

Multiple Target Localization Based on Alternate Iteration in Wireless Sensor Networks

Zhongyou Song, Jie Li
State Grid Chongqing Electric Power Co. Electric Power
Research Institute
Chongqing, China
e-mail: szyread@163.com

Yuanhong Zhong, Yao Zhou
College of Communication Engineering
Chongqing University
Chongqing, China

Abstract - This paper presents a study of multiple target localization in Wireless Sensor Networks (WSNs). A localization method which combines compressive sensing (CS) theory with the alternate iteration method is proposed to improve the localization accuracy. The proposed method firstly divides the location area into discrete grid and transforms multiple target localization problem to the CS problem. And then, traditional localization algorithm based on CS is presented to obtain rough position estimation according to the received signal strengths (RSSs). Finally, the alternate iteration method is presented to further refine the position estimation of target. During alternate iteration process, diamond search is employed to find the positions of targets accurately. Simulation results show that the proposed algorithm overcomes the limitation of traditional CS-based localization algorithms which can only recover targets in the center of grid, and obtains good multiple target localization performance on localization accuracy.

Keywords - wireless sensor networks; compressive sensing; multiple target localization; alternate iteration method

I. INTRODUCTION

Target localization based on WSNs is becoming one of the hot research topics [1-3]. Wireless sensor localization system, a special self-organizing network which fuses wireless communication technology, sensor technology and distributed computing, can be deployed in hazardous area and provides long-term and real-time monitoring and convenient processing of localization data. Localization technology based on WSNs has been widely used in a variety of different fields, such as robot navigation, geographic routing, public safety, environmental monitoring and vehicle tracking, etc. However, challenges come from the limitations of computing ability, communication capacity and energy of the network nodes because of a large number of low-cost sensors, which put a lot of pressure on WSNs.

CS theory [4-7] rising in recent years has brought new opportunities to the field of target localization in WSNs and proves an idea of localization target using the CS theory via spatial sparsity [8, 9]. In target localization algorithm based on CS, sensor nodes firstly only sample a small number of data and complete data compression simultaneously, which is superior to a large number of high-speed sampling based on Nyquist-Shannon sampling theorem. Hence, it reduces the requirement for sensor nodes to simple and cheap. Then, the target locations are recovered from sampling data in data fusion center, which is not limited by the energy and computing ability. By dividing the localization area into discrete grid, the limited number of targets localization problem can be effectively transformed into sparse target localization problem, that provides a certain proof that the CS theory can be applied to the WSNs target localization.

In recent years, numerous research work has been conducted by many scholars [10-16]. Zhang et.al [10] strictly

confirmed the rationality of applying CS theory into localization. They divided WSNs monitoring region to N discrete grid and modeled the positions of target grids to a N -dimensional vector with K -sparse. The greedy matching pursuit method (GMP) was employed to reconstruct sparse signal. Feng et.al [11-13] systematically studied indoor localization technology based on RSS and CS, and proposed a multiple target localization method based on preprocessing of RSS sampling data and l_1 norm optimization. When targets were not in the center of grid, the center of candidate localization result set was selected as the target location, that was a rough estimate method and brought estimation error. He et.al [14] used CS-based method to reduce the communication overhead from $M \times K$ to M (M is the number of sensor nodes, K is the number of target), and proposed iteration backtracking localization algorithm based on CS for the targets that were not in the center of grid, which improved the accuracy of localization. While the performance was significantly affected by noise, and when the target was close to the sensor nodes, the localization result was not accurate.

In this paper, a new method of multiple target localization based on RSS in WSNs is proposed, which combines the CS theory with the alternate iteration method. The main idea of the proposed method includes two stages: coarse localization stage and fine localization stage. In the coarse localization stage, CS theory is presented to collect a small amount of data as far as possible, and l_1 norm optimization was used to recover the rough position of target. In fine localization stage, alternate iteration method is used to further refine the location of the target. The proposed method has good performance when targets are not in the center of grid and solves the problem of the mutual

interference between the adjacent target, so that improves the localization accuracy.

