are roll, pith, and yaw control axis, respectively, $(u_x \quad u_y \quad u_z)$ are the control torques of CMG, $(\theta_x \quad \theta_y \quad \theta_z)$ are the Euler angles of the body axes, $(h_x \quad h_y \quad h_z)$ are the body-axis components of CMG momentum, and $(w_x \quad w_y \quad w_z)$ are the external disturbances modeled as bias plus cyclic terms. Also,

$$\begin{aligned} & w_i = b_{i0} + a_{i1} \sin(\omega_o t) + a_{i2} \sin(2\omega_o t), \\ & w_{bi} \triangleq (-b_i) w_i, \quad b_i = -I_i^{-1} \end{aligned} \tag{3a}$$

$$\begin{aligned} & a_y = -3\omega_o^2 I_y^{-1}(I_z - I_x), \\ & a_{xz21} = -I_x^{-1} \cdot 4\omega_o^2 (I_z - I_y), a_{xz26} = -I_x^{-1} \cdot \omega_o (I_x - I_y + I_z) \\ & a_{xz62} = -I_3^{-1} \cdot \omega_o(-I_x + I_y - I_z), \quad a_{xz26} = -I_3^{-1} \omega_o^2 (I_x - I_y) \end{aligned} \tag{3b}$$

where $(I_x \quad I_y \quad I_z)$ represent the moments of inertia, ($I_{ij} = I_{ji}, i, j = x, y, z, i \neq j$) are the products of inertia, and ω_o is the orbital rate of $0.0011 \ \ rad/s$.

B. Problem Illustration

The ACMM problem of the space station in normal flight motion is to design a control scheme to reject the external disturbance, such that $|\theta_x|, |\theta_z| \le 5\deg$, $|\theta_y| \le 10\deg$, $|\dot{\theta}_i| \le 0.005\deg/s$ with the CMG momentum restriction $h_{i\max} = 1.7236 \times 10^4 \, N \cdot m$. The main difficulty of the ACMM problem is that designers have to deal with the momentum restriction while ensuring the attitude control accuracy.

In this paper, a control scheme based on LESO is designed to solve this problem. And LQR method based on pole placement [4] is adopted to design the controller $\boldsymbol{K}_y, \boldsymbol{K}_{xz}$ of the system shown in Eqs.(1) and (2).

C. Lemmas and Definition

Consider an error dynamic system

$$\dot{\boldsymbol{e}} = \omega_o \boldsymbol{A}\boldsymbol{\varepsilon} + \boldsymbol{B}\frac{h(\boldsymbol{x}) - h(\boldsymbol{z})}{\omega_o^n}. \tag{4}$$

where

$$\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{n+1} \end{bmatrix}^T, \boldsymbol{e} = \boldsymbol{z} - \boldsymbol{x}, \varepsilon_i = \frac{e_i}{\omega_o^{i-1}}, i = 1, 2 \cdots n+1,$$

$\boldsymbol{z}$ is the estimation of $\boldsymbol{x}$ via the LESO, and $\boldsymbol{A}$ is a $(n+1) \times (n+1)$ matrix.

Lemma 1 For Eq.(4), assuming $\boldsymbol{A}$ is Hurwitz, $h(\boldsymbol{x})$ is globally Lipschitz with respect to $\mathbf{x}$. There exists a constant $\omega_o > 0$, such that $\lim_{t \to \infty} \boldsymbol{e}(t) = 0$ [18].

Consider a system shown as

$$\dot{\boldsymbol{\eta}}(t) = \boldsymbol{N}\boldsymbol{\eta}(t) + \boldsymbol{B}\boldsymbol{g}(t)$$

Lemma 2 If $\boldsymbol{N}$ is Hurwitz and $|\boldsymbol{g}(t)| \le \boldsymbol{M}$ for all t, then $\lim_{t \to \infty} \boldsymbol{\eta}(t)$ is bounded [18].

III. DESIGN OF SPACE STATION ACMM STRATEGY

A. LESO Design

Fig. (**1**) shows the space station's ACMM system. It can be observed that the system consists of two main parts: AC passage and MM passage, which play the roles of attitude control and momentum management, respectively. this paragraph, the solid-plug conveying theory of centrifugal extruder is studied by the mathematical analysis.

