An Improved Strategy for the Variable Step-size LMS Algorithm

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***Abstract* — Self-adaptive filtering algorithm is one of the kernel problems that must be addressed to promote the development of Finite Impulse Response (FIR) digital filter. In order to obtain a superior self-adaptive filtering algorithm, an improved strategy is proposed by way of studying the traditional Variable Step-Size Least Mean Square (VSSLMS) algorithm and the function controlled VSSLMS (FCVSSLMS). First, the genetic factor of step-size that is obtained through adjusting the range of the error is proposed to overcome the deficiencies of the traditional experimental method. Second, the concept that past errors influence step-size is proposed to increase the accuracy of step-size. The two ideas form the improved strategy. Through simulating and implementing a self-adaptive FIR digital filter, it is found that the VSSLMS algorithm using the improved strategy has advantages over that algorithm not using it in terms of convergence rate and steady-state error. In this study, the way to determine the value of genetic factor is more flexible than the traditional one and the error can adjust the step-size more exactly. The improved strategy can be also applied to support other self-adaptive algorithm.**

Keywords - step-size; convergence rate; steady-state error

I. INTRODUCTION

Finite impulse response (FIR) digital filter can guarantee strict linear phase, which is important for signals of voice, images and radar, while designing random amplitude-frequency characteristics. Due to fixed filter coefficient of traditional FIR digital filter, parameter of it cannot change as system parameter or external environment changes, making it hard to meet requirements of system by filter output. As a result, self-adaptive FIR digital filter arises [1]. Several new and efficient methods have been proposed to design self-adaptive FIR digital filter [2-4]. Notably, adaptive filtering algorithm is one of the important parts of adaptive filter design. Least mean square (LMS) algorithm, because of its simple structure, small computational complexity and real-time performance, has been widely used to design the FIR digital filter [5-6]. The step-size of LMS algorithm, which controls both the convergent rate and the steady-state error, is a contradictory value in the algorithm's iterative process [7]. Therefore, for getting the LMS algorithm with fast convergent rate and low steady-state error, an efficient improved strategy based on the variable step-size least mean square(VSSLMS) is necessary to study deeply.

The remainder of this paper is organized as follows. Section 2 describes the recent work status quo about LMS and VSSLMS at home and abroad. Section 3 describes the traditional LMS, VSSLMS and FCVSSLMS algorithm firstly, and then proposed the improving strategy after analyzing advantage and disadvantage of those traditional algorithms. Section 4 presents a real experiment to evaluate the performance of the improving strategy. Finally, the conclusions are summarized in Section 5.

II. STATE OF THE ART

In recent years, self-adaptive FIR digital filter has turned to a research focus. One of the most significant research achievements is application of LMS to design of self-adaptive FIR digital filter. LMS algorithm was proposed by Widrow and Hoff in 1960. Because of its simple structure, small computational complexity and real-time performance, it has been widely used in the adaptive beam forming, adaptive noise, noise elimination and the adaptive line enhancement, etc. However, steady-state error and convergence rate of LMS algorithm is paradoxical parameter. Many researchers have made a lot of work on the basis of the traditional algorithm. Some of them put forward some method to speed up the convergence rate and reduced the static error. The one of main solutions is to design LMS algorithm with variable step. A VSSLMS algorithm was proposed by Kwong. VSSLMS has a big step size at the beginning, for a maximum convergence speed, and a much smaller step size after the convergence, for a minimum residual error[8]. Mayyas proposed a variable step-size selective partial update LMS (VSSPULMS) algorithm. Its step-size is controlled by only one parameter, and do not require any a priori information about the statistics of the system environment. The performance of VSSPULMS is superior to VSSLMS[9]. In addition, some function controlled VSSLMS (FCVSSLMS) algorithms have been proposed in recent years [10]. For instance, Some nonlinear function relation between step-size and the error are built based on sigmoid, hyperbolic tangent, arc-tangent, and logarithmic function. However, step-size is usually only related to the error, and has nothing to do with the time in front of the past step-size in FCVSSLMS. Therefore, if interference signal suddenly appears, there will be some errors at the output side.