The rest of this paper is organized as follows. In Section II, we describe the system model and signal attenuation model of multiple target localization. In Section III, traditional CS-based localization algorithm is introduced and the proposed method is described in details in Section IV. Simulation results are showed, analyzed and compared with other algorithms in Section V. Section VI concludes the paper.

II. PROBLEM MODELING

A. System Model

The system model of CS-based localization is shown in Figure 1. Firstly, K targets whose positions are unknown are located in a rectangle area, which is evenly divided into a discrete grid with N points. At the same time, there are M independent sensors whose positions are known in the whole area. The goal is to determine the positions of targets. So the localization problem is transformed into the problem of targets localization based on grid, that is, determine the index of grid point for these targets simultaneously, using the signal intensity received by sensors.

Firstly, targets send signals periodically which are independent and not synchronized. Sensors need to receive signals sent by all the targets in the location area periodically. The sensors collect and accumulate the signals intensity with periodic T . At the end of a cycle, sensors transmit the accumulated signal intensity to the data fusion center respectively, which uses CS-based localization algorithm to determine the specific positions of targets.

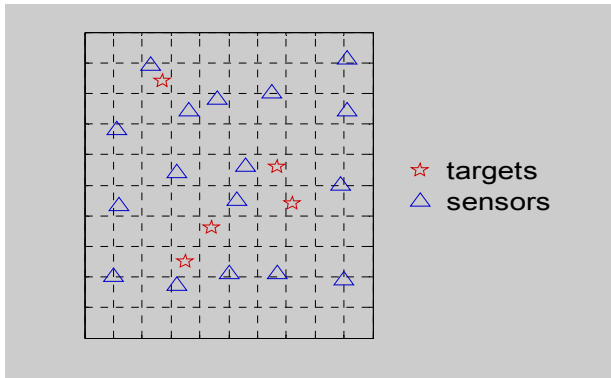


Figure 1. System Model

B. Signal Attenuation Model

The signal intensity inclines with the increase of distance during transmission, due to the influence of environment factors such as obstacle blocking, multipath propagation and so on. According to the difference between the intensity of the sent signal and the received signal, the transmission loss of the signal is calculated, which can be converted to the distance value by using the theoretical or empirical model. A large number of experimental results show that the

relationship between the average RSS and the signal transmission distance can be expressed as [17]:

$$\bar{P} = P_0 - 10n_p \lg(D/D_0) \quad (1)$$

Where \bar{P} is the average RSS (dBm), P_0 is the RSS in reference to the transmission distance D_0 , n_p is the coefficient for path loss (usually between 2 and 4), and D is the real signal transmission distance.

Thus, the RSS sent by target n and received by sensor m is:

$$P_{m,n} = P_0 - 10n_p \lg(D_{m,n}/D_0) \quad (2)$$

Where $P_{m,n}$ is the RSS received by sensor m and sent by grid target n , $D_{m,n}$ is Euclidean distance of sensor m and target n in the grid:

$$D_{m,n} = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2} \quad (3)$$

Where x_m and y_m are the coordinates of sensor m , x_n and y_n are the coordinates of target n .

Finally, the Gauss white noise is superimposed on the measurement result. This mathematical model assumes that the spatial distribution of the RSS is isotropic, that is, there is no difference in RSS in different directions.

III. COMPRESSIVE SENSING LOCALIZATION ALGORITHM

Assume that the locations of targets over the grid are denoted by:

$$X_{N \times 1} = [x_1, x_2, \dots, x_n, \dots, x_N] \quad (4)$$

Where n is the index of the grid point, and when there is a target in the n -th grid point, $x_n = 1$, else, $x_n = 0$. Y are the RSS measurements in an M -dimensional space through accumulating signals intensity collected by sensors. The measurement matrix Φ is conducted through signal intensity received by sensors and sent by targets in each grid. According to the CS theory, the relationship of the measurement matrix $\Phi_{M \times N}$, the measured value $Y_{M \times 1}$ and signal $X_{N \times 1}$ to be recovered can be described as:

$$Y_{M \times 1} = \Phi_{M \times N} X_{N \times 1} \quad (5)$$

Y can also be seen as linear projection of signal X under the measurement matrix Φ . The general process of CS-based localization algorithm can be described as: recover signal X from the measurement result Y .

