A Fuzzy Petri Net Model for Hesitant Decision Making

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***Abstract* — Experts are hesitant to give a decision in a fuzzy decision-making environment. In this study we present a new method for group decision making based on: i) hesitant fuzzy linguistic term sets, ii) using the pessimistic and optimistic attitudes of a decision maker. First we aggregate the fuzzy sets in each hesitant fuzzy linguistic term set into a fuzzy set. The input features in the fuzzy Petri net model can be formulated as uncertain tokens to determine the confidence value. Then we perform minimum and maximum operations among the obtained degrees for membership function to determine the derived value of the rule. The minimum and maximum operations denote the pessimistic and optimistic attitudes of the decision maker, respectively. Finally, for each alternative, the centroid method is used to rank the priority of the obtained confidence values and derive the preference order of the alternatives for the decision maker with pessimistic and optimistic attitudes. Our experimental results demonstrate the promising application of the proposed method in group decision-making problems as a "live" template.**

Keywords - Fuzzy Petri Net; confidence value; fuzzy set

I. INTRODUCTION

In a fuzzy decision-making environment, experts may be hesitant to choose an appropriate linguistic term to assess an alternative in some situations. To address such situations, Torra [1] presented the concept of hesitant fuzzy sets, which were generalizations of fuzzy sets [2]. Rodriguez et al. [3] indicated that hesitant fuzzy sets provided an interesting extension of fuzzy sets. These researchers attempted to manage situations in which a set of values could possibly define the membership value of an element. Farhadinia [4] presented some information measures for hesitant fuzzy sets and interval-valued hesitant fuzzy sets. Torra and Narukawa [5] proposed an extension principle to generalize existing operations on fuzzy sets into hesitant fuzzy sets. Xia and Xu [6] demonstrated several aggregation operators of hesitant fuzzy information for group decision making. The fuzzy logic system has been successfully used in various applications. Meanwhile, large-scale complex systems exist everywhere in society. This scenario encourages complex applications, which have numerous criteria and require real-time responses, of a fuzzy logic system.

The remainder of this paper is organized as follows. Section 2 briefly reviews the concepts of fuzzy sets [2, 19] and PN [20]. Section 3 give the concept of FPN [15, 16] and then present the FPN model of the hesitant decision-making model. Section 4 presents a new method for multi-criteria linguistic decision making based on hesitant fuzzy linguistic term sets using the pessimistic and optimistic attitudes of the decision maker. Conclusions are provided in Section 5.

II. STATE OF THE ART

Based on the concept of hesitant fuzzy sets, Rodriguez et al. [3] presented the concept of hesitant fuzzy linguistic term sets for multi-criteria linguistic decision making. Hesitant fuzzy linguistic term sets provide an interesting research topic on multi-criteria linguistic decision making. These researchers attempted to manage situations in which the evaluating values of the attributes of the alternatives were described by a set of possible linguistic terms represented by fuzzy sets. In recent years, some decision-making methods [3, 7–12] based on hesitant fuzzy linguistic term sets have been presented. Chen and Hong [7] proposed a method for multi-criteria linguistic decision making based on hesitant fuzzy linguistic term sets. Hajlaoui and Halouani [8] suggested a group decision-making method based on hesitant linguistic preference relations. Lee and Chen [9] presented a multi-criteria linguistic decision-making method according to likelihood-based comparison relations of hesitant fuzzy linguistic term sets. Rodriguez et al. [10] exhibited a group decision-making model that dealt with comparative expressions based on hesitant fuzzy linguistic term sets. Wei et al. [11] developed comparison methods for hesitant fuzzy linguistic term sets and proposed a hesitant linguistic weighted averaging operator and a hesitant linguistic ordered weighted averaging operator to address multi-criteria decision-making problems under different situations, in which the importance weights of criteria or experts were known. Zhu and Xu [12] developed two optimization methods for improving the consistency of hesitant fuzzy linguistic preference relations for multi-criteria decision making. The drawback of the methods presented in [9] and [10] is their failure to consider the pessimistic and optimistic attitudes of the decision maker for multi-criteria linguistic decision making based on hesitant fuzzy linguistic term sets. Therefore, we must develop a new method that considers these factors.