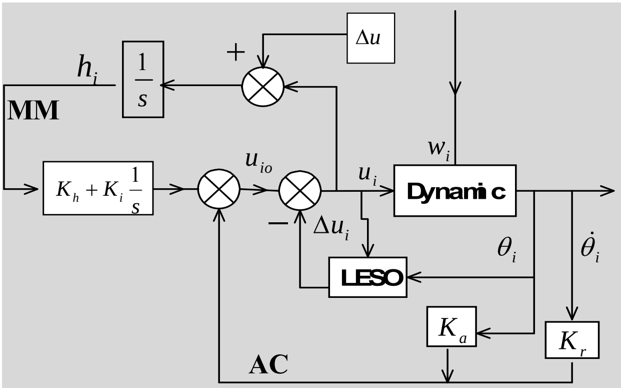

Figure 1. Space Station's ACMM System.

The LESO can treat the dynamic coupling and external disturbances as the total disturbance. Therefore, the observer can be established in each axis to simplify the design process.

As shown in the dynamic model and the system diagram, the external disturbance just loads in AC passage. As a result, it is also the only passage where the LESO should be established

The dynamic expression of the LESO on each axis can be presented as follows

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = x_{i3} + b_i u_i \\ \dot{x}_{i3} = h_i + w_{bi} \\ y_i = x_{i1} \end{cases}. \quad (5)$$

where $i = x, y, z$, x_{i3} is the argument state (the total disturbance) of each axis, $\boldsymbol{x_{ik}} = [x_{i1} \quad x_{i2} \quad x_{i3}]^T$, $h_i = \dot{f}_i$, and

$$\begin{cases} f_x = a_{xz21} \cdot x_{x1} + a_{xz26} \cdot x_{z2} \\ f_y = a_y x_{y2} \\ f_z = a_{xz62} \cdot x_{x2} + a_{xz65} \cdot x_{z1} \end{cases}. \quad (6)$$

The LESO of Eq. (5) can be established as follows

$$\begin{cases} e_{i1} = z_{i1} - y_i \\ \dot{z}_{i1} = z_{i2} - \omega_{ei}\alpha_{i1} \cdot e_{i1} \\ \dot{z}_{i2} = z_{i3} - \omega_{ei}^2\alpha_{i2} \cdot e_{i1} + b_i u_y \\ \dot{z}_{i3} = -\omega_{ei}^3\alpha_{i3} \cdot e_{i1} + \hat{h}_i \end{cases}. \quad (7)$$

where z_{i3} is the estimated disturbance as well as the output of the LESO for each axis, $\boldsymbol{z_{ik}} = [z_{i1} \quad z_{i2} \quad z_{i3}]^T$ and $\omega_{ei} > o$, α_{ij} are coefficients to be tuned to satisfy the equation.

$$s^3 + \alpha_{i1}s^2 + \alpha_{i2}s + \alpha_{i3} = (s+1)^3, \hat{h}_i = h_i(z_i). \quad (8)$$

The disturbance of each axis and the compensation control Δu_{io} can both be calculated by LESO. Compensations are made as follows

$$u_{yo} = -\boldsymbol{K}_y \cdot \boldsymbol{x}_y,$$
$$\boldsymbol{u}_{xzo} = \begin{bmatrix} u_{xo} \\ u_{zo} \end{bmatrix} = -\boldsymbol{K}_{xz} \cdot \boldsymbol{x}_{xz}. \quad (9)$$

$$u_i = u_{io} - \Delta u_{io} = u_{io} - \frac{z_{i3}}{b_i}(i = x, y, z). \quad (10)$$

B. Pitch Compensation Strategy

AC passage and MM passage are coupled via the control signal. It is necessary to investigate how to reasonably compensate disturbance effects in the control signal. In addition, the requirements of the attitude control accuracy and momentum management should both be met to achieve the equilibrium of ACMM.

Combining Eq. (1) with Eq. (10), the following dynamic equation of CMG on the Y-axis is obtained after compensation

$$\dot{h}_y = u_{yo} - \Delta u_{yo}. \quad (11)$$

However, CMG is not subjected to the external disturbance. As such, it is unnecessary to compensate Δu_{yo} in the CMG dynamic equation (i.e., the MM passage). As it is known, the bias of the external disturbance can accumulate on the Pitch axis. When it is offset on MM passage, it would re-accumulate and result in over-compensation. Hence, only the cyclic components of the disturbance need to be offset on MM passage.