The CS-based localization algorithm mainly consists of two stages [4,5]: compressive sampling stage and signal reconstruction stage. In compressive sampling stage, only a

small number of RSS measurements in an M -dimensional space are collected, that are the elements of \mathbf{Y} , and Φ is constructed. The reconstruction stage is to reconstruction \mathbf{X} according to the measurement matrix Φ and the measurement result \mathbf{Y} .

A. Compressive Sampling

Φ is an $M \times N$ matrix, and elements $\varphi_{m,n}$ is the RSS of sensor m and sent by target $n(1 \leq n \leq N)$:

$$\varphi_{m,n} = P_{m,n} \quad 1 \leq m \leq M, 1 \leq n \leq N \quad (6)$$

The procedure of compressive sampling can be described as:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} P_{1,1} & P_{1,2} & \dots & P_{1,N} \\ P_{2,1} & P_{2,2} & \dots & P_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ P_{M,1} & P_{M,2} & \dots & P_{M,N} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \quad (7)$$

Where the compressive measurements \mathbf{Y} are obtained by multiplying the measurement matrix Φ on signal \mathbf{X} , $y_m(1 \leq m \leq M)$ is the sum of the RSS, the number of sensors obeys $M = O(K \log(N/K))$ with $M = N$. \mathbf{X} is a N dimensional vector with K -sparsity, when a target is in the grid $n(1 \leq n \leq N)$, $x_n = 1$, otherwise, $x_n = 0$. So, the multiple target localization problem is transformed into the CS problem that reconstruct N dimension sparse vector based on M measurement results.

There are two ways to generate the measurement matrix Φ . One is generating a measurement matrix according to signal attenuation model. Another method is obtaining the measurement matrix according to the actual test result. In this paper, the measurement matrix Φ is generated based on the signal attenuation model (2).

B. Signal Reconstruction

Signal reconstruction stage is to reconstruct signal \mathbf{X} by the measurement result \mathbf{Y} and measurement matrix Φ . Φ is an $M \times N$ matrix, and the number of equations is much smaller than the unknown number (dimension of \mathbf{X} is much larger than the dimension of \mathbf{Y}), that is a problem of underdetermined linear equations to solve, and the equation has no definite solution. Since the signal \mathbf{X} is K -sparsity (K is much smaller than N), if Φ meets the Restrict Isometry Property(RIP) [18], signal \mathbf{X} can be well recovered by solving the l_1 norm minimum optimization problem through measurement value \mathbf{Y} :

$$\mathbf{X}_{est} = \arg \min \|\mathbf{X}\|_1, \quad s.t. \mathbf{Y} = \Phi \mathbf{X} \quad (8)$$

RIP condition is the sufficient but not necessary condition for the exact reconstruction of the signal. Feng et al [11-13] proposed a sparse target localization algorithm based on Orth. The proposed algorithm preprocess signals by Orth, so that the new measurement matrix satisfies the RIP property. The signal preprocessing procedure of the sparse target localization algorithm based on Orth is listed as follows:

$$\mathbf{Y}' = \mathbf{T}\mathbf{Y} \quad (9)$$

Where \mathbf{Y}' is the received matrix after pretreatment. Let \mathbf{T} be a preprocessing operation on \mathbf{Y} , $\mathbf{T} = \mathbf{Q}\mathbf{A}^+$, where $(\cdot)^+$ is pseudo inverse transform of matrix and $\mathbf{Q} = \text{orth}(\mathbf{A}^T)^T$, where \mathbf{A} is the measurement matrix Φ in this paper, $\text{orth}(\cdot)$ is standardized orthogonal operation of matrix, and $(\cdot)^T$ is the transpose operator of matrix. Matrix \mathbf{Q} is an orthogonal transformation matrix, which can satisfy the RIP property, so as to improve the performance of signal reconstruction, and ultimately improve the accuracy of multiple target localization.