Petri net (PN) theory and fuzzy logic exhibit a graphical and mathematical formulism to model and simulate

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biological systems. Fuzzy PN (FPN) [13] is a successful tool for describing and studying information systems. Incorporating fuzzy logic into FPN has been widely performed to deal with fuzzy knowledge representation and reasoning [13–16]. This procedure has also been proven to be a powerful representation method for the reasoning of a criteria-based system. Such an approach is appropriate in cases where a state of the modeled system corresponds to a marking of the associated FPN. The motivation for developing the FPN model is to fuse the benefits of fuzzy logic (i.e., effectively managing uncertain or corrupted inputs, natural linguistic structure, etc.) with those of FPN techniques.

The advantages of using FPNs in fuzzy criteria-based reasoning systems include the following [15, 17]. (1) The graphical representation of the FPN model can help visualize inference states and modify fuzzy criterion bases. (2) The analytic capability can express the dynamic behavior of fuzzy criteria-based reasoning. The evaluation of markings is used to simulate the dynamic behavior of a system. How conclusions are reached is explained through the movements of tokens in FPNs [17].

In this study, we present a new method for multi-criteria linguistic decision making based on hesitant fuzzy linguistic term sets and the aggregation of fuzzy sets. First, the proposed method aggregates the fuzzy sets in each hesitant fuzzy linguistic term set into a fuzzy set. Then, it performs the minimum and maximum operations among the obtained intervals for each alternative to determine the derived values. The minimum and maximum operations are used to denote the pessimistic and optimistic attitudes of the decision maker, respectively. Finally, for each alternative, the proposed approach uses the centroid method [18] to rank the priority of the obtained values and to derive the preference order of the alternatives for the decision maker with pessimistic and optimistic attitudes. The difference between the proposed method and Chen's method [5] is that the former is based on hesitant fuzzy linguistic term sets, whereas the latter is based on interval-valued fuzzy sets. Moreover, the proposed method is based on FPN to provide the model for reasoning rules. It offers a useful way for multi-criteria linguistic decision making based on hesitant fuzzy linguistic term sets.

III. METHODOLOGY

In this section, we briefly review the concepts of PN [21] and fuzzy sets [2, 20].

A. Petri net and Fuzzy Sets

1) Petri net

PN=(P,T,F,M0) if and only if

- P is a finite non-empty set of places.
- T is a finite non-empty set of transitions, and $P \cap T = \Phi$.
- $F \subseteq (T \times P) \cup (P \times T)$ is a set of arcs, i.e., flow relations.
- $M_0 : P \to \{0,1\}$ is the initial marking.

In the graphic representation, places are drawn as circles or ellipses, and transitions are illustrated as bars. Flow relations between nodes are represented as directed arcs, and the tokens of the markings are shown as dots inside the places. The following symbols are used to represent the pre- and post-set of a node. . is a set of all markings that is reachable from . Transition t is enabled if each $p \in {}^{\bullet}t$ contains one token at the present marking $M \in R(M_0)$. If t is enabled, then it can fire. Consequently, a new marking M' is generated from the current marking M , as represented by $M[t > M'$. When $p \in {}^{\bullet}t - t^{\bullet}$, $M'(p) = M(p) - 1$; when $p \in t^{\bullet} - {}^{\bullet}t$, $M'(p) = M(p) + 1$; otherwise $M'(p) = M(p)$.

2) Fuzzy sets

In 1965, Zadeh proposed theory of fuzzy sets [20]. Let X be the universe of discourse, X={ $\{x_1, x_2, \cdots, x_n\}$. Fuzzy set A in the universe of discourse X can be represented as follows:

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \cdots + \mu_A(x_n)/x_n ,$$

where μ_A is the membership function of fuzzy set A, $\mu_A(x_i)$ is the degree of membership of x_i that belongs to fuzzy set A, and $\mu_A(x_i) \in [0,1]$, i.e.,

$$\mu_A(x) = \begin{cases} 0, & x \le a, \\ x - a/b - a, & a \le x \le b, \\ 1, & b \le x \le c, \\ d - x/d - c, & c \le x \le d, \\ 0, & d \le x. \end{cases} \quad (1)$$

Chen and Chang [20] represented the trapezoidal fuzzy set A shown in Fig. 1 by a quadruple (a_1, a_2, a_3, a_4), where $A = (a_1, a_2, a_3, a_4)$. If $a_2 = a_3$, then trapezoidal fuzzy set A becomes a triangular fuzzy set, as shown in Fig. 2, where $A = (a_1, a_2, a_3, a_4) = (a_1, a_3, a_3, a_4)$.

