# Degradation Model and Security Analysis of Hangers of Suspension Bridges

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***Abstract* — The hangers of a suspension bridge is one of the most important stressed members, the thorough research of the deterioration during operation period and the security analysis after the degeneration of hangers is essential. This is the basis of the strategy of preventive maintenance of suspension bridges. Firstly, the paper studies the four stages of degradation mechanisms of sling and calculation formula of degradation time, and establishes the theoretical relationship between the degree of degradation and resistance reduction coefficient, and then discusses the safety analysis method of sling with broken wire to consider the effect of dangle. Finally, taking an example of the 3rd sling of left Branch Bridge of Ma'anshan bridge, we conducted theoretical calculations of degradation time of sling and security calculations of sling after degradation. The method is purely analytical, and there was an error between the real data and analytical calculations. If we combine the real monitoring data and use it after parameter modification, we can improve greatly the analytical basis of the strategy of preventive maintenance.**

***Keywords-Hanger;Degradation model;Safety analysis***

## I. INTRODUCTION

In recent years, safety and conservation strategies of large suspension bridge are paid more and more attention, the deterioration of important components and the entire bridge has become a hotspot in engineering. JamesM.Stallings and KarlH.Frank analyzed the influencing factors of fatigue life of the hanger using series-parallel model [1]. According to the test results, of the New York Williamsburg main cable, J.Matteo used ductility model of hanger and the brittle model to estimate the carrying capacity of the cable[2]. In China, Lan Chengming established theoretical models calculated fatigue life of parallel wire cable. He considered that the fatigue life of the cable is controlled by a small part of the fatigue life of the shorter wire, 10% broken wires used as life termination of cable is reasonable [3]. Zhu Jinsong combined Matteo's brittle wire model and Monte Carlo method, simulated cable to have m filaments (each filament is divided into n segments) connected in parallel systems, it's also not considered strength degradation of steel due to corrosion and fatigue, the degraded condition of cable is only caused by of broken wires [4]. Based on relevant research, the paper will be further explore the theoretical degradation model of sling, and safety assessment methods of sling after degradation, and apply them to practical engineering of Ma Anshan Bridge.

## II. DEGRADATION MODEL OF SLING AND THE REDUCTION FACTOR

In order to establish the precise degradation model, we must take into account as fully as possible factors leading to Degradation of the sling, and establish degradation model based on different dominant factor in different time periods.

Factors leading to degradation of sling are: sheath damage; wire damage; anchor head injuries. Combined resistance reduction factor Z1, the degradation process will degenerate into the following phases.

### *A. Slings is installed - jacket occur cracks*

At this stage, the wire is without electrochemical corrosion and stress corrosion, therefore only considered natural aging and damage under alternating stress and fatigue damage. The natural aging damage can learn from research results of Liu Yaping to be calculated. Maanshan Bridge, located in the northern subtropical monsoon climate zone, can learn from the results of sub-tropical rainforest climate, do the least-squares fitting and simple derivation, you can get the time t1 from installation to occur cracks of jacket that meet the required expressions :

$$\frac{T}{EA} = 6\times 64.462e^{-0.0397t_1} \quad (1)$$

reduction：

$$t_1 = 25.189\ln(\frac{386.772EA}{T}) \quad (2)$$

### *B. Jacket occur cracks- galvanized coating of steel failure*

This stage jacket has not withstand cable force, the damage is mainly fatigue damage of wire, and length of time tII at this stage is mainly expiration time of the galvanizing layer, can be calculated by:

$$t_{\mathrm{II}} = \sqrt[n]{\frac{C}{C_1}} \quad (3)$$

Where C1 and n is the local parameters of rate of corrosion of galvanized layer, C can be substituted into thickness of the smallest of galvanized steel wire.

This stage the cable resistance reduction factor Z1 is 0.90-0.95, the Z1 can be expressed as a function of time t:

$$Z_1 = 0.95 - 0.05t / t_{\mathrm{II}} \quad (4)$$

### *C. Steel galvanized coating failure - fatigue cracks*

This stage mainly considers time tIII when corrosion fatigue occurs, according to the Faraday's formula of

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corrosion rate of uniform volume, the expression can be deduced:

$$t_{III} = \frac{2\pi nF\rho}{3MI_{po}}(a_{ci}^3 - a_0^3)\exp(\frac{\Delta H}{RT}) \quad (5)$$

This stage of the cable resistance reduction factor Z1 is 0.85-0.90, the Z1 can be expressed as a function of time t:

$$Z_1 = 0.90 - 0.05t / t_{III} \quad (6)$$

*D. Fatigue crack growth - the first floor of wire breakage*

This stage is mainly on account of time tIV after fatigue corrosion of steel occurred. The formula of metal corrosion fatigue crack growth rate of Paris is still available the relations of time and damage can be derived:

$$s_3 = s_{ci} + \int_0^{ft_{IV}} \frac{k}{2\pi\sigma_{ff}^2}(\Delta K - \Delta K_{th})dN \quad (7)$$

Here cannot take 0, and need to take a stage of pitting corrosion depth. If it is determined expression of the crack tip stress by the method combined experiments and experience, can be calculated.