The CS theory indicates that the signal \mathbf{X} can be well recovered given the compressive sampling \mathbf{Y}' , only via an the l_1 norm minimum optimization. In this paper, Basis Pursuit (BP) is employed for the recovery problem from compressive sampling \mathbf{Y}' .

IV. THE PROPOSED LOCALIZATION ALGORITHM

In traditional CS-based localization algorithm, reconstruction matrix is the same as the measurement matrix in signal reconstruction stage. It is noticed that measurement matrix is formed in the actual process of sensors' perception of the intensity of the target signal. If targets are in the center of grid, reconstruction matrix structured according to (2) is equal to actual measurement matrix; while if targets are not in the center of grid, reconstruction matrix structured according to (2) has deviation compared with the actual measurement matrix, which leads to localization error. Moreover, the recovered targets may locate in the vicinity of real grid because of noise and other environmental factors.

In order to solve the shortcomings above, we improve the algorithm to increase the accuracy of localization. The main idea of the proposed method is to combine coarse localization and fine localization. In coarse localization stage, reconstruction matrix structured according to (2) and traditional CS-based localization method is used to obtain the initial positions of targets, which are possible range targets may exist. In fine localization stage, the alternate iteration method is presented to constantly revise reconstruction matrix, so that reconstruction matrix is gradually close to the actual measurement matrix. The diamond search algorithm is presented during the alternate iteration process to search the actual positions of targets in grid.

Now, we introduce the core idea of alternate iteration method. Firstly, select a grid from recovered result of rough localization, and a diamond search method is conducted to find the next candidate. The point whose reconstruction error is minimum in diamond search is chosen as the next candidate position of a target. Meanwhile, a correction of reconstructed matrix based on the candidate is presented. Then, another grid from recovered result is selected to repeat the steps above, until all the recovered result are searched. By now, one iteration process is finished. Finally, steps described above are repeated to adjust the targets positions constantly, so that the reconstruction matrix is gradually close to the actual measurement matrix, until the number of iterations meets the default maximum number of iterations or search of small diamond search pattern (SDSP) of all recovered result are finished.

There are two kinds of search patterns in diamond search method, one is large diamond search pattern (LDSP) composed of nine detection points, the second is small diamond search pattern (SDSP) composed of five detection points, as shown in figure 2.

The main idea of diamond search method can be described as: if there is a potential target in a grid, search for the actual position of the target in the vicinity of the grid, and the position with minimum reconstruction error is selected as a candidate. In detail, the main idea can be described into following steps. Firstly, the search step size g is determined according to the localization accuracy requirement, which divides a grid into a finer grid. Next, the LDSP is carried out to find the next candidate. Reconstructed matrix, recovered signal and reconstruction error are calculated respectively in corresponding nine positions. One-time search process is complete. And then, point in 9 which has the minimum reconstruction error is selected as the next LDSP search center, and the described search process above is carry out to further narrow the scope of the target area. During the search process, the LDSP search is repeated, once minimal reconstruction error points appear in the position of the center point, which means the optimal matching point in the region is found. Finally, LDSP search is used for final localization in the optimal matching point. The point in 5 which has the minimum reconstruction error is selected as the final target position.

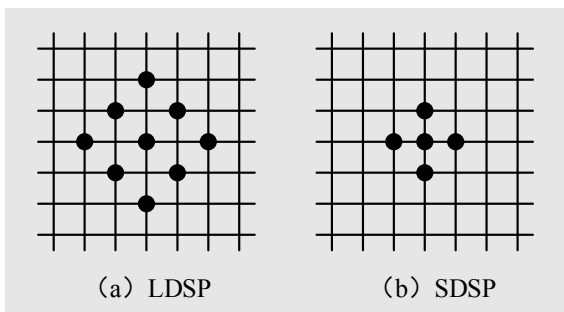


Figure 2. Diamond Search Mode

The minimum deviation $\sum_{n=1}^N E_n$ between the actual reconstruction result and ideal reconstruction result is constructed for the reconstruction error. Ideal reconstruction result is 0 or 1. E_n is the deviation between reconstruction result element x_n and ideal result, which follows [14]:

$$E_n = \begin{cases} |x_n|, & x_n \leq 0.5 \\ |x_n - 1|, & x_n > 0.5 \end{cases} \quad 1 \leq n \leq N \quad (10)$$