B. Formal Definition of FPN

Chen et al. [14] presented a new knowledge representation approach by means of FPN. An FPN model enables a structural representation of knowledge and exhibits a systematic procedure to support the fuzzy reasoning process [15]. The tuple FPN=$(P,T,A,D,M_0,\Pr i,TH,\alpha)$ is an FPN where

- $P=\{p_1,p_2,\cdots,p_n\}$ is a finite non-empty set of places that correspond to the propositions of fuzzy determinations
- $T=\{t_1,t_2,\cdots,t_n\}$ is a finite non-empty set of transitions, $P\cap T=\varphi$, and $t_j\,(j=1,2,\cdots,m)$ corresponds to the execution of FPN criteria.
- $A=(a_{ij})_{n\times m}$ is an incidence matrix that defines the directed arcs between places and transitions, i.e., $a_{ij}=\begin{cases}1, & if\ p_i\in t_j^{\bullet} \\ -1, & if\ p_i\in {}^{\bullet}t_j\end{cases}$.
- $M_0:P\rightarrow\{0,1\}$ is the initial marking.
- $D=\{d_1,d_2,\cdots,d_n\}$is a finite set of propositions of FPN criteria; $P\cap T\cap D=\phi,|P|=|D|$. $d_i\,(i=1,2,\cdots,n)$ corresponds to the proposition that interprets fuzzy linguistic variables, such as vl, med, h, and ah as, in our model.
- $\Pr i=\{r_1,r_2,\cdots,r_m\}$ is a finite set of certainty factors of FPN criteria, and $r_j\,(j=1,2,\cdots,m)$ is the priority of $R_j\,(j=1,2,\cdots,m)$, which indicates the reliability of criteria R_j and $r_j\in[0,1]$.
- $TH:T\rightarrow[0,1]$is a function that assigns a threshold value $TH(t_j)=k_j\in[0,1](j=1,2,\cdots,m)$ to each transition.
- $\alpha:P\rightarrow[0,1]$ is a function that assigns a token value between zero and one to each place with only one token.

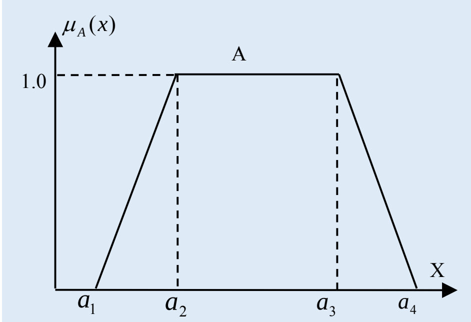

Figure 1. Trapezoidal fuzzy set

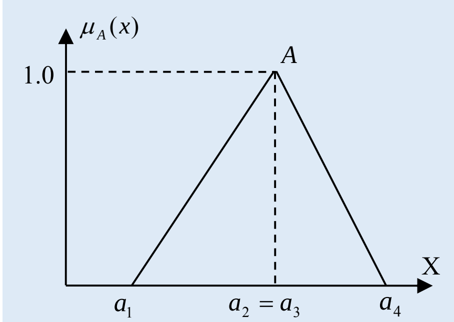

Figure 2. –Figure 2. Triangular fuzzy

In FPN models, a token value in place $p_i\in P$ is denoted by $\alpha(p_i)=y_i,\,y_i\in[0,1]$. This condition indicates that the truth degree of proposition d_i is y_i. Transition t_j is enabled if the input places satisfy: $\forall p_i\in{}^{\bullet}t_j,\,y_i>0$. If a transition is enabled, then it can be fired when the values of the tokens in all the input places of the transition are greater than its given threshold; otherwise, it can always be fired. If transition t_j is fired, then the tokens are removed from the input places and a token is deposited to each output places of t_j.

C. *FPN models of the hesitant decision-making model*

Three kinds of criteria are used in our FPN model: simple fuzzy, composite conjunctive, and composite disjunctive criteria. The simple fuzzy criterion is restricted by only one condition, that is,

$$\forall t\in T,{}^{\bullet}t=\{p_i\},t^{\bullet}=\{p_j\},\Pr i(t)=r.$$

If $M(p_i)=1$, then t is enabled, and the value of the deposited token in p_k, $\alpha(p_k)=\alpha(p_i)\times r$.

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Given the input places of transitions, different propositions for composite criteria can be integrated into hesitant decision making by pessimistic or optimistic attitudes, i.e.

$$\forall t \in T, {}^{\bullet}t = \{p_1, p_2, \cdots, p_j\}, t^{\bullet} = \{p_j\}, \Pr i(t) = r.$$

The general criteria are as follows.