If it is considered that the breakage moment of the first floor of wire is entire hanger failure time, then this phase cable resistance reduction factor is 0-0.85, the can be expressed as a function of time t:

$$Z_1 = 0.85 - 0.85t / t_{IV} \quad (8)$$

Based on period of time of various stages, the wire rope jacket fatigue fracture life t from the start until age can be estimated.

$$t = t_I + t_{II} + t_{III} + t_{IV} \quad (9)$$

## III. SECURITY ANALYSIS OF HANGER

*A. Assessment model of failure and safety of sling*

*1) sling strength model*

Considering the effect of Daniel and rope strength of broken wires equal to the product of the number of wires and the average tensile strength of steel.

$$F = \tilde{n}W_n \quad (10)$$

Where,

F---tensile of sling considered effects of Daniel;

$\tilde{n}$ ---the number of wire not broken;

$W_n$ ---average tensile strength of steel considered effect of Daniel.

*2) Assessment Model of Safe and Failure of sling*

The safety factor of actual tensile of sling under constant loads is failure criterion of cable, it can determine the probability of failure of cable. Performance function of sling failure model is:

$$f\left(A_w, \sigma_u, T_s, \eta\right) = \frac{\tilde{n}A_w\sigma_u}{\left(1+\eta\right)T_D} - \xi \quad (11)$$

Where,

$A_\omega$ ---the cross-sectional area of individual wires;

$\sigma_u$ ---the average tensile strength of steel wire of sling considered the effect of Daniel;

$T_s$ ---cable tension generated during the operation of hanging dead load;

$\eta$ ---ratio of cable force under live load and dead load;

number of no broken wire of sling;

$\xi$ ---safety factor of sling for the design, in general $\xi = 2.5$ .

The formula is performance function of safety failure of sling. Then, after determined the distribution type of each parameter, it can be used in the statistical analysis of Matlab software to generate random sample of the parameters with a certain sample size. The random sample of data used into the performance function and statistic calculations, using Monie-Carlo method to calculate the probability of failure of cable.

*B. Strength analysis of parallel wire of sling based the Daniel effects*

Firstly, defined the ratio of the reduced average intensity wire cable and average tensile strength of the individual wires is the attenuation factor. There are:

$$\tau = \frac{R_D}{R_0} \quad (12)$$

Parallel wire sling system is composed of n filaments, when the sample size is sufficiently large (n> 150), average tensile strength obeys Normal Distribution. Generated a random sample of the normal distribution in MATLAB. In order to effectively analyze the Daniel effects of hangers with different number of wires, different sample sizes were taken for analysis o, calculated correlation parameters.

Then used the following formula to calculate the average tensile strength of steel when wires have different numbers.

$$W_n = 1 - F_Z(x_0) + \frac{c_n}{n} \quad (13)$$

Where,

$W_n$ ---tensile strength, n is the number of wires

Fitting the relationship between the tensile strength of steel wire and wire actual number caused of Daniel effects by regression.

## IV. EXAMPLES

The 3rd sling of left Branch Bridge of Maanshan Bridge is example. Maanshan sling hanged from 2 to 64 points is ordinary sling, each sling consisted by 109 ordinary steel wires. Steel wire is used φ5.0mm high-strength galvanized wire, the standard of tensile strength is not less than 1670MPa, elastic modulus of steel E=2.0×105MPa.

*A. Sling failure elapsed time*

*1) Sling defense system failure experienced time can be divided into two sections* *a) HDPE jacket cracking and aging failure experienced time*

The 3rd sling of left Branch Bridge of Maanshan Bridge is example. According to their results the cable force of the 3rd sling of Ma'anshan Bridge under dead load and live loads of car, T = 1531.56kN ,E=2.0×105MPa, effective tension area A=2 × $10^9$ × π （2.5 × $10^{-3}$ ）$^2$=4.278 × $10^{-3}$m$^2$, according to:

$$t_1 = 25.189\ln(\frac{386.772EA}{T}) \tag{14}$$

Get $t_{I1}$ = 218 (weeks) = 4.2 (years)

*b) The life of anti-corrosion coating*

Based on the durable life specified in the ISO12944 standard, failure time of protective coating system is simply assumed, taking the life of durability of the coating system $t_{I2}$ = 2 (years)

Failure can be obtained from the above sling protection system experienced time:

$$t_I = t_{I1} + t_{I2} = 6.2\text{(years)} \tag{15}$$

*(2) Experienced time from corrode to fail of steel wire galvanized coating*

According to national standards [5] average quality galvanized layer of steel wire of cable and sling is not less than 300g / m2, you can get an average thickness of galvanization layer:

$$b = W / \rho \tag{16}$$

Where.