The value of the reconstructed signal is related to the probability that the target exists in the grid. The bigger the value, the more likely there is a target in the grid. So we only concentrate on the signal which is larger than the threshold value TH , to reduce the complexity of the algorithm.

Assuming that the initial position of a target $(X_{(n,1)}, Y_{(n,1)})$ in grid point n is the center of the grid (X_{Gn}, Y_{Gn}) , the grid width is G , and g is the search step size. In i -th ($i=1,2,3,L$) iteration, the position of the target is adjusted to:

$$(X_{(n,i+1)}, Y_{(n,i+1)}) = (X_{(n)}, Y_{(n)}) + g \times grid \quad (11)$$

Where grid represents the lattice point for diamond template. The lattice point of large diamond search template can be described as:

$$grid = \{(0, -2) (-1, -1) (1, -1) (-2, 0) (0, 0) (2, 0) (-1, 1) (1, 1) (0, 2)\} \quad (12)$$

The lattice point of small diamond search template can be described as:

$$grid = \{(0, -1) (-1, 0) (0, 0) (1, 0) (0, 1)\} \quad (13)$$

The flow chart of alternate iteration is shown in figure 3. The steps are written as follows:

Step1 Select a target from result of coarse localization, initialize target position: $(X_{(n,1)}, Y_{(n,1)}) = (X_{Gn}, Y_{Gn})$.

Step2 Calculate the reconstruction error $\sum_{n=1}^N E_n$. if $\sum_{n=1}^N E_n > ET$ (ET is the tolerance threshold of the error), the target is not recovered accurately, go to step 3, on the other hand, go to step 1.

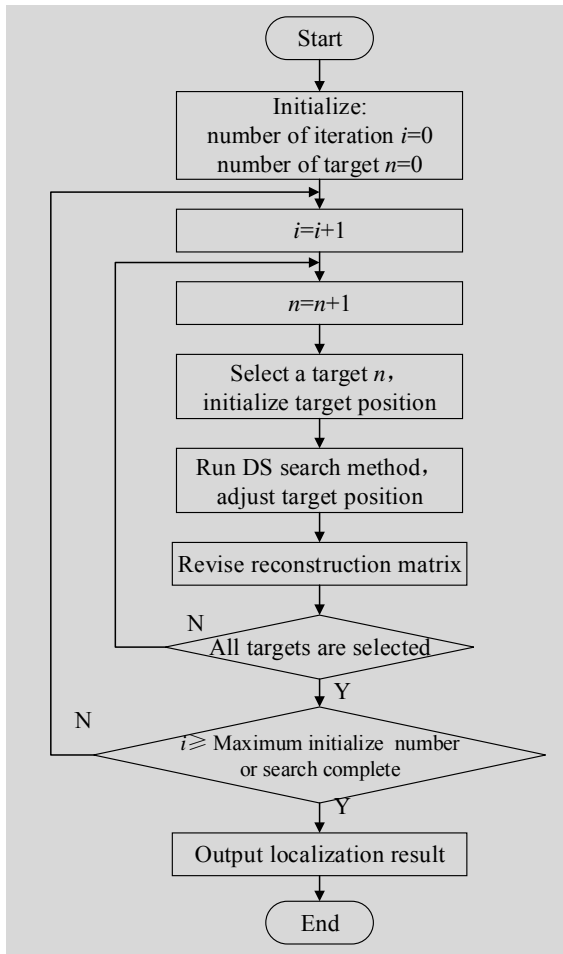


Figure 3. Flow Chart of Alternate Iteration Method

Step3 The target position is adjusted to the corresponding position according to (11) through the diamond search mode respectively, and the reconstruction matrix, the recovered signal X and the reconstruction error are calculated. The point corresponding to the minimum reconstruction error is taken to adjust the target position. It is noticed that if the point of minimum reconstruction error in previous iteration is in the center position, then SDSP search is used now; if the SDSP search is used in previous iteration, then the search of the target is completed, the target position is not adjusted.