In this paper, Based on these FCVSSLMS algorithm described in document 11 to 16 and traditional VSSLMS algorithm, an improvement strategy is proposed to solve the contradiction between convergence rate and steady-state error in LMS algorithm. Meanwhile, A high-speed self-adaptive FIR digital filter based on the LMS algorithm with improvement strategy is successfully achieved in a FPGA chip. It can realize speech de-noising successfully.

III. METHODOLOGY

A. *Self-adaptive FIR Digital Filter*

If x(n) , X(n), ω(n) and y(n) are used to express the input signal at the nth moment, the input vector of filter at the nth moment, the weight vector of filter tap, and the output of filter, then they can be expressed as follows respectively:

$$X(n) = [x(n), x(n-1), \cdots, x(n-M+1)] \quad (1)$$

$$\omega(n) = [\omega_n(0), \omega_n(1), \cdots, \omega_n(M-1)] \quad (2)$$

$$y(n) = \hat{\omega}^T(n) \cdot X(n) \quad (3)$$

where: "*M*" is the order of filter, "^" denotes the estimated current value of weight coefficient and "*T*" denotes transposition.

FIR digital filter can be expressed by Formula (3). For the filter whose *ω* (*n*) is a fixed value, its performance is invariable. For another filter whose *ω* (*n*) can vary as the variation of system parameter to adapt to the performance requirement of system, its performance is changed with the adjustment of *ω* (*n*) and is called self-adaptive FIR digital filter. In theory, self-adaptive digital filter is time-varying and non-linear, however if relation of the output and the input of filter is linear, the filter can also be deemed as linear. In this paper, the widely used direct self-adaptive FIR digital filter is taken as an example for study.

The most commonly used self-adaptive filtering algorithm is LMS algorithm for FIR digital filter. The structure of self-adaptive FIR digital filter based on the LMS algorithm is shown in figure 1, LMS algorithm provides self-adaptive parameter for FIR digital filter, at the same time, the error between the output and desired signal adjust the value of self-adaptive parameter.

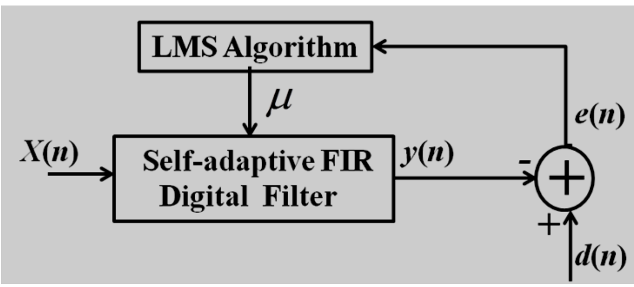

Figure 1 The structure of self-adaptive FIR digital filter based on traditional LMS algorithm.

B. *VSSLMS and FCVSSLMS algorithm*

(1) Traditional VSSLMS algorithm

The LMS algorithm is linear self-adaptive filtering algorithm based on Wiener filtering and least mean square and is commonly used for design of self-adaptive FIR digital filter. Taking *d* (*n*) as desired signal of self-adaptive FIR digital filter and *e* (*n*) as error signal, *e* (*n*) can be expressed as follows:

$$e(n) = d(n) - y(n) \quad (4)$$

Updated weight of filter tap can be expressed as follows:

$$\hat{\omega}^T(n+1) = \hat{\omega}^T(n) + \mu \cdot x(n) \cdot e(n) \quad (5)$$