W = 300g / m2, for the lower limits quality of galvanized layer steel wire;

ρ = 7.2g / cm3, the density of pure zinc coating.

Calculated: b = 41.67um However, taking into account the impact of galvanized coating process itself to the minimum thickness, the coating thickness is generally smaller than the average 15um, therefore C = 26.67um.

According to table of the corrosion rate of the domestic hot-dip galvanizing in all regions [6], n taken 1.088, C1 taken 1.726.

According to the formula：

$$t_{\text{II}} = \sqrt[n]{\frac{C}{C_1}} \tag{17}$$

Calculated:

$$t_{\text{II}} = \sqrt[n]{\frac{C}{C_1}} = \sqrt[1.088]{\frac{26.67}{1.726}} = 12.3 \tag{18}$$

*(3) Experienced time of steel wire body from corrosion pits formed to corrode fatigue occurred*

According to the aforementioned degradation model:

$$t_{\text{III}} = \frac{2\pi nF\rho}{3MI_{po}}(a_{ci}^3 - a_0^3)\exp(\frac{\Delta H}{RT}) \tag{19}$$

Where,

M---the molar mass of material, M = 55.85;

n---the atomic valence, n = 3;

F---Fala Di constant, F = 96514Cal / mole;

ρ---the material density, ρ = 7.8 × 106g / m$^3$;

R---a ventilation coefficient, R = 8.317J / mole•K;

The average annual temperature in Ma Anshan is 15.9 ℃ , T is the absolute temperature, T = 237 + 15.9 = 252.9;

ΔH is steel wire redox reaction activation energy, ΔH = 59.7J / mol;

According to the literature [7], the use of fuzzy - random method gives pits initial size a0, and pitting flow coefficient ,

$$a_0 = 1.5 \times 10^{-6} m \tag{20}$$

$$I_{po} = 80.78c / s \tag{21}$$

$$a_{ci} = \pi\left(\frac{\Delta K_{th}}{2.2K_t\Delta\sigma}\right)^2 \tag{22}$$

According to the literature [8] fitting equation $\Delta K_{th}$ = -3.4324R + 5.5351, where R is a steel wire corrosion fatigue stress ratio, R here take roughly 0.67. Whereby threshold of stress intensity factor amplitude of steel wire $\Delta K_{th}$ = 3.22, $a_{ci}$ = 59.3×$10^{-6}$m.

The above data used into (19), obtained: $t_{\text{III}}$ = 213 (days) = 0.58 (year)

*(4) Experienced time from corrode fatigue crack growth to wire breakage*

When fatigue crack of steel wire extensions to a certain depth, the stress intensity factor of crack tip reaches the threshold $K_I$,

$$K_I = \sigma\sqrt{\pi a}F\left(\frac{a}{d}\right) \tag{23}$$

Assuming crack developing still by spherical approximately, according to the empirical formula:

$$K_I = \sigma\sqrt{\pi a}\left(\frac{0.865}{(a/d)+0.324} + 0.681\right) \tag{24}$$

Can be obtained from the above two equations, $K_{Imin}$ = 0, $K_{Imax}$ = 1.77 .

Based on combining theoretical analysis and experimental data, used the Paris fitting formula to calculate $2\pi\sigma_{ff}^2 = 2.158\times10^{10}MPa$ According to curve of the fatigue experimental results of steel wire in artificial seawater conditions on cold drawn, A.Martin gave the environment constant k = 1.585, ΔK = 1.77MPa.m1 / 2. Diameter of steel wire d = 5mm, pitting depth $a_{ci}$ = 59.3×$10^{-6}$, crack depth of fatigue fracture of steel wire can be regarded a = 2.26mm.