The sum of cyclic disturbance z_{ym} on the pitch axis can be identified by iteration method. And the corresponding control component is $\Delta u_y = z_{ym} / b_y$.

$$\begin{cases} u_y = u_{yo} - \Delta u_y \\ \dot{h}_y = u_y + \Delta u_y \end{cases}. \quad (12)$$

Integrating of the foregoing equation gives

$$h_y = \int (u_y + \Delta u_y) \cdot dt = x_{y4} + \Delta h_y,$$
$$\Delta h_{yi} \triangleq \int \Delta u_y \cdot dt. \quad (13)$$

Incorporating Eqs. (12) and (13) into Eqs. (9) and (10), the compensation strategy of the pitch can be gained as

$$u_{yo} = -\boldsymbol{K}_y \cdot \begin{bmatrix} x_{y1} & x_{y2} & x_{y3} + \Delta h_{yi} & x_{y4} + \Delta h_y \end{bmatrix}^T, \quad u_y = u_{yo} - \Delta u_y. \tag{14}$$

C. Roll/Yaw Compensation Strategy

Through Eqs. (2) and (10), the following dynamic equation of CMG on the X/Z-axis is obtained after compensation

$$\begin{bmatrix} \dot{h}_x - \omega_o h_z \\ \dot{h}_z + \omega_o h_x \end{bmatrix} = \begin{bmatrix} u_x - \Delta u_{xo} \\ u_z - \Delta u_{zo} \end{bmatrix}. \tag{15}$$

Similar as the case of pitch axis, AC passage and MM passage are also coupled via the control input on roll / yaw axis. However, the CMG dynamics (MM passage) of the roll/yaw axis is not subjected to the disturbance. To avoid over-compensation, it is necessary to offset the effects of Δu_{xo} and Δu_{zo} in CMG dynamic equation.

As cyclic terms of the disturbance cause CMG momentum accumulation on the roll/yaw axes, only the bias need to be offset on MM passage. After identifying the bias z_{xc} and z_{zc} , the corresponding compensation control $\Delta u_x = z_{xc} / b_x$ and $\Delta u_z = z_{zc} / b_z$ can be obtained.

In summary, after offset, the MM passage of the X/Z - axis is expressed as

$$\begin{bmatrix} \dot{h}_x - \omega_o h_z \\ \dot{h}_z + \omega_o h_x \end{bmatrix} = \begin{bmatrix} u_x + \Delta u_x \\ u_z + \Delta u_z \end{bmatrix}. \tag{16}$$

Integrating of the foregoing equation gives

$$h_x = \int [\omega_o h_z + (u_x + \Delta u_x)] \cdot dt \overset{\Delta}{=} x_{x4} + \Delta h_x. \tag{17}$$

$$h_z = \int [-\omega_o h_x + (u_z + \Delta u_z)] \cdot dt \overset{\Delta}{=} x_{z4} + \Delta h_z. \tag{18}$$

Substituting Eq.(18) into Eq.(17) gives

$$h_x = \int (\omega_o x_{z4} + u_x) dt + \int (\omega_o \cdot \Delta h_z + \Delta u_x) dt. \tag{19}$$

Combining Eq. (16) with Eq. (19) yields

$$\Delta h_x = \int (\omega_o \cdot \Delta h_z + \Delta u_x) dt. \tag{20}$$

Integrating Eq.(20) gives

$$\Delta h_{xi} = \int \Delta h_x \cdot dt. \tag{21}$$

Similarly, h_z and h_{zi} of the yaw axis are as follows

$$\Delta h_z = \int (-\omega_o \cdot \Delta h_x + \Delta u_z) dt. \tag{22}$$

$$\Delta h_{zi} = \int \Delta h_z \cdot dt. \tag{23}$$

Finally, the ACMM compensation strategy of the space station on roll/yaw axe is obtained as follows

$$\boldsymbol{u}_{xzo} = \begin{bmatrix} u_{xo} \\ u_{zo} \end{bmatrix} = -\boldsymbol{K}_{xz} \cdot \left[x_{x1}, x_{x2}, x_{x3} + \Delta h_{xi}, x_{x4} + \Delta h_x, x_{z1}, x_{z2}, x_{z3} + \Delta h_{zi}, x_{z4} + \Delta h_z \right]^T,$$

$$\boldsymbol{u}_{xz} = \begin{bmatrix} u_{xo} \\ u_{zo} \end{bmatrix} - \begin{bmatrix} \Delta u_x \\ \Delta u_z \end{bmatrix}. \tag{24}$$