In Formula (5), μ represents constriction factor in LMS algorithm, also known as step-size parameter. According to Formula (5), final form of LMS algorithm only includes multiplication and addition operation, so the algorithm is easily to be realized using hardware. When μ is invariable, it is called fixed step-size LMS（FSSLMS） algorithm and the value of μ is usually obtained through multiple experiment. In the operation processing of the FSSLMS algorithm, the step-size cannot be adjusted according to actual situations. If the step-size is too small, convergence rate will be lower. And if it is too large, there will lead to output offset. When μ is variable, it is called VSSLMS algorithm. First, μ *is* large at the initial stage of VSSLMS algorithm operation, so convergence rate is raised. Then after finding the better value, μ is decreased gradually, so convergence rate is slowed down to prevent imbalanced output. Updated μ value can be expressed as follows:

$$\mu(n+1) = \alpha\mu(n) + \beta e^2(n) \quad (6)$$

Formula (6) shows that $\mu(n+1)$ is related to not only the value of μ and $e(n)$ before update but also the value of α and β. α is the genetic factor of step-size, and β is the genetic factor of error. rate of change of $\mu(n+1)$ is related to α^n. If α is too large, then μ (*n*+1) will not be able to converge. While α is too small, μ (*n*+1) will change too rapidly to lead to imbalanced output during the depth convergence. Therefore, the range of α is generally taken as $0<\alpha<1$ and $\alpha\to1$. As β determines the degree of μ (*n*+1) being affected by *e* (*n*), the value of β is generally small and taken as $0<\beta<1$ and $\beta\to0$ to ensure $\beta e^2(n)\to0$ during depth convergence. In general, the value of α and β are obtained through multiple experiments. At the initial stage of LMS algorithm operation, as *e* (*n*) is relatively large, so convergence rate of *ω* (*n*+1) is rapid. With *e*(*n*) becoming smaller, the algorithm reaches depth convergence gradually. At last, the optimal solution of weight of filter tap is obtained, and then the algorithm stops.

The traditional VSSLMS algorithm has been widely applied in such fields as image processing and audio processing, but there are still weaknesses, such as:

1) μ $(n+1)$ is related to error signal $e(n)$ of the nth moment and has no relation with the previous error, the characteristic causes poor anti-jamming capability of the traditional VSSLMS algorithm.

2) α and β can be obtained only after multiple experiments. After the system parameter or the environment changes, the new value of α and β must be obtained through multiple experiments, this leads to poor stability of the traditional VSSLMS algorithm.

(2) FCVSSLMS algorithm

FCVSSLMS algorithm is superior to VSSLMS in the performance of the steady-state error and convergence rate. The functions of Sigmoid, hyperbolic tangent, arc-tangent, and logarithmic are always used to control the value of μ. To the four functions, μ is expressed sequentially as follows: formula (7), (8), (9) and (10).

$$\mu(n) = c \cdot \left(\frac{1}{1+\exp[-d \cdot |e(n)|^r]} + 0.5\right) \tag{7}$$

$$\mu(n) = c \cdot \lg[d \cdot |e(n)|^r] \tag{8}$$

$$\mu(n) = c \cdot [\arctan(d \cdot |e(n)|^r)] \tag{9}$$

$$\mu(n) = c \cdot \left\{1 - \frac{3}{2+\exp[d \cdot |e(n)|^r]}\right\} \tag{10}$$

In the four formulas, there are three parameters: c, d and r. They adjust $\mu(n)$ together: c is used to control the change of the overall shape, d is used to control the change speed of the bottom, and r is to control range. The three parameters are usually obtained through many experiments, so FCVSSLMS algorithm is not very convenient in practical application.

C. *Improved Strategy*

(1) Improved VSSLMS algorithm

Considering the above-mentioned weaknesses of the traditional VSSLMS, an improved VSSLMS algorithm is proposed in this paper to improve anti-jamming capability and stability.

Aiming at the first weakness of traditional VSSLMS filtering algorithm, Formula (6) is turned to:

$$\mu(n+1) = \alpha\mu(n) + \beta \left|\prod_{i=n-P}^{n} e(i)\right| \tag{11}$$

Formula (11) shows that μ $(n+1)$ is related to error signal of the previous $(P+1)$ moment and P is a parameter designed by the designer. The larger the P value is, the stronger the relevance between $\mu(n+1)$ and error signal of each previous moment is and the stronger the anti-jamming capability is. However, the larger the P value is, more multiplying units are needed to realize the filter. Thus, the designer can obtain appropriate P value by weighing the performance and the cost of Hardware.