The above data substituted into the formula：

$$s_3 = s_{ci} + \int_0^{ft_{\text{IV}}}\frac{k}{2\pi\sigma_{ff}^2}(\Delta K - \Delta K_{th})dN \tag{25}$$

Regarded stress cycles when steel wire fatigue fracture

$$N = 3.36 \times 10^{6} cycle \tag{26}$$

Regarded stress cycle frequency of sling wire

$$f = 1/60 (cycle/s) \tag{27}$$

Calculated: $t_{IV} = N / f = 6.4 (years)$ (28)

*B. Security Analysis of sling wire*

*(1) Strength analysis of the wire sling based the Daniel effects*

Steel wire Analyzed is ordinary steel wire of Ma'anshan Bridge, steel wire is φ5.0mm high-strength galvanized steel wire, standard tensile strength of steel wire is not less than 1670MPa.

Firstly, determining the distribution function of tensile strength of steel wire, $y_u$ = 1670MPa, according to the literature [9] whichever coefficient of variation ξ = 0.05, the standard deviation of the tensile strength of the steel wire:

Then wire tensile strength x~N(1670，6972.25)

Probability density function:

$$f_Z(x) = \frac{1}{\sqrt{2\pi} \times 83.5} e^{\frac{(x-1670)^2}{2\times 83.5^2}} \tag{29}$$

Distribution function:

$$F_Z(x) = \frac{1}{\sqrt{2\pi} \times 83.5} \int_{-\infty}^{x} e^{\frac{(t-1670)}{2\times 83.5^2}} dt \tag{30}$$

Then generate a random sample of the normal distribution in MATLAB. In order to effectively analyzed the effect Daniel when slings have the number of different steel wire, Samples capacity which were taken in this 10,50,100,150,200 five cases were analyzed. Sample statistic data is processed solved Daniel effect parameters as shown in Table 1.

TABLE I. DANIEL EFFECT PARAMETERS

| n / root | x0/MPa | Fz | fz | a/MPa | Cn/MPa |
|---|---|---|---|---|---|
| 10 | 1457.4 | 0.0054 | 1.8802 | 236.4654 | 548.3231 |
| 50 | 1466.2 | 0.0072 | 2.4423 | 293.2517 | 1043.627 |
| 100 | 1466.9 | 0.0077 | 2.5091 | 296.4251 | 1329.124 |
| 150 | 1468.7 | 0.0079 | 2.6213 | 301.7756 | 1548.899 |
| 200 | 1468.7 | 0.0079 | 2.6213 | 301.7756 | 1700.131 |

Substituted into the formula：

$$W_n = 1 - F_Z(x_0) + \frac{c_n}{n} \tag{31}$$

Calculated the average tensile strength of steel wire of sling when sling have different number of steel wire, the results in Table 2.

TABLE II. THE AVERAGE TENSILE STRENGTH OF STEEL WIRE OF SLING

| n / root | 10 | 50 | 100 | 150 | 200 |
|---|---|---|---|---|---|
| $W_n$ | 1503.325 | 1475.759 | 1468.154 | 1466.598 | 1464.237 |

The following figure can be obtained using polynomial regression to process data:

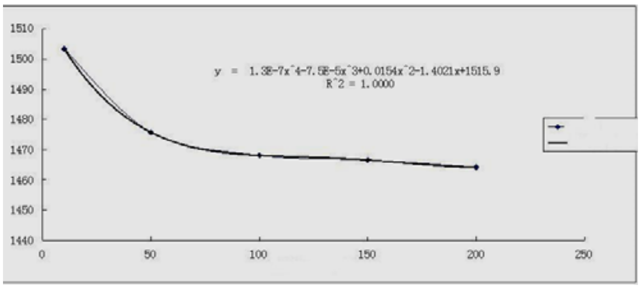


Figure 1. the average tensile strength of steel wire of sling with different numbers steel wire.

The relationship between the actual tensile strength and number of steel wires caused by Daniel effect can be got by regression fitting:

Where, $W_n$ is the tensile strength, n is the number of steel.

*(2) Safety failure probability analysis of sling wire*

The tensile actual safety factor of sling under dead loads is failure criterion, used it to determine the failure probability of sling. Performance function of sling failure model is：

$$f\left(A_w, \sigma_u, T_s, \eta\right) = \frac{\tilde{n} A_w \sigma_u}{\left(1+\eta\right) T_D} - \xi \tag{32}$$

Determining statistical properties of parameters of the function that determine the distribution type, mean, standard deviation, coefficient of each parameter. According to the results, statistic properties of parameters Aw, Wn, TD, η of the performance function will be described.

Area of the sling wire is average cross-sectional area 19.6, the tensile strength is 1670MPa, the coefficient of variation of tensile strength and cross-sectional area is 0.05. So, the results of statistical properties of the four sample statistics of No. 3 sling of Maanshan left Branch Bridge Bridge is: Aw~N (19.6,0.98) (coefficient of variation of 0.05); σu~N (1670,83.5) (coefficient of variation 0.05); η~N (0.815,0.013) (coefficient of variation 0.016); TD (variation coefficient 0.02).