IV. STABILITY OF THE ACMM SYSTEM

A. Convergence of the LESO

Let $\boldsymbol{e_i} = [e_{i1} \quad e_{i2} \quad e_{i3}]^T$, where $e_{ij} = z_{ij} - x_{ij}, i = x, y, z$. The error dynamic of the LESO for each axis can be obtained from Eqs.(5) and (7) as

$$\begin{cases} \dot{e}_{i1} = e_2 - \omega_{ei}\alpha_{i1} \cdot e_{i1} \\ \dot{e}_{i2} = e_3 - \omega_{ei}^2 \alpha_{i2} \cdot e_{i2} \\ \dot{e}_{i3} = h_i - \omega_{ei}^3 \alpha_{i1} \cdot e_{i1} \end{cases}. \tag{25}$$

Proposition 1 For the 3rd-LESO shown as Eq.(7), suppose that the matrix $\boldsymbol{A_{ei}} (i = x, y, z)$ is Hurwitz. Then, there exists a constant $\omega_{ei} > 0$ such that $\lim_{t \to \infty} \boldsymbol{e}_i(t) = 0$.

Proof Let $\varepsilon_{ij} = \dfrac{e_{ij}}{\omega_{ei}^{j-1}}$, $\boldsymbol{\varepsilon_i} = \begin{bmatrix} \varepsilon_{i1} & \varepsilon_{i2} & \varepsilon_{i3} \end{bmatrix}$, Eq. (9) can be rewritten as

$$\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{i}} = \omega_{ei} \boldsymbol{A_{ei}} \boldsymbol{\varepsilon_i} + \boldsymbol{B_{ei}} \frac{h_i - \hat{h}_i}{\omega_{ei}^2}, \tag{26}$$

where $\boldsymbol{A_{ei}} = \begin{bmatrix} -\alpha_{i1} & 1 & 0 \\ -\alpha_{i2} & 0 & 1 \\ -\alpha_{i3} & 0 & 0 \end{bmatrix}, \boldsymbol{B_{ei}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

From (5), it can be found that h_i is Lipschitz. Since $\boldsymbol{A_{ei}}$ is Hurwitz, according to Lemma 1, it can be concluded that for the 3rd-LESO (7), $\lim_{t \to \infty} \boldsymbol{e}_i(t) = 0$.**Q.E.D.**

B. Stability of the Closed-loop System

With the final effective compensations (Eqs.(14) and (24)), the closed-loop system can be written in the state space form as

$$\begin{cases} \dot{\boldsymbol{x}}_j = \boldsymbol{A}_j \boldsymbol{x}_j + \boldsymbol{B}_j \boldsymbol{u}_j + \boldsymbol{w}_{bj} \\ \boldsymbol{u}_{jo} = -\boldsymbol{K}_j (\boldsymbol{x}_j + \boldsymbol{\Delta H}_j) \\ \boldsymbol{u}_j = \boldsymbol{u}_{jo} - \boldsymbol{\Delta u}_j \end{cases}. \tag{27}$$

where $j = y, xz$,

$$\boldsymbol{\Delta H}_y = \begin{bmatrix} 0 & 0 & \int \Delta h_y & \Delta h_y \end{bmatrix}^T,$$

$$\boldsymbol{\Delta H}_{xz} = \begin{bmatrix} 0 & 0 & \int \Delta h_x & \Delta h_x & 0 & 0 & \int \Delta h_z & \Delta h_z \end{bmatrix}^T.$$

The case we discuss in this paper is the normal flight mode of the space station. The desired attitude and the angular velocity corresponding to each axis are 0 . Offsetting compensation Δu_i of each axis in MM passage can be turned into the problem of tracking $-\Delta u_i$. In other words the desired CMG momentum and its integral are $-\Delta h_i$ and $-\int \Delta h_i$, respectively. Therefore, the reference input of each axis is $\boldsymbol{r}_i = \begin{bmatrix} 0 & 0 & -\int \Delta h_i & -\Delta h_i \end{bmatrix}^T$.