Step4 Revise reconstruction matrix according to the adjusted position.

Step5 Select another point, repeat step 1 to step 4, until all targets searching is completed. By now, one iteration process is finished.

Step6 Repeat steps 1 to 5, until the number of iterations meets the maximum number of iterations or search of small diamond search pattern (SDSP) of all recovered result are finished.

Step7 Output the localization result.

Compared with the traditional CS-based localization algorithm, alternate iteration method solves the problem

when targets are not in the center of grid and recovered positions locate in the vicinity of real grid through continuous correction to the reconstruction matrix, which improve the localization accuracy and reduce the mutual interference between targets, but at the increased cost of computation. At the same time, the localization accuracy of proposed method is determined by the search step. LDSP is used first for finding candidate, which avoids searching tending to a region at the very start to lead the search process trapped in a local minimum. SDSP mode can improve the accuracy. In addition, the diamond search method is not limited to search in one grid, which is very effective when the target positions are recovered in the vicinity of real grid.

V. SIMULATION RESULTS AND ANALYSIS

Simulations in MATLAB are conducted to study and analyze the effectiveness and properties of the proposed localization approach based on CS theory. Localization errors with respect to the number of targets and measurement noise are considered. BP is employed for reconstruction. An $50m \times 50m$ area is divided into 14×14 grid. The sensor number M is 49, and the maximum iteration number is 6. For the employed signal strength and distance model: $P_0 = -40dBm$, $n_p = 2$, $D_0 = 1$.

A. Targets are in the Center of Grid

In the first simulation, 15 targets whose locations are unknown are located on the center of grid randomly. Y are obtained by 49 sensors accumulating the RSS from these 15 targets. The CS-based localization algorithm is used to calculate the index of grid point under three different situations (absence of noise, SNR=5dB and SNR=20dB). The simulation result in figure 4 shows that the CS-based localization algorithm can achieve a high level accuracy under three situations. With the noise is increased, the localization effect is getting poor, but all the targets can be located to the real grid or adjacent grid. The localization result is worse when the targets gather together, because of the mutual influence of the targets.

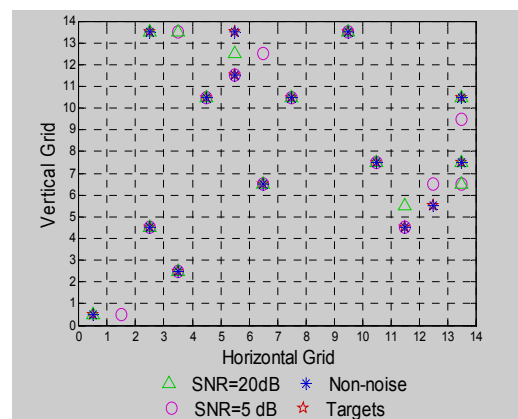


Figure 4. Localization Result when Targets Are in the Center of Grid under Different SNR

The curve of the localization error varies with the target number under different signal to noise ratio is obtained by 200 Monte-Carlo experiments. The number of sensors is fixed at 49, and the number of target ranges from 1 to 20. The localization error is defined as the average Euclidean distance between the real positions and the recovered positions. Figure 5 shows that localization error increases as the number of targets increases. Meanwhile, localization error increases as the SNR decreases. Sparsity of the signal becomes worse as the increase of number of targets, that is, CS-based localization algorithm can obtain higher localization accuracy in less number of targets which has high degree of sparsity. When target number is fixed at 20, the localization error is about 1.2m in noise free case; under low noise interference, the localization error reaches about 1.5 meters; under the strong noise interference, positions can be recovered within 1.7m localization error.