Aiming at the second weakness of traditional VSSLMS filtering algorithm, Formula (11) is turned to:

$$\mu(n+1) = \alpha\mu(n) + (1-\alpha)^Q \left|\prod_{i=n-P}^{n} e(i)\right| \tag{12}$$

$$\alpha = \begin{cases} 1, & e(n) < z_0 \\ 0.99, & z_0 < e(n) < z_1 \\ 0.98, & z_1 < e(n) < z_2 \\ 0.97, & z_2 < e(n) < z_3 \end{cases} . \tag{13}$$

In Formula (12), Q is a parameter designed by the designer. the faster the convergence rate is, the easier imbalance occurs. Multiple experiments proves that 3-6 is the reasonable value range, the range not only makes proper convergence rate, but also ensures items in Formula (8) relevant to error signal to approach 0 during depth convergence.

Formula (13) describes the value of α. Numerous experiments conducted by researchers proves that value of α should approach to 1. The value of α is obtained by subsection function based on the situation of error signal. When the error is relatively large, make α equal to 0.97 and $\mu(n+1)$ is greatly affected by error signal at this moment. as gradual convergence of the algorithm, the error becomes smaller ,the value of α approaches to 1, and μ(n+1) remains stable. Wherein, z0, z1, z2 and z3 are preset according to system performance.

(2) Improved FCVSSLMS algorithm

In the formula (7), (8), (9) and (10), the parameters value of c, d and r depends on the experiment results of many times. In addition, if the external environment change, the parameters value will be no longer appropriate. There will bring a lot of inconvenience in practical application. Therefore, some improvement was proposed as follows:

1) Based on design experience, c is assigned for 500, r is assigned for 2, and e (n) is changed to $\prod_{i=n-P}^{n} e(i)$ in the formula (7), (8), (9) and (10). P is a parameter designed by the designer.

2) Making (7), (8), (9) and (10) transform as follows:

$$\mu(n+1) = \alpha\mu(n) + (1-\alpha)^Q \cdot \left(\frac{1}{1+\exp[-d \cdot \left|\prod_{i=n-P}^{n} e(i)\right|^r]} + 0.5\right) \tag{14}$$

$$\mu(n+1) = \alpha\mu(n) + (1-\alpha)^Q \lg[d \cdot \left|\prod_{i=n-P}^{n} e(i)\right|^r] \tag{15}$$

$$\mu(n+1) = \alpha\mu(n) + (1-\alpha)^Q [\arctan(d \cdot \left|\prod_{i=n-P}^{n} e(i)\right|^r)] \tag{16}$$

$$\mu(n+1) = \alpha\mu(n) + (1-\alpha)^Q \{1 - \frac{3}{2 + \exp[d \cdot \left|\prod_{i=n-P}^{n} e(i)\right|^r]}\} \tag{17}$$

In the formula (14), (15), (16) and (17), the value of *α, P, Q* is identical with formula (12) and (13). The improved FVVSSLMS has the following characteristics:

1) The value of P and Q are obtained depending on designer’s experience and don't have be the optimal value.

2) The value of α are changed adaptively in the processing of algorithm operation.

3) The value of μ(n) is not only related with e(n) but also with μ(n+1), So the improved algorithm can overcome the interference signal effectively.

(3) Improved strategy

The improved strategy can be obtained as follows:

$$\mu(n+1) = \alpha\mu(n) + (1-\alpha)^Q f(\left|\prod_{i=n-P}^{n} e(i)\right|^r) \tag{18}$$

In formula (18), $f(\left|\prod_{i=n-P}^{n} e(i)\right|^r)$ can be sigmoid, hyperbolic tangent, arc-tangent, and logarithmic function, etc. The value α is got by formula (13), the value of *P* and *Q* are obtained depending on designer’s experience.