- A composite conjunctive criterion,which expresses the pessimistic attitude toward the criteria.

If $M(p_1) \wedge M(p_2) \wedge \cdots \wedge M(p_j) = 1$, then t is enabled. After transition t is fired,

$$\alpha(p_k) = \min\{\alpha(p_i), i = 1,2,\cdots, j\} \times r \tag{2}$$

- A composite disjunctive criterion, which expresses the optimistic attitude toward the criteria.

If $M(p_1) \vee M(p_2) \vee \cdots \vee M(p_j) = 1$, then t is enabled. After transition t is fired,

$$\alpha(p_k) = \max\{\alpha(p_i), i = 1,2,\cdots, j\} \times r \tag{3}$$

t is enabled.

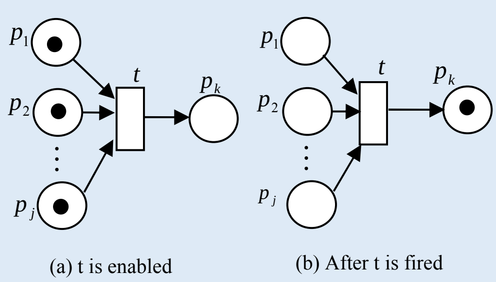

Figure 3. Composite conjunctive criterion.

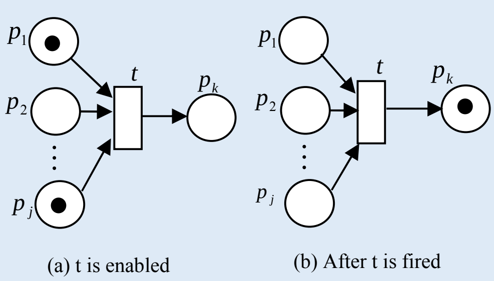

Figure 4. Composite disjunctive criterion.

D. New method for hesitant group decision making

As illustrated in Fig. 5, the model is used to describe the inference reasoning system. The figure presents the contents of the reasoning rules applied in the model. The properties of the proposition set of places, as well as the logic input and output transitions, are described as follows.

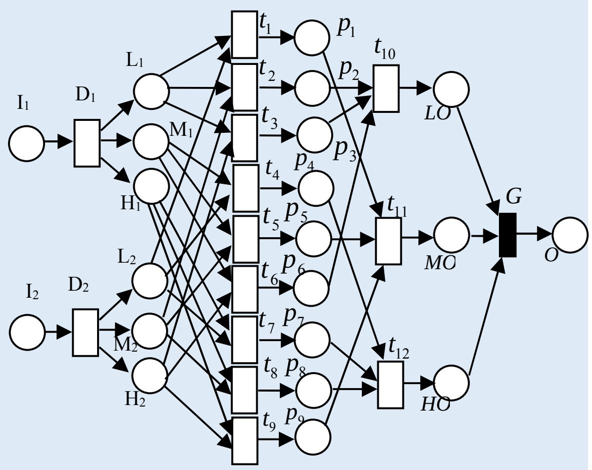

Figure 5. Fuzzy Petri Net Model of Rule-based Reasoning

I_1 and I_2 represent “input data.” Each place has a token value to provide the information on the normalized value of the alternatives.

The first transitions, D_1 and D_2, represent the distribution function transition of input information. By using the membership function in Eq. (1), we derive the membership degree of the input value that belongs to the linguistic evaluation, such as vl, l, m, h, and ah as, in our model.

L_1, M_1, H_1, L_2, M_2, and H_2 are places that hold the value of the deposited token for comparison.

Transitions t_1, t_2,…, t_9 are composite conjunctive criteria, which express the pessimistic attitude toward a decision. Each of these transitions provides the “minimum” operation in Eq. (2) to the values of two antecedent places.

Places p_1, p_2,…, p_9 hold the results of the “minimum” operation.

Transitions t_{10}, t_{11}, t_{12} are the composite disjunctive criteria, which express the optimistic attitude toward the decision. Each of these transitions provides the “maximum” operation in Eq. (3) to the values of two antecedent places.

Places LO, MO, and HO hold the results of the “maximum” operation.

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Transition G provides the likelihood-based comparison to obtain the final value.

Place O holds the final confidence value of the alternatives.