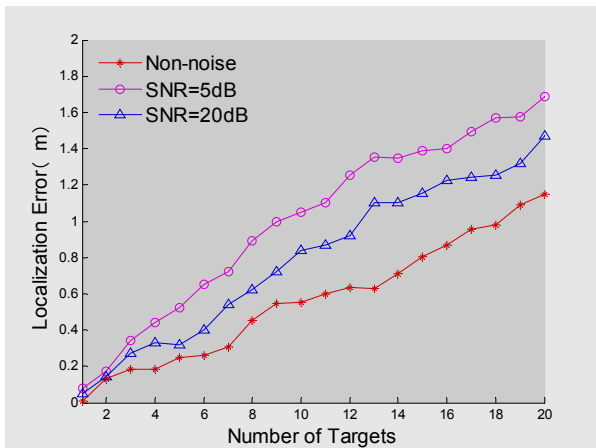


Figure 5. Curves of Localization Error Changing with Target Number under Different SNR3.

B. Targets are not in the Center of Grid

For the case that targets are not in the center of grid, the iteration backtracking algorithm [14], the traditional CS-based localization algorithm and the proposed algorithm in this paper are employed and compared to examine the performance of proposed method this paper. SNR is equal to 25dB, and target number is fixed at 7. Figure 6 shows that the proposed algorithm in this paper achieves better accuracy than other techniques under the same parameter settings. The traditional CS-based localization algorithm can only recover the targets accurately which are on the center of grid, and the localization error is high when the targets are not in the center of grid. The iteration backtracking algorithm can obtain good result when targets are not in the center of grid, but it only searches target in one grid, which cannot solve the case when a target is recovered in adjacent grid of actual target in rough localization. The proposed algorithm avoids the limitation of searching in one grid, which can search target in the vicinity of grid where is likely to exist target. At the same time, the position of the target is constantly updated

in the process of alternate iteration, which reduces the interference of the mutual influence on the localization precision.

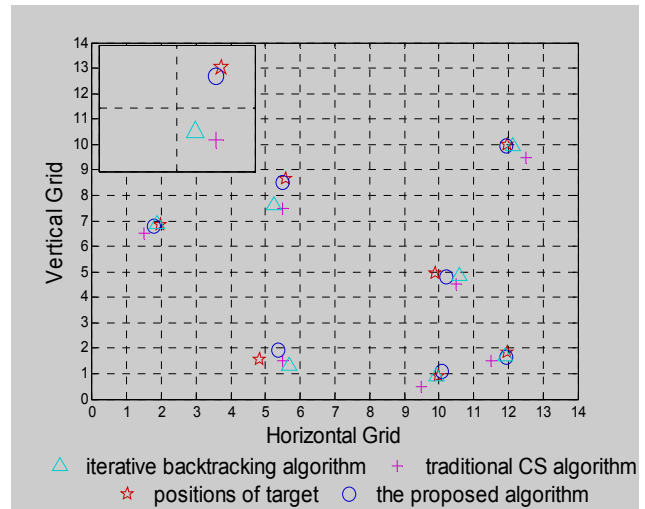


Figure 6. Comparison of Localization Result of Different Localization Algorithms.

VI. CONCLUSION

In this paper, we have proposed a new method of multiple target localization based on CS theory in WSNs, which combines the CS theory with the alternate iteration method. The proposed method divides localization process into two stages: coarse localization stage and fine localization stage. Traditional CS-based localization algorithm is presented for coarse localization firstly. In fine localization stage, alternate iteration method is proposed for searching accurate positions, which combined diamond search to find the target accurately. The influence of target number and SNR on the target localization performance have been analyzed by simulation, and the localization performance of the traditional CS-based localization algorithm, the iteration backtracking algorithm and the proposed algorithm have been compared. The simulation results have demonstrated that the proposed method outperforms the algorithms described above. In the future, we will work towards reducing the computational complexity.

ACKNOWLEDGMENT

This work is funded by National Natural Science Foundation of China under Grant No.61501069.