The track error and its differential corresponding to each axis are described as

$$\boldsymbol{\xi}_j = \boldsymbol{r}_j - \boldsymbol{x}_j = \begin{bmatrix} -x_{j1} & -x_{j2} & -\int \Delta h_j - x_{j3} & -\Delta h_j - x_{j4} \end{bmatrix}^T = -(\boldsymbol{X}_j + \boldsymbol{\Delta H}_j). \quad (28)$$

$$\dot{\boldsymbol{\xi}}_j = \dot{\boldsymbol{r}}_j - \dot{\boldsymbol{x}}_j = \begin{bmatrix} -\dot{x}_{j1} & -\dot{x}_{j2} & -\Delta h_j - \dot{x}_{j3} & -\frac{d\Delta h_j}{dt} - \dot{x}_{j4} \end{bmatrix}^T. \quad (29)$$

Applying Eq. (29) into Eqs.(16) and (26) gives

$$\begin{aligned} \boldsymbol{u}_{jo} &= \boldsymbol{K}_j \boldsymbol{\xi}_j, \\ \boldsymbol{u}_j &= \boldsymbol{K}_j \boldsymbol{\xi}_j - \boldsymbol{\Delta u}_j. \end{aligned} \quad (30)$$

Theorem 1 For system (27), if the LESO is stable, the LQR based controller can make the closed system of ACMM bounded.

Proof Substituting Eqs. (27) and (30) into Eqs. (1) and (2), we can obtain

$$\begin{cases} \dot{\xi}_{y1} = \xi_{y2} \\ \dot{\xi}_{y2} = a_y \xi_{y1} - b_y \boldsymbol{K}_y \boldsymbol{\xi}_y + b_y \Delta u_y - w_{by} \\ \dot{\xi}_{y3} = \xi_{y4} \\ \dot{\xi}_{y4} = -\boldsymbol{K}_y \boldsymbol{\xi}_y \end{cases}. \quad (31)$$

$$\begin{cases} \dot{\xi}_{x1} = \xi_{x2} \\ \dot{\xi}_{x2} = a_{xz21} \cdot \xi_{x1} + a_{xz26} \cdot \xi_{z2} - b_x (u_{xo} - \Delta u_x) - w_{bx} \\ \dot{\xi}_{x3} = \xi_{x4} \\ \dot{\xi}_{x4} = \omega_o \xi_{z4} - u_{xo} \\ \dot{\xi}_{z1} = \xi_{z2} \\ \dot{\xi}_{z2} = a_{xz62} \cdot \xi_{x2} + a_{xz65} \cdot \xi_{z1} - b_z (u_{zo} - \Delta u_z) - w_{bz} \\ \dot{\xi}_{z3} = \xi_{z4} \\ \dot{\xi}_{z4} = -\omega_o \xi_{x4} - u_{zo} \end{cases}. \quad (32)$$

Eqs. (31) and (32) can be rewritten in the following form

$$\dot{\boldsymbol{\xi}}_j = (\boldsymbol{A}_j - \boldsymbol{B}_j \boldsymbol{K}_j) \boldsymbol{\xi}_j + \boldsymbol{B}_{\xi j} \boldsymbol{\Delta u}_j - \boldsymbol{w}_{bj}, \ j = y, xz. \quad (33)$$

where

$$\boldsymbol{B}_{\xi y} = \begin{bmatrix} 0 & by & 0 & 0 \end{bmatrix}^T,$$

$$\boldsymbol{B}_{\xi xz} = \begin{bmatrix} 0 & b_x & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_z & 0 & 0 \end{bmatrix}^T.$$

The idea of pole-assignment based LQR is to place the eigenvalues of the closed-loop system in the left half-plane of the complex plane [3, 4]. So $(\boldsymbol{A}_j - \boldsymbol{B}_j \boldsymbol{K}_j)$ is Hurwitz.

The external disturbance w_{bi} can be decomposed as the sum of the bias w_{ic} and the cyclic term w_{im}. And their LESO-estimated values are z_{ic} and z_{im}.

$$\begin{aligned} w_{bi} &= w_{ic} + w_{im}, \\ z_{i3} &= z_{ic} + z_{im}. \end{aligned} \quad (34)$$

The estimation errors of w_{bi} can be denoted as

$$e_{i3} = z_{i3} - w_{bi} = z_{ic} - w_{ic} + z_{im} - w_{im} \triangleq e_{ic} + e_{im}. \quad (35)$$

Since LESO is stable, $|e_{ic}|$ and $|e_{im}|$ are bounded.

Since $\Delta u_y = z_{ym} / b_y$, it follows that

$$\boldsymbol{B}_{\xi y} \boldsymbol{\Delta u}_y - \boldsymbol{w}_{by} = \begin{bmatrix} 0 & e_{ym} - w_{yc} & 0 & 0 \end{bmatrix}^T, \quad (36)$$

Let $\boldsymbol{B}_{\xi y} \boldsymbol{\Delta u}_y - \boldsymbol{w}_{by} \triangleq \zeta_y$, and combine it with Eq.(3a), the following equation can be given

$$|\zeta_y| \le |e_{ym}| + |b_y||b_{y0}|$$

According to Lemma 2, $\lim_{t\to\infty} \boldsymbol{\xi}_y(\mathbf{t})$ is bounded.