Distribution of the parameters is known, used Matlab to generate random samples of the parameters: the average cross-sectional area $A_w$ = normrnd (19.6,0.98,100,1), different number of broken wires are a random sample of average tensile strength of steel, non-broken wire $W_{109}$ = normrnd (1470.8,3.45,100,1), off 10 $W_{99}$ =normrnd(1471.32,2.83,100,1), off 20 $W_{89}$=normrnd(1472.6Wn2.56Wn100Wn1), off 30 W79= normrnd(1474.80,2.67,100,1), off 40

$W_{69}$=normrnd(1478.22,3.29,100,1) and off 50 $W_{59}$=normrnd(1480.46,3.80,100,1) and so on.

Similarly random sample of cable tension under dead load can be produced when the number of broken wires is different.

The sample values are brought into the equation to calculate the value of the function f, due to the sample size of each parameter was 100; the number of wire broken wires followed by 0,10,20,30,40,50 six kinds situations to consider, each case strength steel and dead load cable tension was simulated 100 times. Therefore, the word that random sample of data one by one brought into to the performance function to calculate the value of the performance function is very large. To simplify the calculation process, converting a random sample of data into a matrix form, using the matrix operation to complete.

$$f_{10} = (a_{10} * w_{10}) / (b_1 * t_{10}) / 1000 - 2.5 \quad (33)$$

Where,

$a_{10}$=99 * Aw;

$b_1 = \eta + 1$, generated in matlab ;

$w_{10}$---random sample of average tensile strength of steel when the number of broken wires is 10 and t10 is random sample of force of cable under dead load when the number of broken wires is 10.

According to the above method the value of f can be calculated when the number of broken wires are 0,20,30,40,50. To simplify the calculations, the following statement in matlab can be used to identify number which is greater than zero, and statistics.

Statistic the number of f which is greater than, equal to, less than zero. Supposed there are m f <0, then the failure probability of sling can be calculated by the following equation.

$$P(f < 0) = \frac{m}{100} \quad (34)$$

The results of Statistics calculations in the following table:

Table 3. Failure probability of broken wire of NO. 3 hanger

TABLE III. FAILURE PROBABILITY OF BROKEN WIRE OF NO. 3 HANGER

| Broken wires | Simulation times | f>0 | f<0 | Failure probability |
|---|---|---|---|---|
| 0 | 100 | 100 | 0 | 0 |
| 10 | 100 | 93 | 7 | 0.07 |
| 20 | 100 | 48 | 42 | 0.42 |
| 30 | 100 | 10 | 90 | 0.90 |
| 40 | 100 | 0 | 100 | 1 |
| 50 | 100 | 0 | 100 | 1 |

Simulating failure probability of the 3rd sling of Ma'anshan Bridge by Monte-Carlo method - broken wire curves, using the least squares to regress.

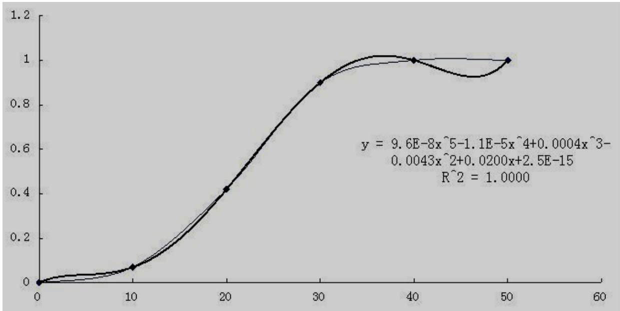


Figure 2. the plot of failure probability of broken wire of NO. 3 hanger

Thereby obtaining functional relationship between the probability of failure and the number of broken wires of the 3rd sling.

$$P_f = 9.6\times10^{-8} n^5 - 1.1\times10^{-5} n^4 + 0.0004 n^3 - 0.0043 n^2 + 0.0200 n + 2.5\times10^{-15} \quad (35)$$

Where, $P_f$ is the broken wire failure probability of the 3rd sling of Ma'anshan Bridge considered the Daniel effect; n is the number of broken wires.

When the number of broken wires are between 0 and 40, you can use the above formula to estimate the probability of failure of broken wires.

## V. CONCLUSION

Theoretical studies and numerical examples in this paper showed that: degradation mechanisms of the four stages and degradation time calculation formula of sling degradation and safety analysis method of the sling after broken wire considered the Daniel effect, had a certain rationality and feasibility, if combining it with the actual monitoring data to update model, it will be able to provide a reliable basis for preventive maintenance strategy better.

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