REFERENCES

[1] Pottie G. J., "Wireless Sensor Networks", Information Theory Workshop, 2012, pp. 139-140.
 [2] Patwari N., Ash J. N., Kyperountas S., Hero A. O. I., Moses R. L. & Correal N. S., "Locating the Nodes: Cooperative Localization in

- Wireless Sensor Networks”, *IEEE Signal Processing Magazine*, vol. 22, no. 4, 2005, pp. 54-69.
- [3] Patwari N., Hero A. O. I., Perkins M., Correal N. S. & O’Dea, R. J., “Relative Location Estimation in Wireless Sensor Networks”, *IEEE Transactions on Signal Processing*, vol. 51, no. 8, 2003, pp. 2137-2148.
- [4] Donoho D. L., “Compressed Sensing”, *IEEE Transactions on Information Theory*, vol. 52, no. 4, 2006, pp. 1289-1306.
- [5] Candès E. J., Romberg J. & Tao T., “Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information”, *IEEE Transactions on Information Theory*, vol. 52, no. 2, 2006, pp. 489-509.
- [6] Candès E. J., & Tao T., “Near Optimal Signal Recovery from Random Projections: Universal Encoding Strategies”, *IEEE Transactions on Information Theory*, vol. 52, no. 12, 2007, pp. 5406-5425.
- [7] Donoho D. L., and Tsaig Y., “Extensions of Compressed Sensing”, *Signal Processing*, vol. 86, no. 3, 2006, pp. 549-571.
- [8] Cevher V., Duarte M. F., & Baraniuk R. G., “Distributed Target Localization via Spatial Sparsity”, *European Signal Processing Conference*, Switzerland: Lausanne, 2008, pp. 25-29.
- [9] Liu K., Yu J. J. & Huang Q. H., “Bi-object Device-free Localization Based on Compressive Sensing”, *Journal of Electronics & Information Technology*, vol. 36, no. 04, 2014, pp. 862-867.
- [10] Zhang B., Cheng X., & Zhang N., et. al. “Sparse Target Counting and Localization in Sensor Networks Based on Compressive Sensing”, *Proceedings-IEEE INFOCOM*, 2011, April 10-15, pp. 2255-2263.
- [11] Feng C., AU W. S. A., Valaee S., & TAN Z., “Received-signal-strength Based Indoor Positioning Using Compressive Sensing”, *IEEE Transactions on Mobile Computing*, vol. 11, no. 12, 2012, pp. 1983-1993.
- [12] AU W. S. A., Feng C., & Valaee S., “Indoor Tracking and Navigation Using Received Signal Strength and Compressive Sensing on a Mobile Device”, *IEEE Transactions on Mobile Computing*, vol. 12, no. 10, 2013, pp. 2050-2062.
- [13] Feng C., Valaee S., & Tan Z., “Multiple Target Localization Using Compressive Sensing”, *Global Telecommunications Conference, GLOBECOM 2009*, IEEE. IEEE, 2009, pp. 1-6.
- [14] He F. H., “Multiple Target Localization via Compressed Sensing in Wireless Sensor Networks”, *Journal of Electronics & Information Technology*, vol. 32, no. 3, 2012, pp. 716-721.
- [15] Nikitaki S., & Tsakalides P., “Localization in Wireless Networks via Spatial Sparsity”, *Asilomar Conference on Signals, Systems and Computers*, 2010, November 7-10, pp. 236-239.
- [16] Liu H., Darabi H., Banerjee P., & Liu J., “Survey of Wireless Indoor Positioning Techniques and Systems”, *IEEE Transactions on Systems, Man, and Cybernetics, Part C*, vol. 37, no. 6, 2007, pp. 1067-1080.
- [17] Candès E. J., & Plan Y., “A Probabilistic and RIPless Theory of Compressed Sensing”, *IEEE Transactions on Information Theory*, vol. 57, no. 11, 2011, pp. 7235-7254.
- [18] Candès E. J., “The Restricted Isometry Property and its Implications for Compressed Sensing”, *Academie Des Sciences*, vol. 346, no. 1, 2006, pp. 589-592.