Let $\boldsymbol{B}_{\xi xz} \boldsymbol{\Delta u}_{xz} - \boldsymbol{w}_{bxz} \stackrel{\Delta}{=} \begin{bmatrix} \boldsymbol{\zeta}_x \\ \boldsymbol{\zeta}_z \end{bmatrix}$, according to $\Delta u_x = z_{xc} / b_x$ and $\Delta u_z = z_{zc} / b_z$, it follows that

$$\begin{aligned} \boldsymbol{\zeta}_x &= \begin{bmatrix} 0 & e_{xc} - w_{xm} & 0 & 0 \end{bmatrix}^T, \\ \boldsymbol{\zeta}_z &= \begin{bmatrix} 0 & e_{zc} - w_{zm} & 0 & 0 \end{bmatrix}^T. \end{aligned} \quad (37)$$

Since $|w_{im}| \le |a_{i1}| + |a_{i2}|$, one has

$$\begin{aligned} |\boldsymbol{\zeta}_x| &\le |e_{xc}| + |a_{x1}| + |a_{x2}|, \\ |\boldsymbol{\zeta}_z| &\le |e_{zc}| + |a_{z1}| + |a_{z2}|. \end{aligned} \quad (38)$$

According to Lemma 2, $\lim_{t\to\infty} \begin{bmatrix} \boldsymbol{\xi}_x(t) \\ \boldsymbol{\xi}_z(t) \end{bmatrix}$ is bounded, and the closed-loop system is bounded as well. **Q.E.D.**

V. SIMULATIONS

A. System Parameters

For comparison and analysis, the parameters of the Phase 1 Space Station specified in [2-5] are adopted, as shown in TABLE I. The initial conditions are

$$\boldsymbol{x}_i(0) = [-0.004 \quad -2e\text{-}5 \quad 0 \quad 0]^T, i = x, y, z.$$

The maximum momentum of CMG is $h_{i\max} = 1.7236 \times 10^4 \ N.m$. The initial estimated value of states in the LESO is set to be 0. The values of corresponding parameters are as follows:

$$\omega_{ei} = 1.25, \alpha_{i1} = 1.6, \alpha_{i2} = 0.16, \alpha_{i3} = 0.013.$$

It is clear that A_{ei} is Hurwitz, so the LESO designed for Phase 1 is stable.

TABLE I. PARANETERS OF PHASE1

Inertia (kg·m2)			
I_x	6.8170×10^7	I_{xy}	-0.0528×10^7
I_y	1.4643×10^7	I_{xz}	0.0217×10^7
I_z	7.9410×10^7	I_{yz}	0.0217×10^7
Aerodynamic Torque (N·m)			
w_x	$1.36+\sin(\omega_o t)+0.68\sin(2\omega_o t)$		
w_y	$5.44+2.72\sin(\omega_o t)+0.68\sin(2\omega_o t)$		
w_z	$1.36+\sin(\omega_o t)+0.68\sin(2\omega_o t)$		

The controller gains of the space station calculated by LQR method are showed in TABLE II.

TABLE II. GAINS OF CONTROLLER

K_y^T	K_{xz}^T	
	-7.66×10	3.58×10^2
	-9.11×10^4	-5.25×10^4
-2.98×10^2	-1.93×10^{-6}	1.05×10^{-5}
-1.79×10^5	3.86×10^{-3}	-6.01×10^{-4}
-2.01×10^{-6}	2.67×10^2	-1.30×10^3
-5.5×10^{-3}	-2.01×10^5	-2.92×10^5
	-2.25×10^{-6}	-5.82×10^{-7}
	-2.98×10^{-3}	2.16×10^{-3}

B. Simulation Results

Benefited from the fact that LESO can manage the coupling axes to simplify the design process, and successfully reduce the dimensions of the control gains.