IV. Result Analysis And Discussion

A. Simulation Analysis of the Improved VSSLMS Algorithm

A 20 order self-adaptive FIR digital filter was realized respectively through traditional VSSLMS algorithm and the improved algorithm. Figure 2 shows the comparative result of the convergence rate and the weight of the filter by using two algorithms. Traditional algorithm converged after 40 times iteration while improved algorithm converged after 30 times iteration. Moreover, figure 2 showed us the mighty shock and overshoot of the weight of the filter in the processing of the traditional algorithm operation, but in the improved algorithm operation, the weight is changed to a optimal value stably.

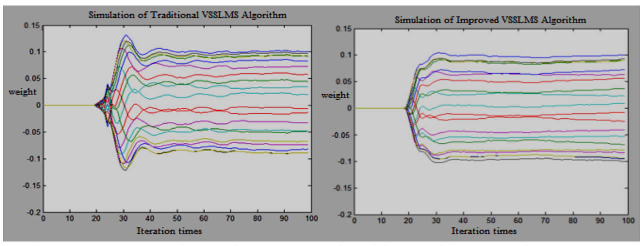

Figure 2. Comparison of VSSLMS algorithm and improved VSSLMS algorithm.

In Figure 3, anti-jamming capability of these two algorithms are compared, the output interference signal of improved algorithm is lower obviously than the traditional one. From these two figures, it can be seen that the improved VSSLMS algorithm is obviously superior to the traditional one in terms of convergence rate and anti-jamming capability.

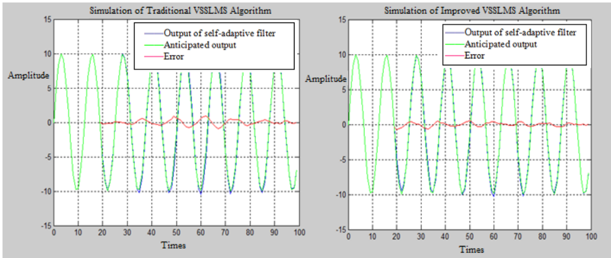

Figure 3. Comparison of traditional VSSLMS algorithm and improved VSSLMS algorithm in terms of anti-jamming capability.

Therefore, the VSSLMS algorithm adopting the improved strategy is superior to the traditional VSSLMS algorithm.

B. Simulation analysis of the Improved FCVSSLMS Algorithms

In order to compare the steady-state error and convergence rate of the four FCVSSLMS algorithm and their improved algorithms, A 20 order self-adaptive FIR digital filter was realized respectively using the four FCVSSLMS algorithm and their improved algorithms. Table 1 describes the results of the comparison: After improving the four FCVSSLMS algorithms, their steady-state error reduce, and their convergence rate increase.

TABLE I Performance Comparision Between The Four Fcvsslms Algorithm And Their Improved Algorithmsn

Algorithm	**Iterations Times**	**Minimum of |e(n)|**
Sigmoid controlled VSSLMS	58	0.035
Improved Sigmoid controlled VSSLMS	40	0.015
Hyperbolic tangent controlled VSSLMS	47	0.024
Improved hyperbolic tangent controlled VSSLMS	37	0.011
Arc-tangent controlled VSSLMS	48	0.025
Improved arc-tangent controlled VSSLMS	37	0.012
Logarithmic controlled VSSLMS	45	0.022
Improved logarithmic controlled VSSLMS	35	0.009

In order to more clearly compare, after simulation, Figure 4 was obtained. The average performance of the four improved FCVSSLMS algorithms is superior to the four original FCVSSLMS algorithms.