After the construction of the FPN model, the beliefs (truth degree) of the propositions/predicates mapped at the appropriate places are initialized. The implementation of the FPN is realized by firing transitions (rules). Transition t fires immediately as soon as it is enabled.

E. Algorithm for predicting confidence value

Algorithm 1. The reasoning algorithm for predicting confidence value

STEP 1: The FPN model of rules is modeled in the knowledge base.

STEP 2: The required data (I1 and I2) for the model are entered.

STEP 3: The membership degree of the proposition of variables is calculated.

STEP 4: Firing strength is calculated through the composition AND operator (MIN).

STEP 5: The maximum firing strength is calculated through the composition OR operator (MAX).

STEP 6: A conclusion of the output is calculated through the "center of gravity" formula [19].

STEP 7: The value of the model that corresponds to the output of the transition is obtained.

IV. RESULT ANALYSIS AND DISCUSSION

The fuzzy membership function of the input is described in Fig. 6. The different variables are listed in Table 1, and the membership functions of the candidates are listed in the third column.

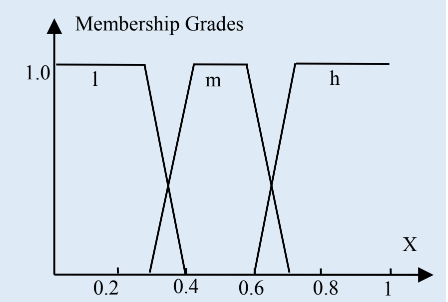

Figure 6. Three linguistic terms

Step 1: The FPN model of the rules is modeled as shown in Fig. 7.

Step 2: The crisp data of different variables x1, x2, x3 are entered; each datum has two candidates.

Step 3: The membership degree of the corresponding data is calculated as shown in Eq. (1).

Step 4: The MIN operator is used to obtain the input of the MAX operator. For x1, the rule involves reasoning by two candidates as follows:

p_1: Min (0.5, 0) =0,
p_2: Min (0.5, 0) =0,
p_3: Min (0.5, 1) =0.5,
p_4: Min (0.5, 0) =0,
p_5: Min (0.5, 0) =0,
p_6: Min (0.5, 1) =0.5,
p_7: Min (0, 0) =0,
p_8: Min (0, 0) =0,
p_9: Min (0, 1) =0.

Step 5: The MAX operator is used to obtain the input of the "center of gravity" operator, as follows:

LO: Max (0, 0, 0.5, 0.5) =0.5,
MO: Max (0, 0) =0,
HO: Max (0, 0, 0) =0.

TABLE I. MEMBERSHIP FUNCTION DEGREE OF THE INPUT VALUE

Alternative	Candidate	Membership function value		
		l	m	h
x_1	0.35	0.5	0.5	0
	0.72	0	0	1
x_2	0.52	0	1	0
	0.37	0.3	0.7	0
x_3	0.28	1	0	0
	0.46	0	1	0

By performing the same operations on the other variables, i.e., x2 and x3, we derive the results: x2= (0, 0, 0.7), x3= (1, 0, 0).

Step 6: According to the centroid technique [19], we obtain the output of the calculation of the MAX operation, as follows:

For $x_1 : \dfrac{\int_{0.35}^{0.72} 0.5xdx}{\int_{0.35}^{0.72} dx} = 0{,}2675$,

For $x_2 : \dfrac{\int_{0.37}^{0.52} 0.7xdx}{\int_{0.37}^{0.52} dx} = 0{,}3115$,

For $x_3 : \dfrac{\int_{0.28}^{0.46} xdx}{\int_{0.28}^{0.46} dx} = 0{,}37$.

Step 7: We can obtain the final decision from the calculation results.

From the results of the example, we determine that the confidence value of the three variables is $x_1 < x_2 < x_3$; thus, we choose x_3 as the final decision.

V. CONCLUSION

In this study, we presented a new method for multi-criteria linguistic decision making based on the aggregation of linguistic terms represented by fuzzy sets in hesitant fuzzy linguistic term sets. The reasoning rules were modeled by the FPN model, in which the minimum and maximum operations denoted the pessimistic and optimistic attitudes of the decision maker, respectively. The algorithm presented in this study was validated through an example. From this example, we could obtain the final decision by calculating the confidence values of the three variables. But this reasoning model could only test in sevaral decision making, not on large amount of decision making. In the future work, we plan to use some software tools to give the method of processing large amount of decision making.

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