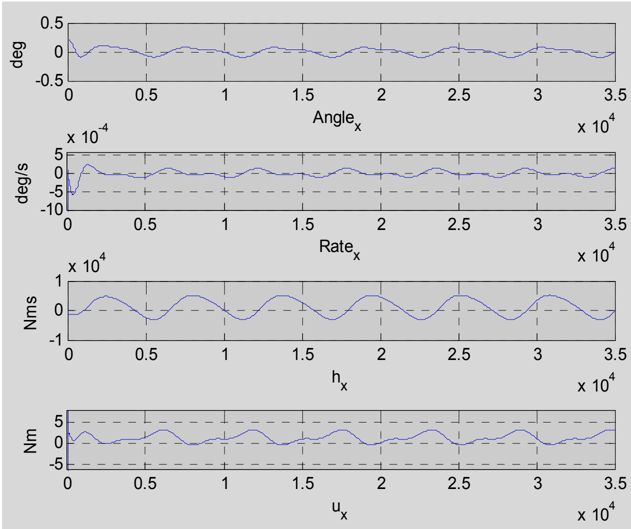

Fig.2 Roll Response.

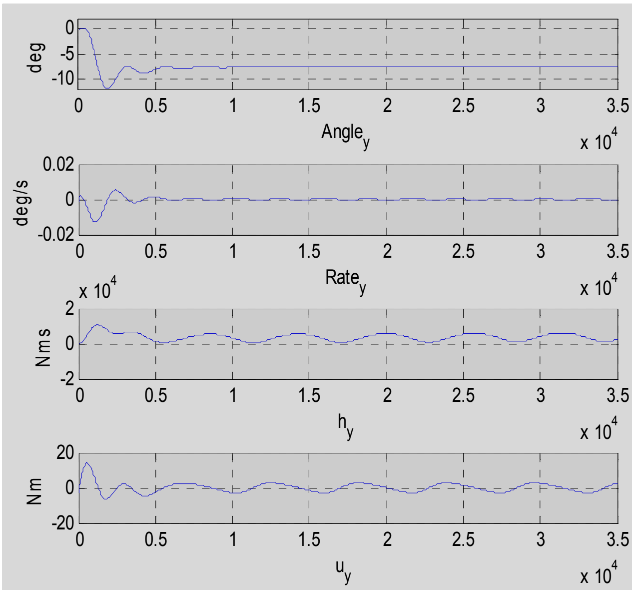

Fig.3 Pith Response.

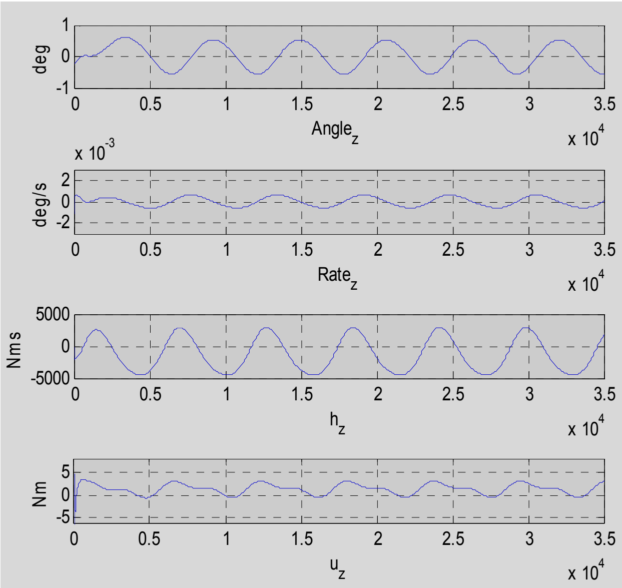

Fig.4 Yaw Response.

Fig. 2-4 depict the simulation results of the Euler angles θ_i, the angular velocities $\dot{\theta}_i$, the CMG momentums h_i, and the control torques u_i on the roll, pitch, and yaw axis, respectively. It can be observed that all CMG momentums are within the envelop range of $h_{i\max}$. The specified results are shown in TABLE III**.**

TABLE III. RESULTS COMPARISON

	LESO	**IMP**
θ_x(deg)	$-0.1 \sim 0.1$	$-0.75 \sim 0.25$
$\dot{\theta}_x$(deg/ s)	$(-1.4 \sim 1.4)\times 10^{-4}$	$(0.5 \sim 2)\times 10^{-3}$
θ_y(deg)	-7.65	-8
$\dot{\theta}_y$(deg/ s)	$(-3 \sim 3)\times 10^{-5}$	2×10^{-4}
θ_z(deg)	$-0.25 \sim 0.82$	-1.25
$\dot{\theta}_z$(deg/ s)	$(-6 \sim 6)\times 10^{-4}$	$(0.5 \sim 2)\times 10^{-3}$

Table III presents comparisons of the control effects between the LESO-based method and the IMP-based LQR method. It can be seen that: (1) The LESO with LQR controller can better reject the disturbance to balance the attitude control accuracy and the momentum management restriction than the latter control method. (2) The LESO is able to upgrade the stability of each axis by one grade. To be more specific, the attitude on the yaw axis oscillates within a much smaller range, and the control accuracy of the roll axis is comparatively increased.