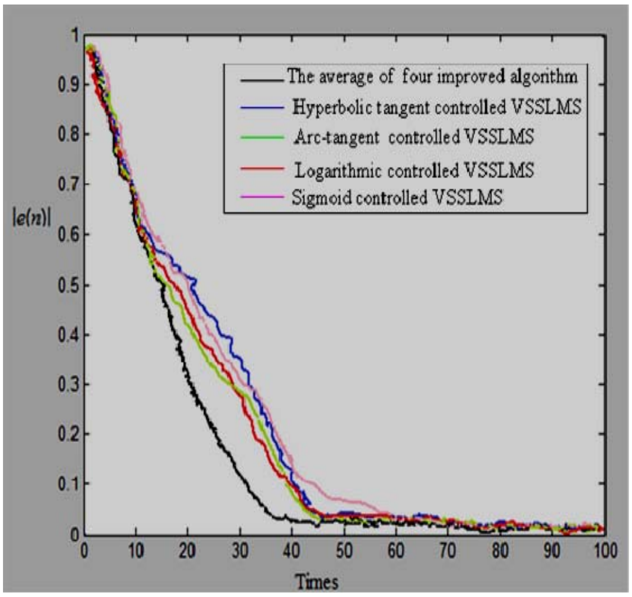

Figure 4. The comparison between the performance of the four FCVSSLMS algorithm and the average performance of their improved algorithm.

Therefore, the four common FCVSSLMS algorithms adopting the improved strategy is superior to these algorithms not adopting the improved strategy.

C. FPGA Implementation of the FIR Digital Filter Based Improved FCVSSLMS Algorithm

It can be found from Formula (3) that the realization of FIR digital filter using FPGA mainly involves multiplication and accumulation operation. If requirement for filtering speed is not very demanding, the design scheme of serial accumulation can be adopted. while the requirement for filtering speed is high, parallel accumulation scheme must be adopted so as to obtain faster speed by more hardware resources. Distributed algorithm is taken as the example in this paper to realize high-speed FIR filter for design, through which FIR digital filter can be realized by taking advantage of numerous look-up tables in FPGA. Details of the design approach are introduced in multiple references.

When distributed algorithm is adapted to design FIR digital filter, make B as digit of binary system of the input signal and then the b^{th} of input vector of filter can be expressed as:

$$X_b(n) = [x_b(n), x_b(n-1), \cdots, x_b(n-M+1)], \qquad 0 < b < B-1 \tag{19}$$

Substitute Formula (10) into Formula (3), and then:

$$y(n) = -2^{B-1} \cdot \hat{\omega}^T(n) \cdot X_{B-1}(n) + \sum_{b=0}^{B-2} 2^b \cdot \hat{\omega}^T(n) \cdot X_b(n) \tag{20}$$

According to Formula (3), the input vector is input into filter in a serial way based on digit, while after Formula (3) is converted to Formula (18), the input vector is input into the filter in a parallel way, and then Formula (20) is the expression for distributed algorithm to realize FIR self-adaptive filter.

According to the above analysis, FPGA is adopted to design the high-speed self-adaptive FIR digital filter, hardware structure of which is as shown in Figure 5. Firstly, design series-to-parallel module to realize series-to-parallel convergence of the input signal, and then input signal in a parallel way and make parallel operation in filtering algorithm so as to realize high-speed filtering. The signal from series-to-parallel module is put into ROM based on digit. The ROM is constituted by look-up tables in FPGA chip. Input corresponding $X_b(n)$ into B ROMs and then the corresponding $X_b(n)\hat{\omega}^T(n)$ can be obtained by the look-up tables. Conduct accumulation operation for all $X_b(n)\hat{\omega}^T(n)$ by pop-line adder and then $y(n)$ can be acquired, at the same time $y(n)$ is put into the improved LMS algorithm module. The improved LMS algorithm module is constituted by multiplication and addition operation, which can be realized by hardware multiplier and summator respectively. Input signals in LMS algorithm module includes desired signal $d(n)$, input signal $x(n)$ and FIR output signal $y(n)$, the output signal is the weight vector of filter.

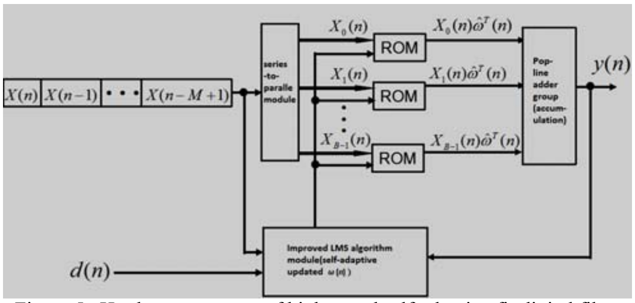

Figure 5. Hardware structure of high-speed self-adaptive fir digital filter

FIR low pass filter which can eliminate high-frequency noise in speech signal is taken as example to realize one high-speed self-adaptive FIR digital filter.

In order to ensure successful hardware design, MATLAB software is compiled for FIR low pass filter based on improved logarithmic controlled VSSLMS algorithm. Figure 6 is the frequency spectrum of audio and noise before filtering, Figure 7 is the audio frequency spectrum after filtering , in which high-frequency noises are eliminated.

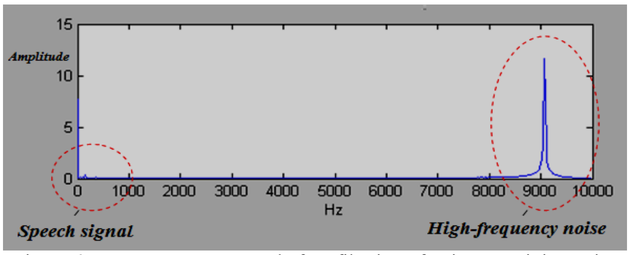

Figure 6. Frequency spectrum before filtering of voice containing noise

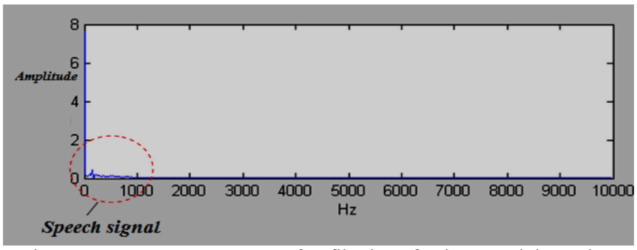

Figure 7. Frequency spectrum after filtering of voice containing noise.

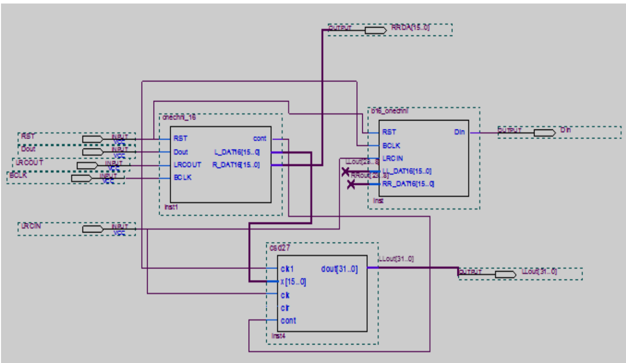

Figure 8 FPGA Implementation of Improved Logarithmic Controlled VSSLMS Algorithm.

After it is verified by matlab verification algorithm, high-speed self-adaptive FIR digital filter based on improved logarithmic controlled VSSLMS filtering algorithm is realized through FPGA chip (model: EPEC6Q240C8) and audio encoding and decoding chip TLV320AIC23B. The circuit diagram is as shown in Figure 8.

Experiments have proved that high-frequency noises were eliminated successfully via the circuit, and the response time and filtering effect was increased corresponding to traditional VSSLMS filtering algorithm.

V. CONCLUSION

This paper proposed an improving strategy for VSSLMS algorithm. The strategy involves two main ideas. In the first part, closely related to the genetic factor of step-size is the range of the error. This thought makes the method to get the genetic factor more flexible. In the second part, the step-size is connected not only the present error but also the past errors. This idea makes the value of the step-size more accurate. The experiment result shows that the strategy can get more precise weight for the FIR filter. Also, through simulating and implementing a self-adaptive FIR digital filter, it is found that the VSSLMS algorithm using the improvement strategy has advantages over that algorithm not using it in terms of convergence rate and anti-jamming capability. The results of this study have significant potential to support many self-adaptive algorithms. Further research based on this improving strategy is necessary, especially for the relationship of the step-size and the error.

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