VI. CONCLUSIONS

In this paper, a LESO is adopted to estimate the disturbance of the space station. Based on the working mechanism of the external disturbance, reasonable compensating strategies are deduced to achieve ACMM. The LESO can estimate the dynamic coupling of the ACMM system, thus greatly simplify the design process. The proposed method can well reject the disturbance while avoiding the calculation of high-dimensional feedback controller, and well balance the control accuracy and momentum management restriction with LQR controller. Furthermore, the stabilities of the designed LESO and the closed-loop system have been proved theoretically. Finally, the simulation results show that the proposed control scheme achieves better control effects than traditional IMP-based LQR control method.

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REFERENCES

[1] V. N. Branetz, V. P. Legostayev, and B. Y. Chertok, “The system of the Mir station motion control”. IAF-88-334.

[2] J.T. Harduvel, “Continuous Momentum Management of Earth-Oriented Spacecraft. Journal of Guidance, Control and Dynamics”, vol.15, no6, pp.1417-1426,1992.

[3] B. Wie, K.W. Byun, and V.W. Warren, “New Approach to Attitude/Momentum Control for the Space Station”, Journal of Guidance, Control and Dynamics, vol.12, no.5, pp.714-722, 1989.

[4] J.W. Sunkel, L.S. Shieh, “Multistage Design of an Optimal Momentum Management Controller for the Space Station”, Journal of Guidance, Control and Dynamics, vol.14, no.3, pp.492-502, 1991.

[5] J. Mapar, “Innovative Approach to the Momentum Management Control for Space Station Freedom”, Journal of Guidance, Control and Dynamics, vol.16, no.1, pp.175-181, 1993.

[6] G.J. Bennett, K.P. Sebelius, and Barth A.L, “ISS Russian Segment Motion Control System Operating Strategy During Orbiter Repair Maneuver”, AIAA-2005-5855.

[7] J. Zhang, Y. He, and J. Zhang, “Attitude Control and Momentum Management of Inertially Oriented Space Station”, in Proc.19th IFAC Symposium on Automatic Control in Aerospace, Würzburg, 2013: pp.1-3.

[8] J. Han, “Auto-disturbance rejection control and its applications”, Control Decis, 13(1): 19–23, 1998 [in Chinese].

[9] J. Han, “From PID to active disturbance rejection control”, IEEE Trans Ind Electron, vol.56, no.3, pp. 900–906, 2009,.

[10] Y. Huang and W. Xue, “Active disturbance rejection control: Methodology and theoretical analysis”, ISA Transactions, vol.53, vol.4, pp.963-976, 2014.

[11] Y. Huang, K. Xu, J. Han, and J. Lam, “Flight control design using extended state observer and non-smooth feedback”, IEEE Conference on Decision and Control, Orlando, 2001, pp.223–228.

[12] B.Yan, et al, “Fault diagnosis for a class of nonlinear systems via ESO”, ISA Transactions, vol.47, no.4, pp.386-394, 2008.

[13] B.X.S. Alexander, R. Rarick, and L. Dong, “A Novel Application of An Extended State Observer for High Performance Control of NASA’s HSS Flywheel and Fault Detection”, in Proc. American Control Conference, Washington, 2008, pp.5216-5221.

[14] L. Dong, D. Avanesian, “Drive mode control for vibrational MEMS gyroscopes”, IEEE Trans Ind Electron, vol.6, no.4, pp.956–63, 2009.

[15] S.E. Talole, J.P. Kolhe, and S.B. Phadke, “Task-independent robotic uncalibrated hand-eye coordination based on the extended state observer”, IEEE Trans Ind Electron, vol.57, no.4, pp.1411–1419, 2010.
[16] X. Yang, Y. Huang, “Capabilities of Extended State Observer for Estimating Uncertainties”, in Proc.American Control Conference, St. Louis, 2009, pp.3700-3705.
[17] Q. Zheng, L.Q. Gao, Z. Gao, “On Stability Analysis of Active Disturbance Rejection Control for Nonlinear Time-Varying Plants with Unknown Dynamics”, in Proc.46th IEEE Conference on Decision and Control, New Orleans,12-14, pp.3501-3506, 2007.